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ANDHRA PRADESH, INDIA

COURSE INTRODUCTION

Presented By:
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GIET(A)

Lecture Details:
Course Introduction
FM & HM /Mechanical Engineering, I -Semester.

Contents

Course Information

- **Course Pre requisites**
- **Course Objectives**
- **Course Outcomes**
- **Syllabus**

Course?

- What is "**Fluid Mechanics & Hydraulic Machinery**"?

References

- **Text Books**
- **Reference Books**
- **Web References**

Course Pre- requisites

Requires knowledge of

- Engineering Mechanics

Course Objectives

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Expected to:

- Understand the properties of fluids, its kinematic and dynamic behavior through various laws of fluids like continuity, Euler's, Bernoulli's equations, energy and momentum equations.
- Further, the student shall able to understand the theory of boundary layer, working and performance characteristics of various hydraulic machines like pumps and turbines.

Course Outcomes

On Completion of the course, the student will be able to

- Describe the properties of fluids and Explain the mechanics of fluids at rest and in motion by observing the fluid phenomena.
- Distinguish the types of flows and continuity equation.
- Derive Euler’s Equation of motion and Deduce Bernoulli’s equation.
- Examine energy losses in pipe transitions and Sketch energy gradient lines.
- Describe Basic working of hydraulic turbines and hydraulic pumps.

Syllabus

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
- FLUID DYNAMICS
- CLOSED CONDUIT FLOW
- BOUNDARY LAYER THEORY AND APPLICATIONS
- BASICS OF TURBO MACHINERY
- HYDRAULIC TURBINES
- PERFORMANCE OF HYDRAULIC TURBINES
- CENTRIFUGAL PUMPS
- RECIPROCATING PUMPS

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Syllabus(Cont…)

UNIT-I :

FLUID STATICS

Dimensions and units: physical properties of fluids- specific gravity, viscosity surface tension- vapor pressure and their influence on fluid motion- atmospheric, gauge and vacuum pressure – measurement of pressure- Piezometer, U-tube and differential manometers.

Syllabus(Cont..)

UNIT-II:

FLUID KINEMATICS:

Stream line, path line and streak lines and stream tube, classification of flows-steady & Unsteady, uniform, non uniform, laminar, turbulent, rotational, and ir-rotational flows-equation of continuity for one dimensional flow.

FLUID DYNAMICS: surface and body forces –Euler’s and Bernoulli’s equations for flow along a stream line, momentum equation and its application on force on pipe bend.

Syllabus(Cont..)

UNIT-III:

CLOSED CONDUIT FLOW: Reynold's experiment- Darcy Weisbach equation- Minor losses in pipes- pipes in series and pipes in parallel- total energy line-hydraulic gradient line. Measurement of flow: pilot tube, venturi meter, and orifice meter, Flow nozzle.

BOUNDARY LAYER THEORY AND APPLICATIONS: Concepts of boundary layer, boundary layer thickness and equations, momentum integral equation, boundary layer separation and its control, Cavitation. Circulation, Drag and lift on immersed bodies, Magnus effect.

UNIT-IV

BASICS OF TURBO MACHINERY: hydrodynamic force of jets on stationary and moving flat, inclined, and curved vanes, jet striking centrally and at tip, velocity diagrams, work done and efficiency, flow over radial vanes.

Syllabus(Cont..)

HYDRAULIC TURBINES: classification of turbines, impulse and reaction turbines, Pelton wheel, Francis turbine and Kaplan turbine-working proportions, work done, efficiencies , hydraulic design – draft tube-theory- functions and efficiency.

PERFORMANCE OF HYDRAULIC TURBINES: Geometric similarity, Unit and specific quantities, characteristic curves, governing of turbines, selection of type of turbine, cavitation, water hammer.

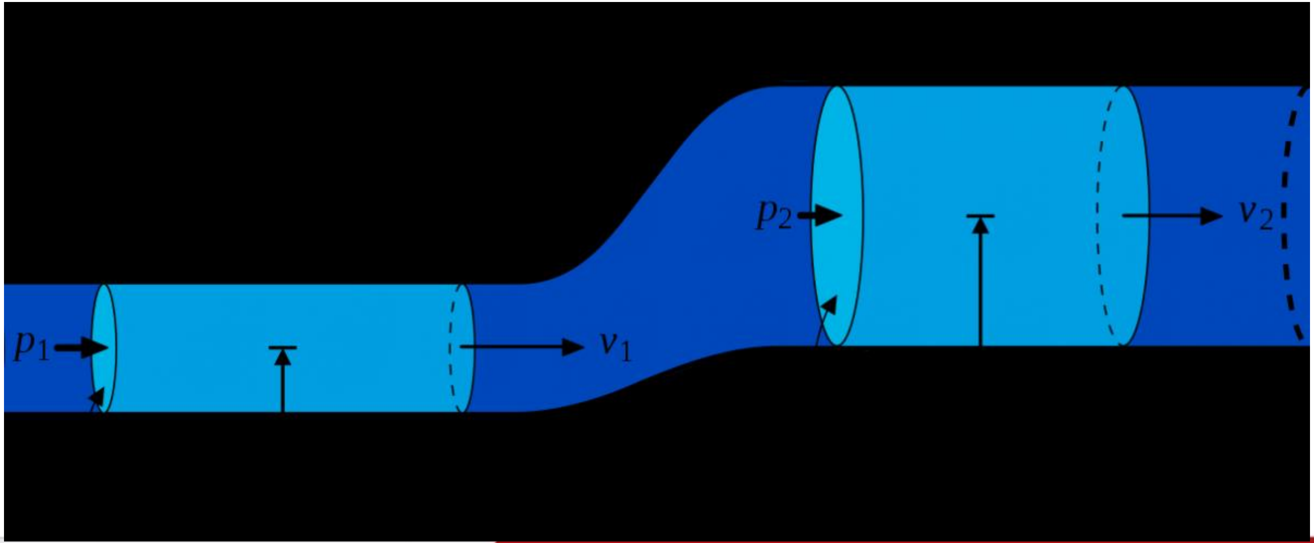
UNIT-V:

CENTRIFUGAL PUMPS: classification, working, work done – manomertic head- losses and efficiencies specific speed- pumps in series and parallel-performance characteristic curves, NPSH.

RECIPROCATING PUMPS: Working, Discharge, slip, indicator diagrams.

What is “**Fluid Mechanics & Hydraulic Machinery**”?

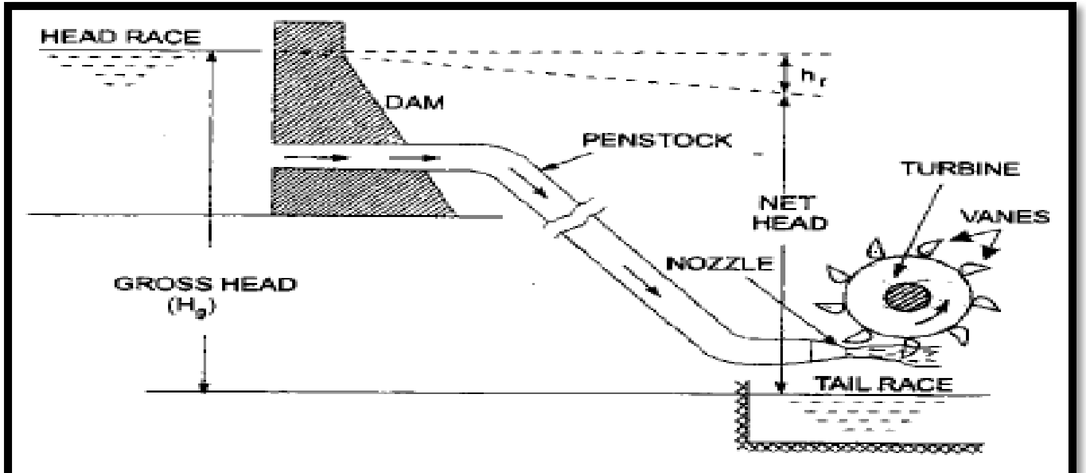
<u>Fluid mechanics</u> is the study of fluid behavior (liquids, gases, blood, and plasmas) at rest and in motion. Fluid mechanics has a wide range of applications in mechanical and chemical engineering, in biological systems, and in astrophysics



What is “**Fluid Mechanics & Hydraulic Machinery**”?(Cont…)

<u>Hydraulic Machines :</u>

Those machines which convert either hydraulic energy (energy possessed by water) into mechanical energy (P.E + K.E) or Mechanical energy into hydraulic energy



Layout of a hydro-eletric power plant.

Text Books

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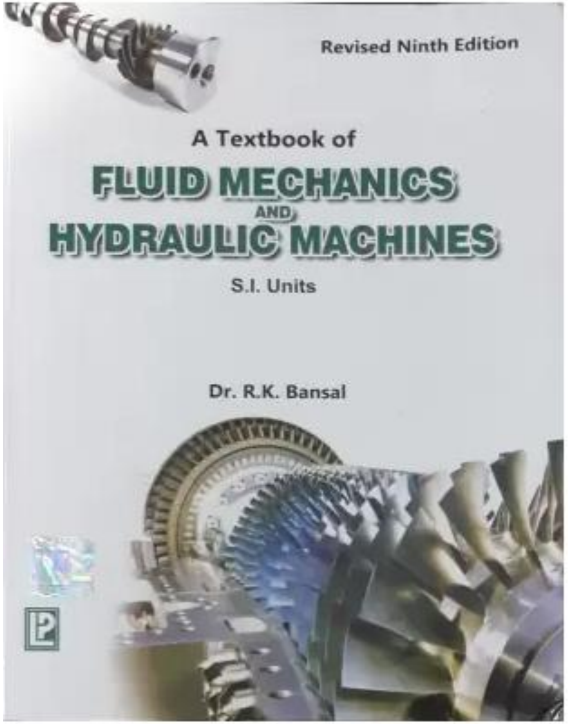
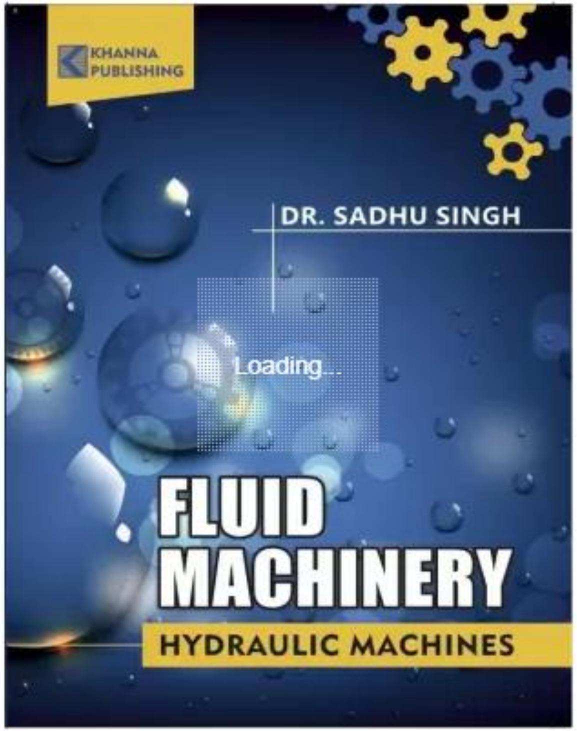
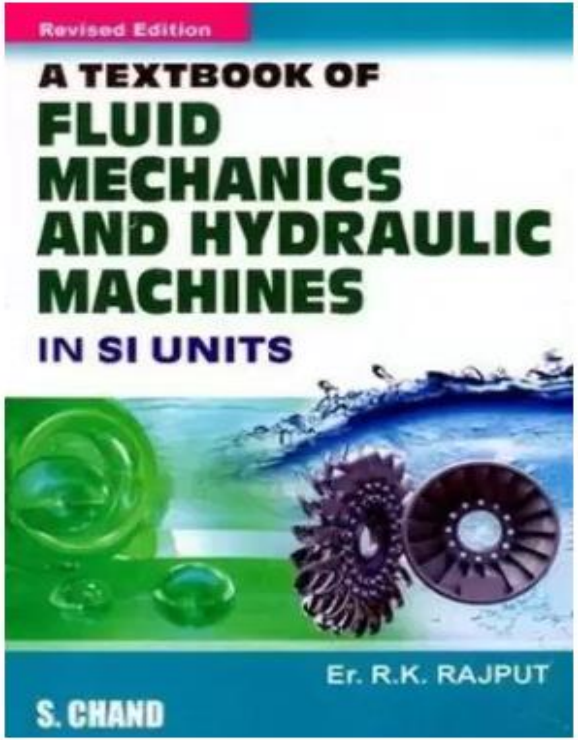
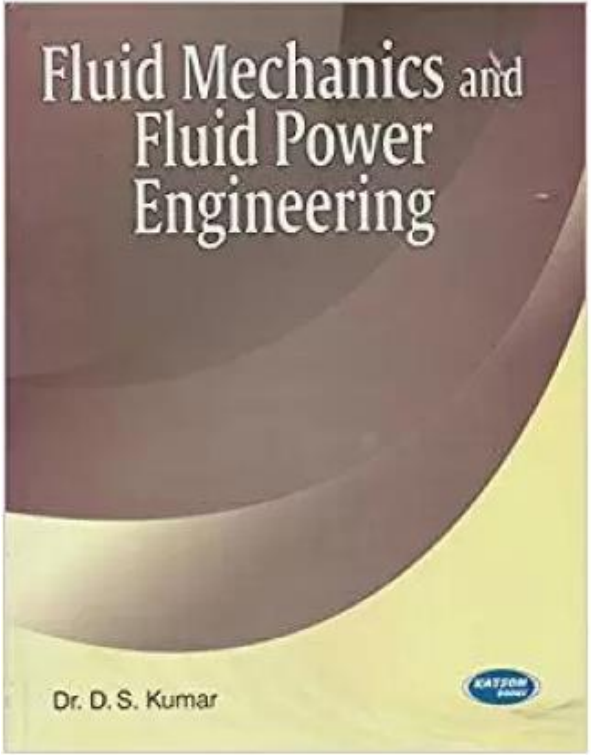
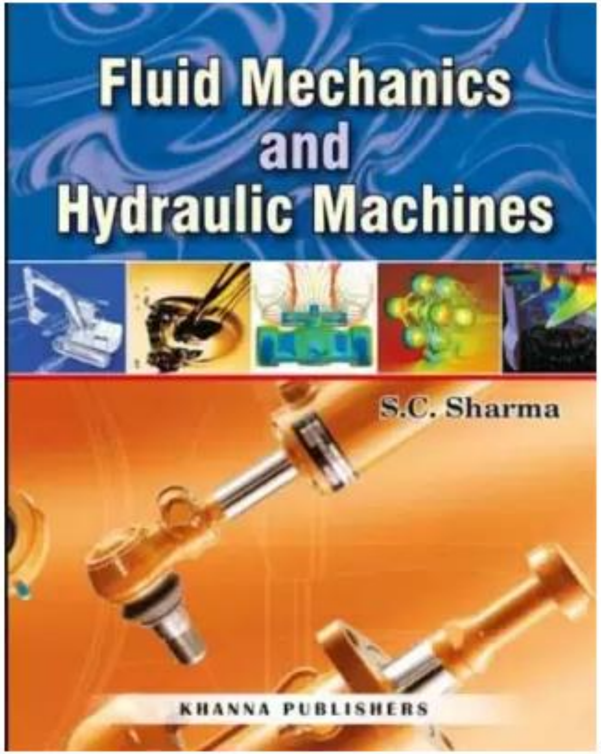


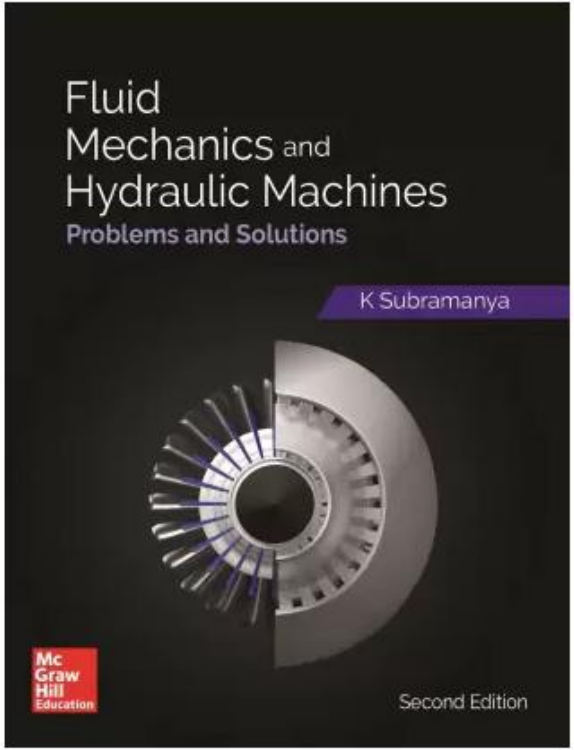




Reference Books







Web References

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1	https://nptel.ac.in/courses/112/105/112105171/
2	https://nptel.ac.in/courses/112/105/112105287/
3	https://cosmolearning.org/courses/fluid-mechanics/video-lectures/

INTRODUCTION TO FLUID MECHANICS

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Lecture Details:
Introduction to Fluid Mechanics
FM & HM /Mechanical Engineering, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
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- RECIPROCATING PUMPS

Contents

- Introduction
- What is Fluid?
- Difference between Liquid and Gas
- Applications of Fluid Mechanics
- Fluid Flow Examples
- Summary

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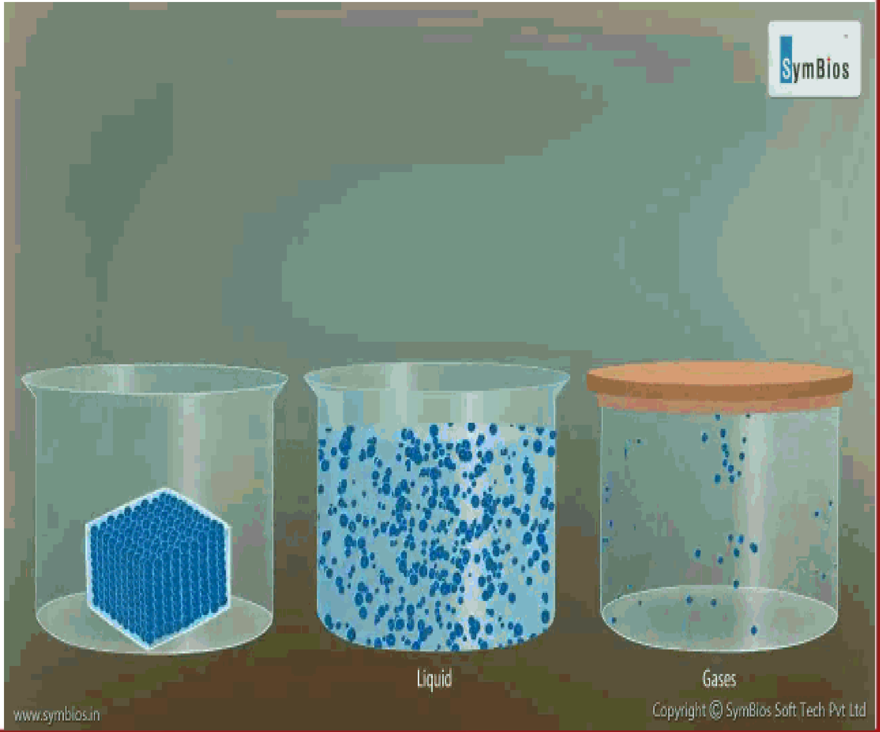
Introduction

- Fluid mechanics is the science which deals with the action of forces on fluids at rest as well as in motion
- If the fluids are at rest, the study of them is called **fluid statics**
- If the fluids are in motion, where pressure forces are not considered, the study of them is called **fluid Kinematics**
- If the fluids are in motion and the pressure forces are considered, the study of them is called **fluid dynamics**

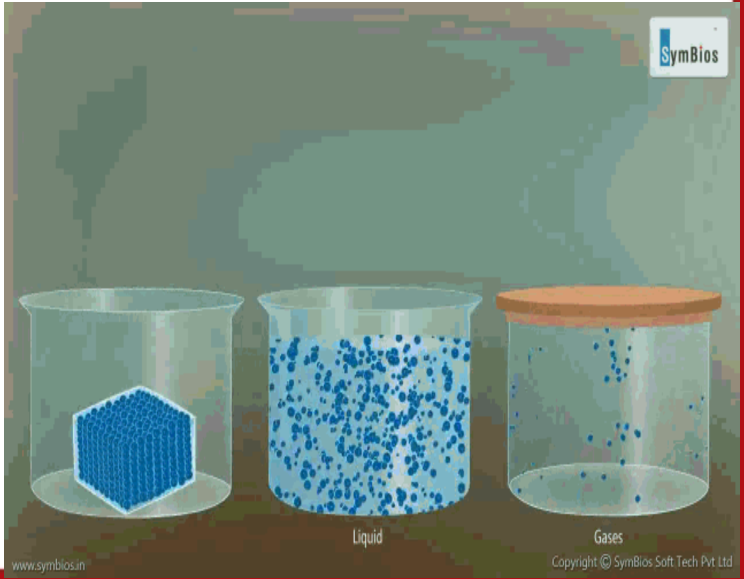
What is Fluid?

- Matter exists in two states- solid state and fluid state
- This classification of matter is based on the spacing between different molecules of matter as well as on the behavior of matter when subjected to stresses



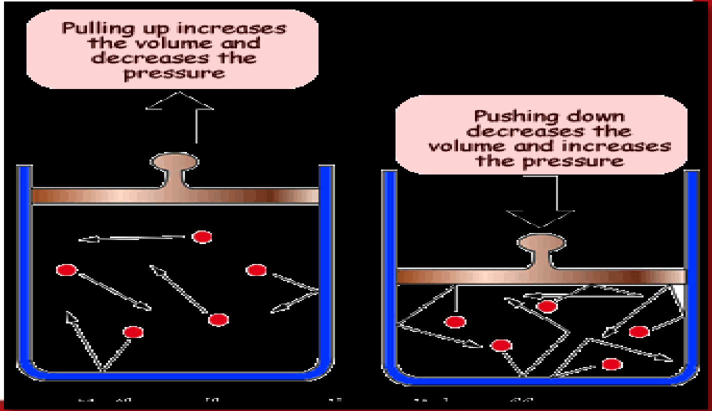
What is Fluid?(Cont...)

- In solid state molecules are closely packed, solids possess compactness and rigidity
- The molecules in fluid can move more freely within the fluid mass and therefore the fluids do not possess any rigidity
- Thus Fluid exist in two form:- Liquid and Gas



Difference between Liquid and Gas

- Liquids flow and take the shape of their container but maintain a constant volume. Examples : Water, Milk, Kerosene , Petrol, emulsions etc
- Gases expand to fill the available volume
- Liquids are incompressible While the gases are compressible



Applications of Fluid Mechanics

- Fluids are the principle transport media and hence play a central role in nature (winds, rivers, ocean , blood etc.)
- Fluids are a source of energy generation (power)
- They have several engineering applications

– Mechanical engineering

– Electrical engineering

– Chemical Engineering

– Aerospace engineering

Applications of Fluid Mechanics(Cont…)

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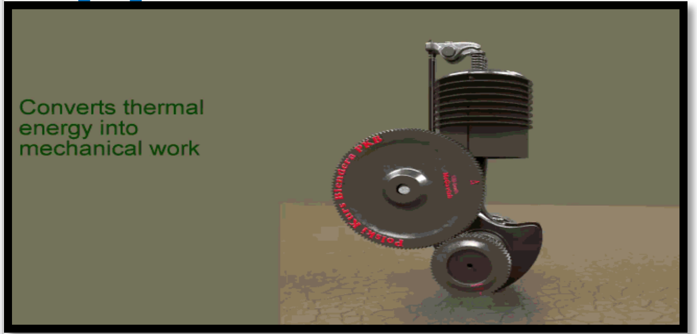
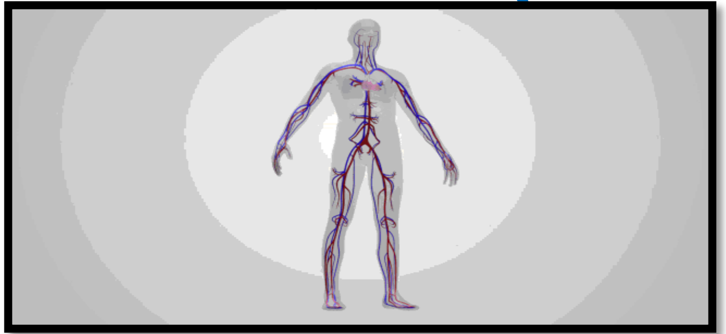
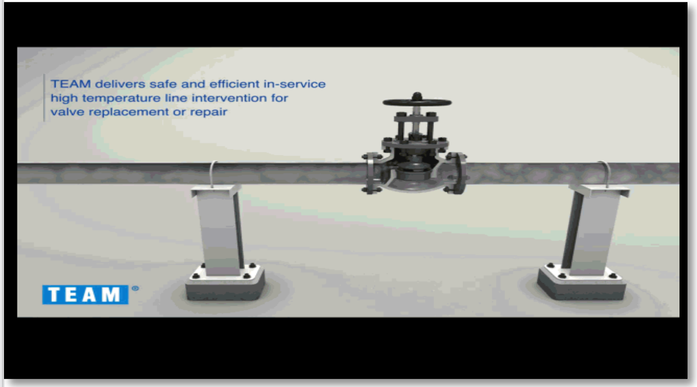


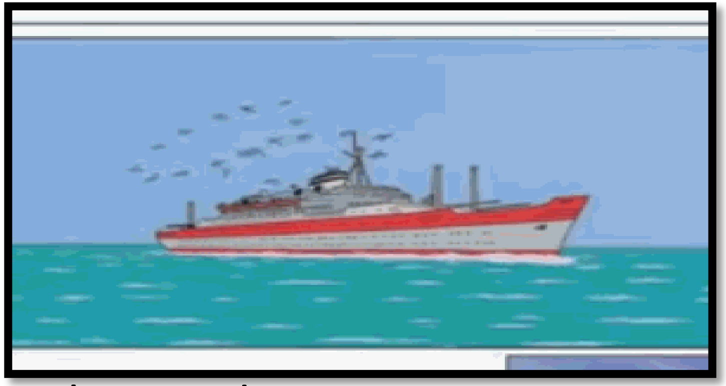
Automobiles



Medical Science



Piping Design



Ships and Boats

Applications of Fluid Mechanics(Cont…)

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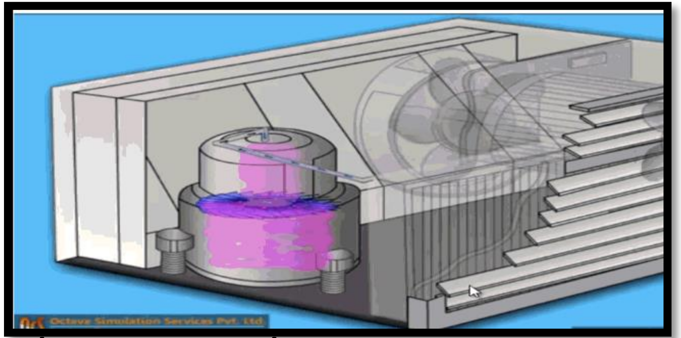
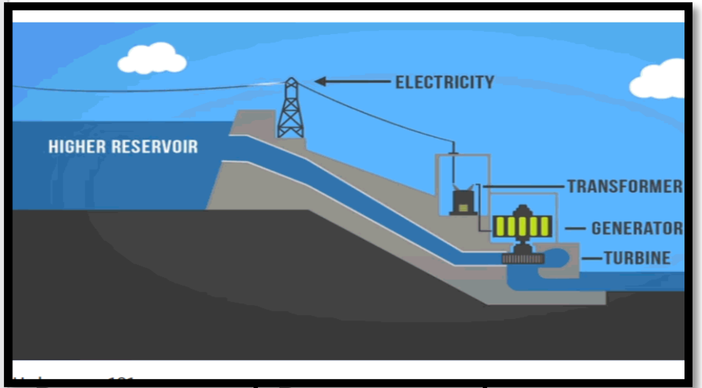

Electric Appliances

Aircrafts



Power and Process plants


Fire Safety

Applications of Fluid Mechanics(Cont…)

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Nature

Fluid Flow Examples

Leaking crude oil from the grounded tanker *Argo Merchant* (Nantucket Shoals 1976)

Fluid Flow Examples (Cont…)

Smoke plumes

Fluid Flow Examples (Cont...)

Turbulent Jet impinging into fresh water

Summary

- Classification of fluid mechanics
- Difference between liquid and gas
- Applications of fluid mechanics
- Fluid flow examples

PROPERTIES OF FLUID:PART-1

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Lecture Details:
Unit-1 (Fluid Statics) , Properties of Fluid
FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

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PROPERTIES OF FLUID:PART-1

Contents

- Density or Mass Density
- Specific Weight or Weight Density
- Specific Volume
- Specific Gravity or Relative Density(S)
- Viscosity
- Newton’s Law of Viscosity
- Kinematic viscosity
- Summary

Density or Mass Density(ρ)

- Density is the mass per unit volume of a fluid. In other words, it is the ratio between mass (m) and volume (V) of a fluid
- Density is denoted by the symbol 'ρ'. Its unit is kg/m3

Mathematically, mass density is written as:

$$\rho = \frac{Mass\ of\ the\ fluid}{Volume\ of\ the\ fluid} = \frac{m}{V}$$

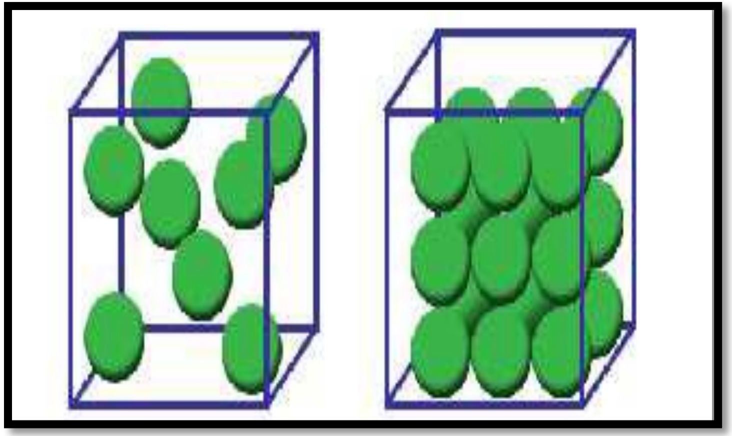

Fig: Different molecules arrangement in a same volume of space

Problem:1

A quantity of helium gas at 0°C with a volume of 4.00 m^3 has a mass of 0.712 kg at standard atmospheric pressure. Determine the density of this sample of helium gas?

Given Data:

Volume $V = 4\ m^3$

Mass $m = 0.712\ kg$

$$\text{Density } \rho = \frac{m}{V} = \frac{0.712}{4} = \mathbf{0.178}\ \frac{kg}{m^3}$$

Specific Weight or Weight Density(w)

- Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume and it is denoted by the symbol $w(N/m^3)$
- Thus mathematically

$$w = \frac{Weight\ of\ fluid}{Volume\ of\ fluid} = \frac{(Mass\ of\ fluid)X\ Acceleration\ due\ to\ gravity}{Volume\ of\ fluid}$$

$$= \frac{Mass\ of\ fluid\ X\ g}{Volume\ of\ fluid} \qquad \left[\text{ since } \boldsymbol{\rho} = \frac{\boldsymbol{Mass\ of\ the\ fluid}}{\boldsymbol{Volume\ of\ the\ fluid}}\right]$$

$$w = \rho\ X\ g$$

$$\mathbf{w} = \boldsymbol{\rho g}$$

Specific Volume

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- It is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume. It is expressed in m^3/kg

- Specific Volume = $\dfrac{\text{Volume of the Fluid}}{\text{Mass of the Fluid}}$

$$= \frac{1}{\text{mass of the fluid/volume of the fluid}}$$

$$= \frac{1}{\rho}$$

Specific Gravity or Relative Density(S)

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- Specific gravity is defined as the ratio of the weight density (or density) of a fluid to the weight density (or density) of a standard fluid
- For liquids the standard fluid is water and for gases the standard fluid is air

Mathematically,

$$\textbf{S (for liquids)} = \frac{\textit{Weight density (density) of liquid}}{\textit{Weight density (density) of water}}$$

$$\textbf{S (for gases)} = \frac{\textit{Weight density (density) of gas}}{\textit{Weight density (density) of air}}$$

Problem:2

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Calculate the specific weight, density and specific gravity of one litre of a liquid which weighs 7 N.

Solution. Given :

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \quad \left(\because \ 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3\right)$$

$$\text{Weight} = 7 \text{ N}$$

$$(i) \ \text{Specific weight } (w) = \frac{\text{Weight}}{\text{Volume}} = \frac{7 \text{ N}}{\left(\frac{1}{1000}\right) \text{m}^3} = \mathbf{7000 \ N/m^3. \ Ans.}$$

Problem:2(Cont...)

(ii) Density (ρ) $= \dfrac{w}{g} = \dfrac{7000}{9.81}$ kg/m^3 = **713.5 kg/m^3. Ans.** $\quad w = \rho g$

(iii) Specific gravity $= \dfrac{\text{Density of liquid}}{\text{Density of water}} = \dfrac{713.5}{1000}$ $\{\because$ Density of water = 1000 kg/m$^3\}$

= **0.7135. Ans.**

Problem:3

Calculate the density, specific weight and weight of one litre of petrol of specific gravity = 0.7

Solution. Given : Volume = 1 litre = $1 \times 1000\ cm^3 = \frac{1000}{10^6}\ m^3 = 0.001\ m^3$

Sp. gravity $S = 0.7$

(*i*) *Density* (ρ)

Using equation Specific gravity $= \frac{\text{Density of liquid}}{\text{Density of water}}$

Density (ρ) $= S \times 1000\ kg/m^3 = 0.7 \times 1000 =$ **$700\ kg/m^3$. Ans.**

Problem:3(Cont…)

(ii) Specific weight (w)

Using equation $w = \rho \times g = 700 \times 9.81 \text{ N/m}^3 = \mathbf{6867 \text{ N/m}^3}$. **Ans.**

(iii) Weight (W)

We know that specific weight $= \dfrac{\text{Weight}}{\text{Volume}}$

or $w = \dfrac{W}{0.001}$ or $6867 = \dfrac{W}{0.001}$

$\therefore$ $W = 6867 \times 0.001 = \mathbf{6.867 \text{ N}}$. **Ans.**

Viscosity

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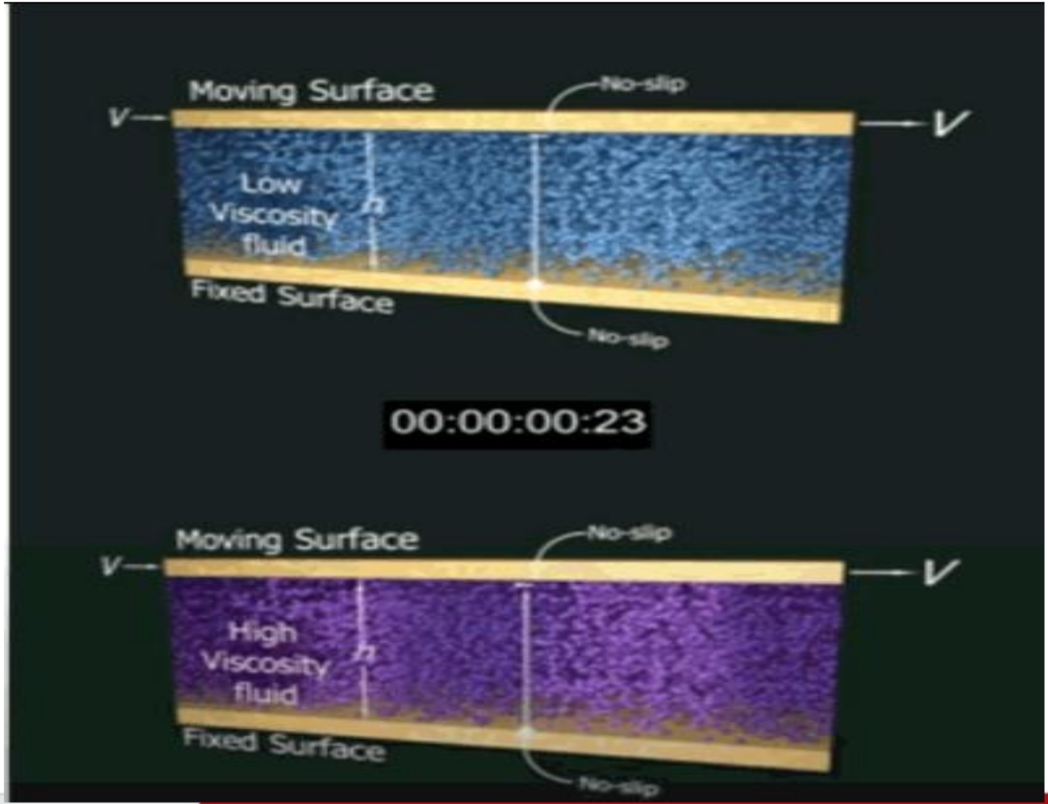
- Viscosity is a measure of a fluid's resistance to flow
- It describes the internal friction of a moving fluid
- A fluid with large viscosity resists motion because its molecular makeup gives it a lot of internal friction
- A fluid with low viscosity flows easily because its molecular makeup results in very little friction when it is in motion

Viscosity(Cont…)



Viscosity(Cont…)

- Gases also have viscosity, although it is a little harder to notice it in ordinary circumstances

Fig. Different fluid flow behavior in a same type of glass tube

Newton’s law of viscosity

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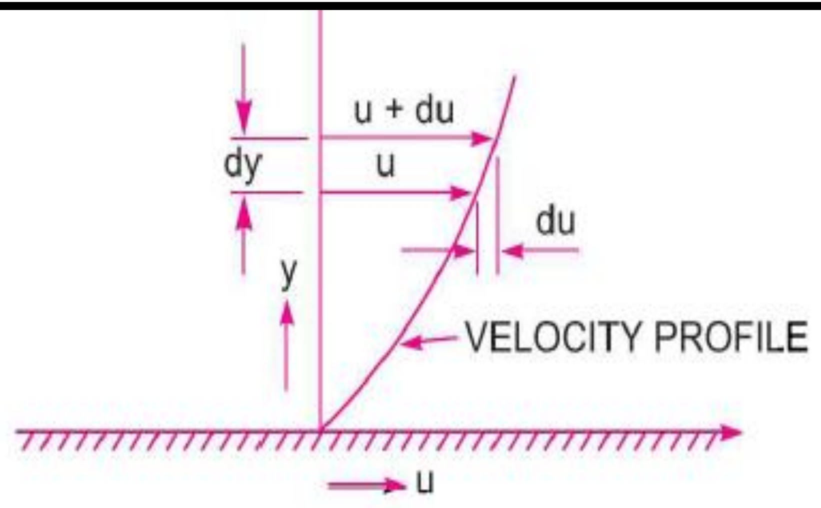
- It states that the shear stress on a fluid element layer is directly proportional to the rate of shear strain
- dy = Distance between adjacent fluid layers.
- du = Velocity difference between adjacent fluid layers.

$$\tau \propto \frac{du}{dy}$$



Velocity variation near a solid boundary.

Newton's law of viscosity(cont…)

- After removing proportionality the equation becomes

$$\tau = \mu \frac{du}{dy}$$

- The constant of proportionality is called the coefficient of dynamic viscosity or coefficient of viscosity
- *Where* τ = shear stress

$\frac{du}{dy}$ = Velocity Gradient or Rate of shear strain or Rate of shear deformation

μ = coefficient of viscosity or coefficient of dynamic viscosity

Units of Viscosity

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$\therefore$ MKS unit of viscosity $= \dfrac{\text{kgf-sec}}{\text{m}^2}$

CGS unit of viscosity $= \dfrac{\text{dyne-sec}}{\text{cm}^2}$

$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{\text{Force/Area}}{\frac{du}{dy}} = \frac{\text{Force/Area}}{\frac{1}{sec}} = \frac{\text{Force} \times \text{Sec}}{\text{Area}}$$

The unit of viscosity in CGS is also called Poise which is equal to $\dfrac{\text{dyne-sec}}{\text{cm}^2}$.

SI unit of viscosity = Ns/m^2 = Pa s.

$$\boxed{1\ \text{kgf} = 9.8\ \text{N}}$$

One poise $= \dfrac{1}{10}\dfrac{\text{Ns}}{\text{m}^2}$.

$$\boxed{1\ \text{N} = 10^5\ \text{dynes}}$$

Kinematic viscosity

- The kinematic viscosity(ϑ) (also called "momentum diffusivity") is the ratio of the dynamic viscosity(μ) to the density of the fluid(ρ)

Mathematically, $$\boldsymbol{\vartheta} = \frac{\boldsymbol{\mu}}{\boldsymbol{\rho}}$$

Kinematic viscosity(Cont…)

Units:

In MKS and SI units: **m²/sec** , $\vartheta = \frac{\mu}{\rho} = \frac{\frac{\text{Force} \times \text{Sec}}{\text{Area}}}{\frac{mass}{Area\ X\ Lenth}} = \frac{\text{mass X } a \text{X SecX length}}{\text{mass}}$

In CGS units: **cm²/s,**

This CGS units of Kinematic viscosity is also known as Stroke

Thus, one stoke $= cm^2/s = \left(\frac{1}{100}\right)^2 m^2/s = 10^{-4}\, m^2/s$

Centistoke means $= \frac{1}{100}$ stoke.

Problem:4

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If the velocity distribution over a plate is given by $u = \frac{2}{3}y - y^2$ *in which u is the velocity in metre per second at a distance y metre above the plate, determine the shear stress at* $y = 0$ *and* $y = 0.15$ *m. Take dynamic viscosity of fluid as 8.63 poises.*

Solution. Given : $u = \frac{2}{3}y - y^2 \quad \therefore \quad \frac{du}{dy} = \frac{2}{3} - 2y$ $\qquad \tau = \mu \frac{du}{dy}$

$$\left(\frac{du}{dy}\right)_{\text{at } y=0} \text{ or } \left(\frac{du}{dy}\right)_{y=0} = \frac{2}{3} - 2(0) = \frac{2}{3} = 0.667$$

Also $$\left(\frac{du}{dy}\right)_{\text{at } y=0.15} \text{ or } \left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2 \times .15 = .667 - .30 = 0.367$$

Problem:4(Cont…)

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Value of $\mu = 8.63$ poise $= \dfrac{8.63}{10}$ SI units $= 0.863$ N s/m^2

Now shear stress is given by equation as $\tau = \mu \dfrac{du}{dy}$.

(*i*) Shear stress at $y = 0$ is given by

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.863 \times 0.667 = \mathbf{0.5756\ N/m^2.\ Ans.}$$

(*ii*) Shear stress at $y = 0.15$ m is given by

$$(\tau)_{y=0.15} = \mu \left(\frac{du}{dy}\right)_{y=0.15} = 0.863 \times 0.367 = \mathbf{0.3167\ N/m^2.\ Ans.}$$

Summary

- Density is the mass per unit volume of a fluid
- Specific weight of a fluid is the ratio between the weight of a fluid to its volume
- Volume per unit mass of a fluid is called specific volume
- Specific gravity is the ratio of the weight density of a fluid to the weight density of a standard fluid
- Viscosity is a measure of a fluid's resistance to flow
- Newton’s law of viscosity states that the shear stress on a fluid element layer is directly proportional to the rate of shear strain
- The kinematic viscosity is the ratio of the dynamic viscosity(μ) to the density of the fluid (ρ)

PROPERTIES OF FLUID:PART-2

Presented By:

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Mechanical Engineering

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Lecture Details:

Unit-1 (Fluid Statics) , Properties of Fluid

FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

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PROPERTIES OF FLUID:PART-2

Contents

- Variation of viscosity with Temperature
- Types of Fluids
- Summary

Problem:1

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The dynamic viscosity of an oil, used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4 m and rotates at 190 r.p.m. Calculate the power lost in the bearing for a sleeve length of 90 mm. The thickness of the oil film is 1.5 mm.

Solution. Given :

Viscosity $\mu = 6$ poise

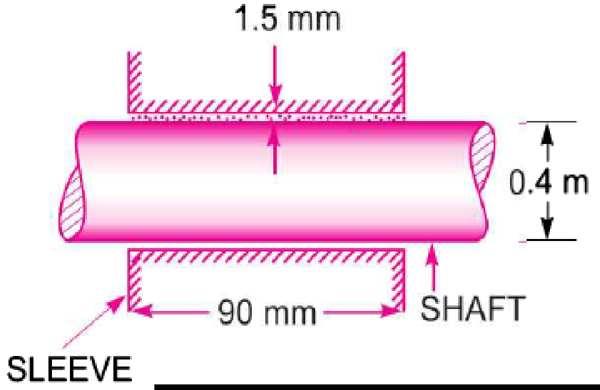
$$= \frac{6}{10}\frac{\text{N s}}{\text{m}^2} = 0.6\ \frac{\text{N s}}{\text{m}^2}$$

Dia. of shaft, $D = 0.4$ m

Speed of shaft, $N = 190$ r.p.m

Sleeve length, $L = 90 \text{ mm} = 90 \times 10^{-3}$ m

Thickness of oil film, $t = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m



Power lost $= \dfrac{2\pi NT}{60}$

Problem:1(Cont…)

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Tangential velocity of shaft, $u = \frac{\pi D N}{60} = \frac{\pi \times 0.4 \times 190}{60} = 3.98 \text{ m/s}$

Torque on the shaft, $T = \text{Force} \times \frac{D}{2}$

Using the relation $\tau = \mu \frac{du}{dy}$

Shear force on the shaft, $F = \text{Shear stress} \times \text{Area}$

where du = Change of velocity $= u - 0 = u = 3.98$ m/s

dy = Change of distance $= t = 1.5 \times 10^{-3}$ m

$$\tau = 10 \times \frac{3.98}{1.5 \times 10^{-3}} = 1592 \text{ N/m}^2$$

This is shear stress on shaft

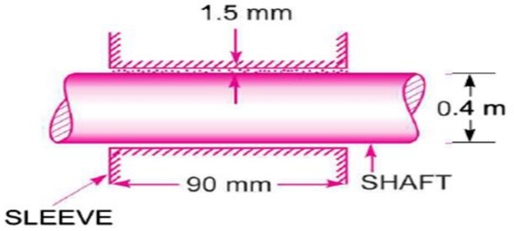
Problem:1(Cont…)

$\therefore$ Shear force on the shaft, F = Shear stress × Area

$$= 1592 \times \pi D \times L = 1592 \times \pi \times .4 \times 90 \times 10^{-3} = 180.05 \text{ N}$$

Torque on the shaft, $T = \text{Force} \times \dfrac{D}{2} = 180.05 \times \dfrac{0.4}{2} = 36.01 \text{ Nm}$

$\therefore$ *Power lost $= \dfrac{2\pi NT}{60} = \dfrac{2\pi \times 190 \times 36.01}{60} =$ **716.48 W. Ans.**



Problem:2

A plate 0.025 mm distant from a fixed plate, moves at 60 cm/s and requires a force of 2 N per unit area i.e., 2 N/m^2 to maintain this speed. Determine the fluid viscosity between the plates.

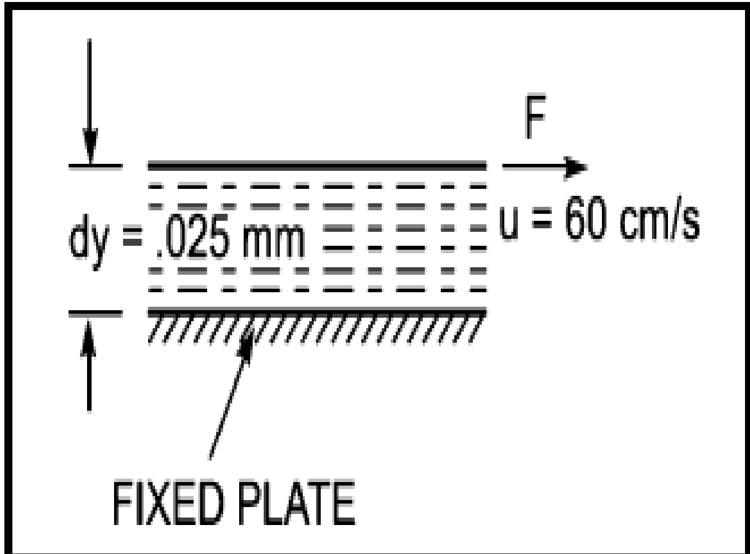
Solution. Given :

Distance between plates, $dy = .025$ mm

$= .025 \times 10^{-3}$ m

Velocity of upper plate, $u = 60$ cm/s $= 0.6$ m/s

Force on upper plate, $F = 2.0\ \dfrac{\text{N}}{\text{m}^2}$.



Problem:2(Cont...)

Let the fluid viscosity between the plates is μ.

we have $\tau = \mu \dfrac{du}{dy}$.

where du = Change of velocity = $u - 0 = u = 0.60$ m/s

dy = Change of distance = $.025 \times 10^{-3}$ m

τ = Force per unit area = $2.0 \dfrac{\text{N}}{\text{m}^2}$

$$\therefore \quad 2.0 = \mu \frac{0.60}{.025 \times 10^{-3}} \quad \therefore \quad \mu = \frac{2.0 \times .025 \times 10^{-3}}{0.60} = 8.33 \times 10^{-5} \frac{\text{Ns}}{\text{m}^2}$$

$= 8.33 \times 10^{-5} \times 10$ poise = $\mathbf{8.33 \times 10^{-4}}$ **poise. Ans.**

Problem:3

Two large plane surfaces are 2.4 cm apart. The space between the surfaces is filled with glycerine. What force is required to drag a very thin plate of surface area 0.5 square metre between the two large plane surfaces at a speed of 0.6 m/s, if :

(i) the thin plate is in the middle of the two plane surfaces, and

(ii) the thin plate is at a distance of 0.8 cm from one of the plane surfaces ? Take the dynamic viscosity of glycerine = 8.10×10^{-1} N s/m^2.

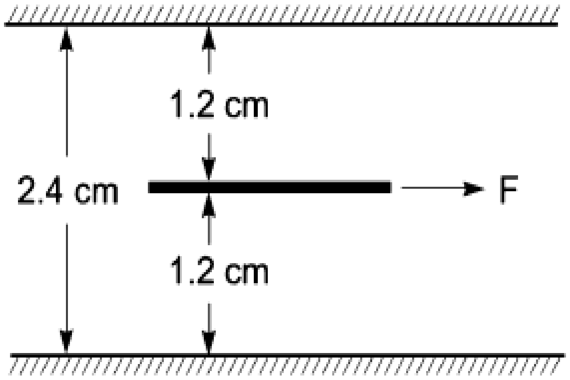
Solution. Given :

Distance between two large surfaces = 2.4 cm

Area of thin plate, $A = 0.5\ \text{m}^2$

Velocity of thin plate, $u = 0.6$ m/s

Viscosity of glycerine, $\mu = 8.10 \times 10^{-1}$ N s/m^2



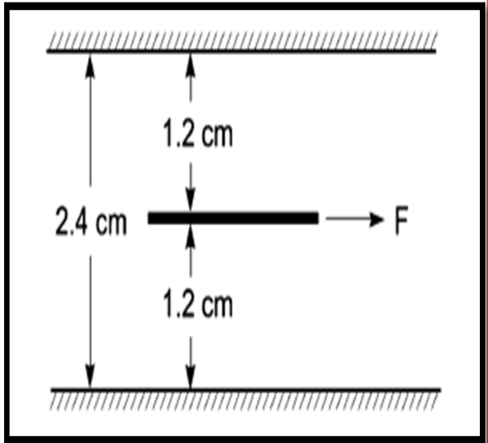
Problem:3(Cont...)

Case I. When the thin plate is in the middle of the two plane surfaces

Let F_1 = Shear force on the upper side of the thin plate

F_2 = Shear force on the lower side of the thin plate

F = Total force required to drag the plate



Then $$F = F_1 + F_2$$

The shear stress (τ_1) on the upper side of the thin plate is given by equation,

Problem:3(Cont...)

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$$\tau_1 = \mu \left(\frac{du}{dy} \right)_1$$

where du = Relative velocity between thin plate and upper large plane surface
= 0.6 m/sec
dy = Distance between thin plate and upper large plane surface
= 1.2 cm = 0.012 m (plate is a thin one and hence thickness of plate is neglected)

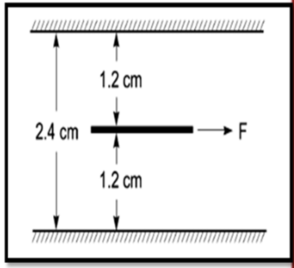
$$\therefore \quad \tau_1 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{.012} \right) = 40.5 \text{ N/m}^2$$

Now shear force, F_1 = Shear stress × Area

$$= \tau_1 \times A = 40.5 \times 0.5 = 20.25 \text{ N}$$

Problem:3(Cont...)

Similarly shear stress (τ_2) on the lower side of the thin plate is given by



$$\tau_2 = \mu \left(\frac{du}{dy}\right)_2 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.012}\right) = 40.5 \text{ N/m}^2$$

$\therefore$ Shear force, $F_2 = \tau_2 \times A = 40.5 \times 0.5 = 20.25$ N

$\therefore$ Total force, $F = F_1 + F_2 = 20.25 + 20.25 =$ **40.5 N. Ans.**

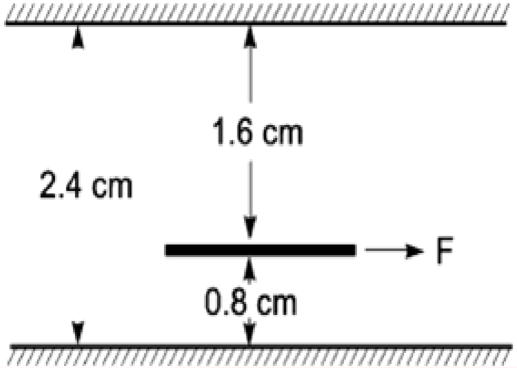
Case II. When the thin plate is at a distance of 0.8 cm from one of the plane surfaces.

Let the thin plate is at a distance 0.8 cm from the lower plane surface.

Then distance of the plate from the upper plane surface

$= 2.4 - 0.8 = 1.6$ cm $= .016$ m

(Neglecting thickness of the plate)

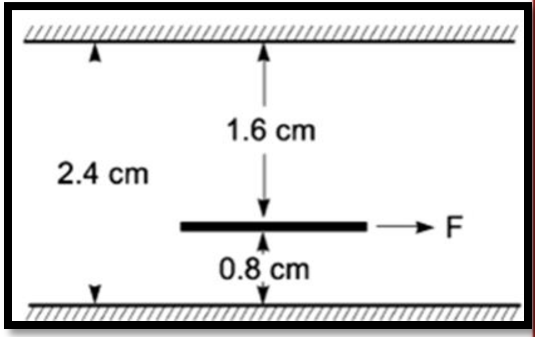


Problem:3(Cont...)

The shear force on the upper side of the thin plate,

$$F_1 = \text{Shear stress} \times \text{Area} = \tau_1 \times A$$

$$= \mu \left(\frac{du}{dy}\right)_1 \times A = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.016}\right) \times 0.5 = 15.18 \text{ N}$$



The shear force on the lower side of the thin plate,

$$F_2 = \tau_2 \times A = \mu \left(\frac{du}{dy}\right)_2 \times A$$

$$= 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.8/100}\right) \times 0.5 = 30.36 \text{ N}$$

$\therefore$ Total force required $= F_1 + F_2 = 15.18 + 30.36 =$ **45.54 N. Ans.**

Variation of viscosity with Temperature

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- The relation between viscosity and temperature for liquids and gases are

(i) For Liquids: $\boldsymbol{\mu = \mu_0 \left(\frac{1}{1+\alpha t+ \beta t^2}\right)}$

Where, μ = Viscosity of liquid at t^0C, in poise, μ_0 = Viscosity of liquid at 0^0 C , in poise

α, β = Constants for the liquid

For water μ_0 = 1.79X 10^{-3} poise,

α= 0.03368, β =0.000221

Variation of viscosity with Temperature (Cont…)

(ii) For gases:

$$\boldsymbol{\mu = \mu_0 + \alpha t - \beta t^2}$$

Where, μ_0 = 0.000017 poise,

α= 0.000000056

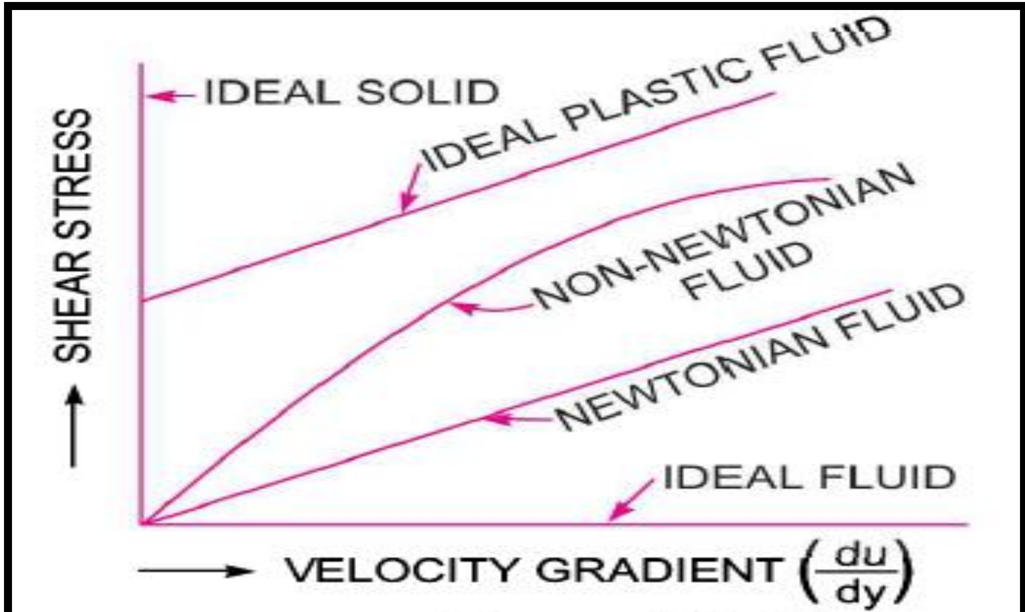
β =0.1189 X 10^{-9}

Types of Fluids

- Fluids can be classified into five basic types.

They are:

1. Ideal Fluid
2. Real Fluid
3. Newtonian Fluid
4. Non-Newtonian Fluid
5. Ideal-plastic Fluid



Types of fluids.

Types of Fluids (Cont…)

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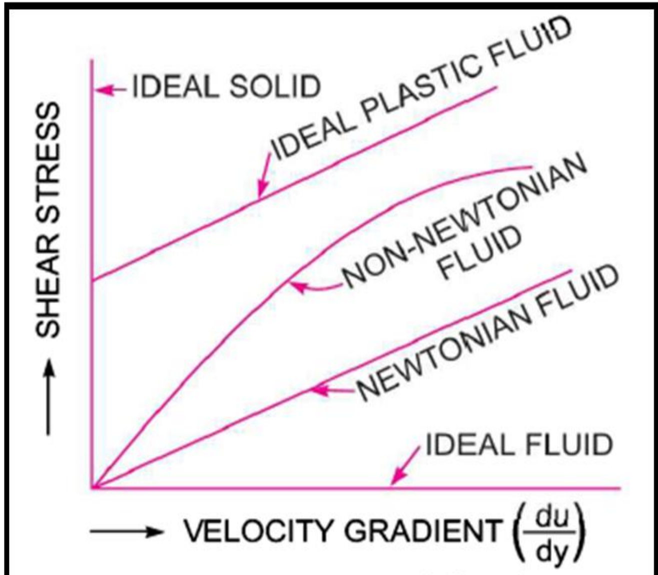
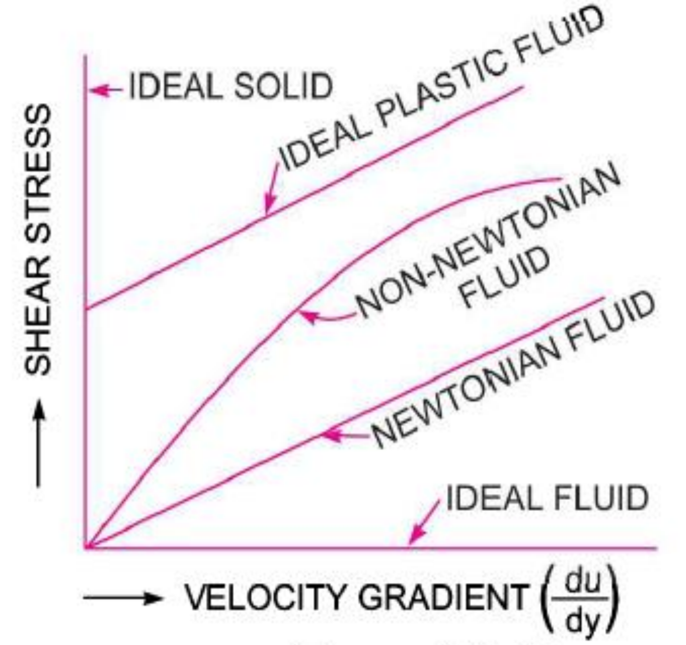
- **Ideal Fluid:** Which is incompressible and is having no viscosity
- **Real Fluid:** Which possesses viscosity. All the fluids in actual practice are real fluids
- **Newtonian Fluid:** A real fluid, in which the shear stress is directly proportional to the rate of shear strain



Types of fluids.

Types of Fluids (Cont…)

- **Non-Newtonian Fluid:** A real fluid , in which the shear stress is not proportional to the rate of shear strain(or velocity gradient)
- **Ideal Plastic Fluid:** A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain



Types of fluids.

Summary

- Some Extra problems are solved related to viscosity
- The relation between viscosity and temperature for liquids $\boldsymbol{\mu} = \boldsymbol{\mu_0}\left(\frac{\mathbf{1}}{\mathbf{1}+\boldsymbol{\alpha t}+\boldsymbol{\beta t^2}}\right)$
- The relation between viscosity and temperature for gases $\boldsymbol{\mu} = \boldsymbol{\mu_0} + \boldsymbol{\alpha t} - \boldsymbol{\beta t^2}$
- Fluids can be classified as Ideal Fluid, Real Fluid, Newtonian Fluid, Non-Newtonian Fluid and Ideal-plastic Fluid

PROPERTIES OF FLUID: PART-3

Presented By:
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GIET(A)

Lecture Details:
Unit-1 (Fluid Statics) , Properties of Fluid
FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
- FLUID DYNAMICS
- CLOSED CONDUIT FLOW
- BOUNDARY LAYER THEORY AND APPLICATIONS
- BASICS OF TURBO MACHINERY
- HYDRAULIC TURBINES
- PERFORMANCE OF HYDRAULIC TURBINES
- CENTRIFUGAL PUMPS
- RECIPROCATING PUMPS

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PROPERTIES OF FLUID:PART-3

Contents

- Compressibility and Bulk Modulus
- Surface Tension
- Capillary Action
- Vapor Pressure
- Cavitation
- Summary

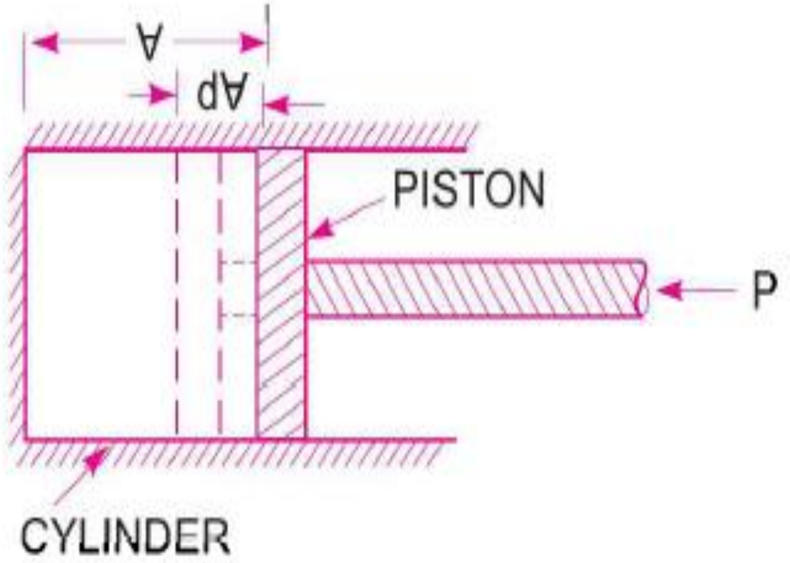
Compressibility and Bulk Modulus

- Compressibility is the reciprocal of the bulk modulus of elasticity
- Bulk modulus is defined as the ratio of compressive stress(Increase of Pressure) to volumetric strain

Bulk Modulus $K = \dfrac{\textit{Increase of Pressure}}{\textit{Volumetric Strain}}$

Volumetric Strain = - $\dfrac{d\forall}{\forall}$

-ve sign means the volume decreases with increase of pressure.

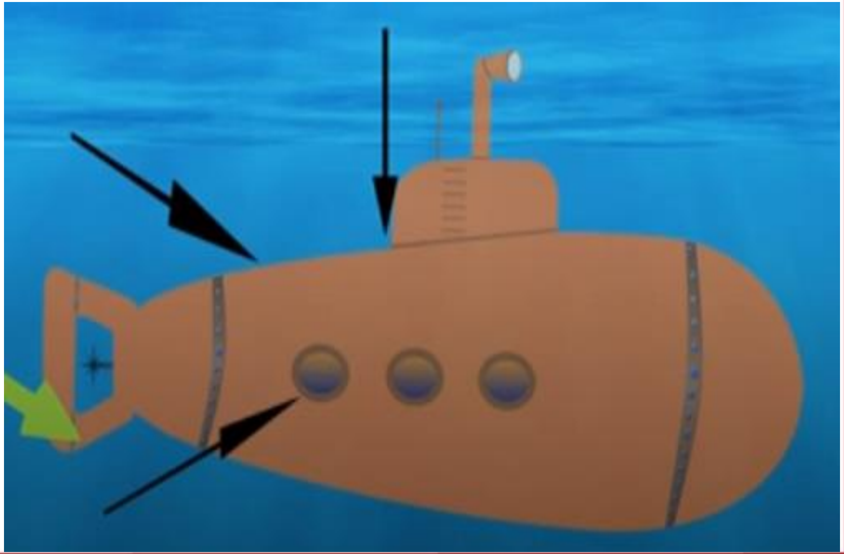


Compressibility and bulk modulus(Cont…)

$$\text{Bulk Modulus } K = \frac{\textit{Increase of Pressure}}{\textit{Volumetric Strain}}$$

$$= \frac{dp}{\frac{-d\forall}{\forall}} = -\frac{dp}{d\forall}\forall$$

$$\textbf{Compressibility} = \frac{1}{K}$$



Problem:1

Determine the bulk modulus of elasticity of a liquid, if the pressure of the liquid is increased from 70 N/cm^2 to 130 N/cm^2. The volume of the liquid decreases by 0.15 per cent.

Solution. Given :

Initial pressure $= 70 \text{ N/cm}^2$

Final pressure $= 130 \text{ N/cm}^2$

$\therefore \quad dp =$ Increase in pressure $= 130 - 70 = 60 \text{ N/cm}^2$

Decrease in volume $= 0.15\%$

$$\therefore \qquad -\frac{d\forall}{\forall} = +\frac{0.15}{100}$$

Problem:1 (Cont...)

Bulk modulus, K is given by equation as

$$K = \frac{dp}{-\dfrac{d\forall}{\forall}} = \frac{60\ \text{N/cm}^2}{\dfrac{.15}{100}} = \frac{60 \times 100}{.15} = \mathbf{4 \times 10^4\ N/cm^2.\ Ans.}$$

Surface Tension

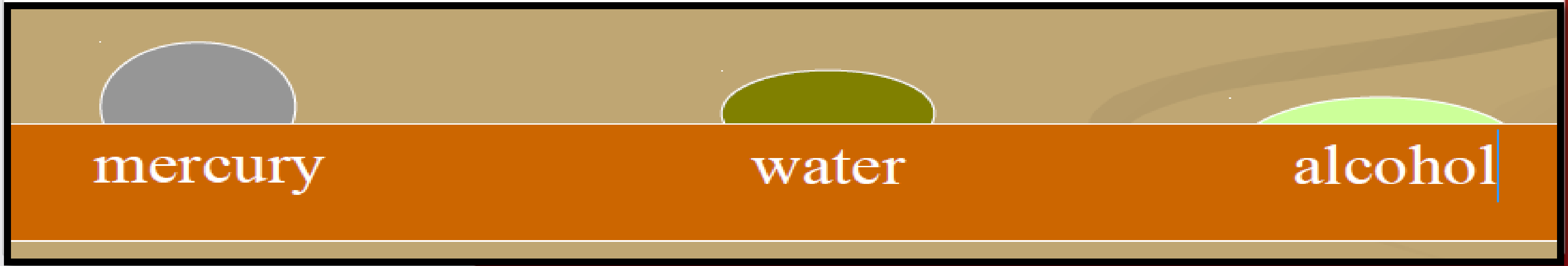
- Ever wonder why water beads up on a car, how some insects can walk on water, how bubbles hold themselves together?



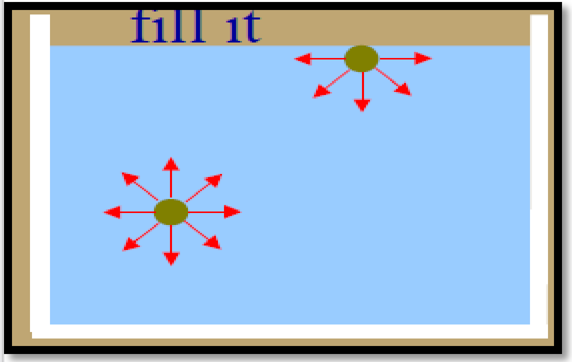
- The answer is surface tension: Because of cohesion between its molecules, a substance tends to contract to the smallest area possible

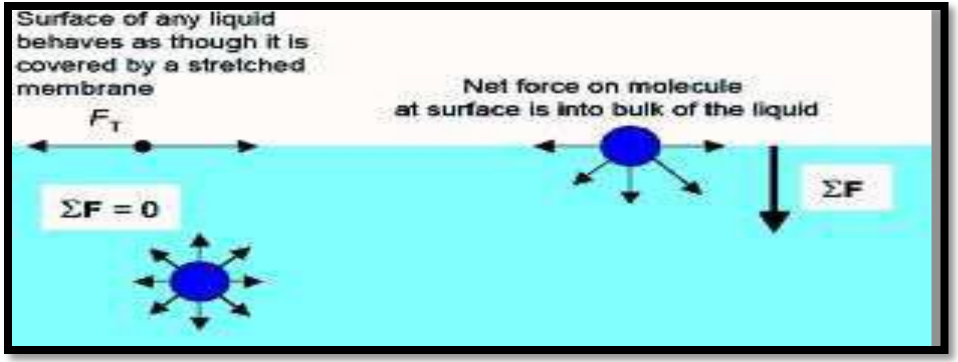
Surface Tension (Cont…)

- Water on a waxed surface, forms round beads. More weak bounds can be formed between molecules, if they were arranged on flat layer
- Cohesive forces are greater in mercury than in water, so it forms a more spherical shape
- Cohesive forces are weaker in alcohol than in water, so it forms a more flattened shape



Surface Tension (Cont…)





- A molecule in fluid is pulled in all directions by its neighbors with approximately equal strength, so the net force on it is about zero
- This is not the case at the surface. Here the net force on a molecule is downward. Thus, the layer of molecules at the surface are slightly compressed
- Surface tension can be defined as the force per unit length holding a surface together

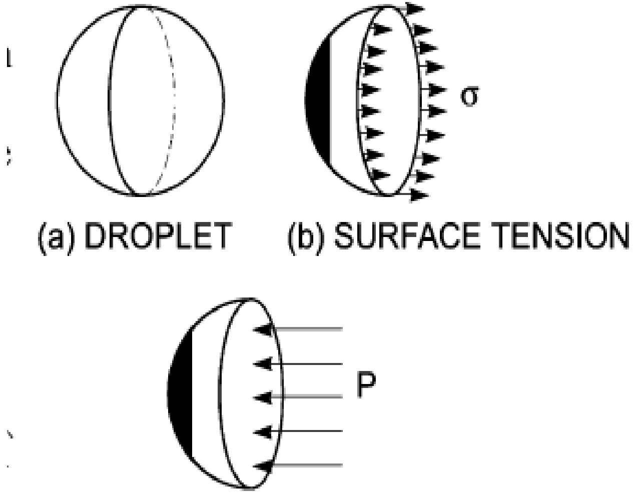
Surface Tension on Liquid Droplet

- Consider a small spherical droplet of a liquid of radius ‘r’
- On the entire surface of the droplet, the tensile force due to surface tension will be acting
- Let **σ** = Surface tension of the liquid

  P =Pressure intensity inside the droplet

  (in excess of the outside pressure intensity)

  d= Dia. of droplet



(a) DROPLET (b) SURFACE TENSION

(c) PRESSURE FORCES

Surface Tension on Liquid Droplet(Cont...)

- Surface tension acting around the circumference = σ X Circumference

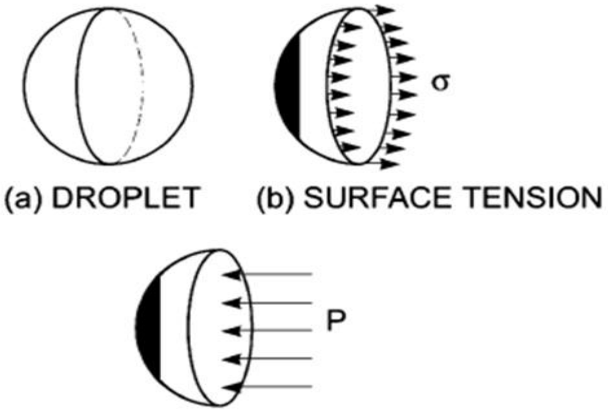
  = σ Xπd

- Pressure force on the area $\frac{\pi}{4}d^2$ = P x $\frac{\pi}{4}d^2$
- The two forces will be equal and opposite under equilibrium conditions

$$\mathbf{P \times \frac{\pi}{4} d^2 = \sigma \times \pi d}$$

$$\mathbf{P = \frac{\sigma \times \pi d}{\frac{\pi}{4} d^2}}$$

$$\mathbf{P = \frac{4\,\sigma}{d}}$$



(a) DROPLET (b) SURFACE TENSION

(c) PRESSURE FORCES

Surface Tension on Hollow Bubble

- A Hollow bubble like a soap bubble in air has two surfaces in contact with air. One is inside and other is outside
- Thus two surfaces are subjected to surface tension

$$P \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

$$P = \frac{2\sigma \pi d}{\frac{\pi}{4} d^2}$$

$$P = \frac{8\sigma}{d}$$

Surface Tension on a Liquid Jet

- Consider the equilibrium of the semi jet
- Force due to pressure = P x area of semi jet

  = P x L x d

- Force due to surface tension = σ x 2L
- Equating the forces

$$P \times L \times d = \sigma \times 2L$$

$$P = \frac{\sigma \times 2L}{L \times d} \Rightarrow \mathbf{P} = \frac{2\sigma}{d}$$

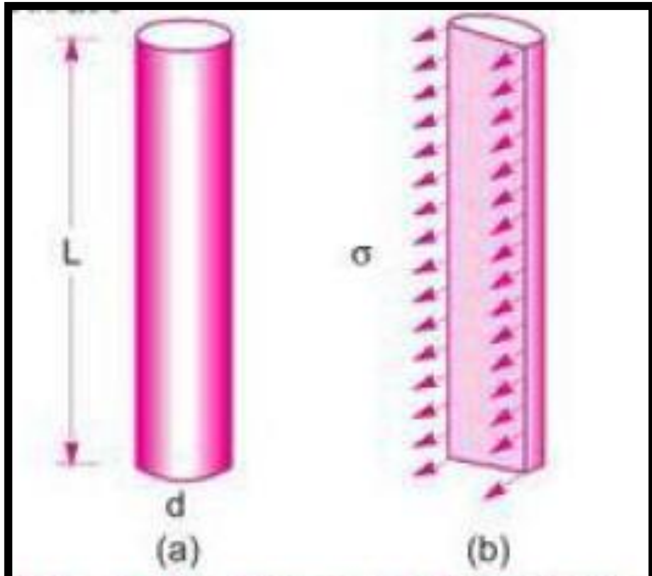


Fig. 1.12 Forces on liquid jet.

Problem:2

Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m² above atmospheric pressure.

Solution. Given :

Dia. of bubble, $d = 40 \text{ mm} = 40 \times 10^{-3}$ m

Pressure in excess of outside, $p = 2.5 \text{ N/m}^2$

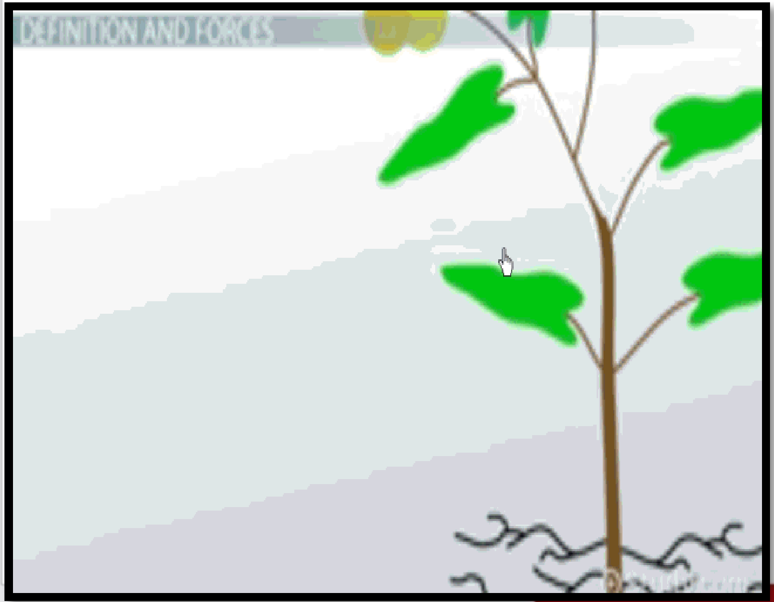
For a soap bubble, using equation , we get

$$p = \frac{8\sigma}{d} \quad \text{or} \quad 2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}}$$

$$\sigma = \frac{2.5 \times 40 \times 10^{-3}}{8} \text{ N/m} = \mathbf{0.0125 \text{ N/m. Ans.}}$$

Capillary Action

- How do trees pump water from the ground to their highest leaves? Why does liquid wax rise to the tip of a candle?



Capillary Action(Cont…)

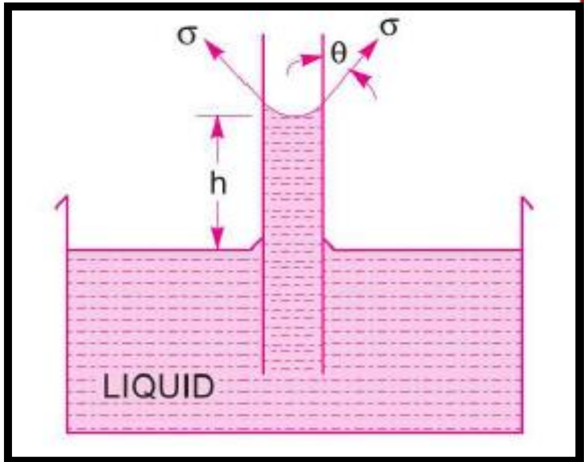
- These are all examples of capillary -It is due to adhesion and cohesion
- Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid
- The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression

Expression for Capillary Rise

- Let h= height of the liquid in the tube
- Under a state of equilibrium the weight of liquid of height *h* is balanced by the force at the surface of the liquid in the tube
- But the force at the surface of the liquid in the tube is due to surface Tension
- $\theta = Angle\ of\ Contact\ between\ liquid\ and$
  $glass\ tube$
- The weight of liquid of height h in the tube

  = (Area of tube x h) x ρ x g

  = $\frac{\pi}{4} d^2$x h x ρ x g



Expression for Capillary Rise (Cont...)

- Vertical component of the surface tensile force $= \sigma \times circumference \times \cos\theta$

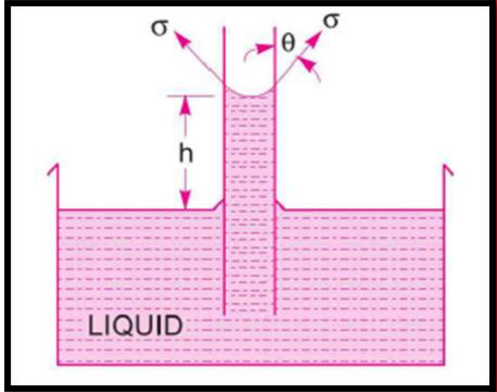
$$= \sigma \times \pi d \times \cos\theta$$

For equilibrium ,

$$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos\theta$$

$$h = \frac{\sigma \times \pi d \times \cos\theta}{\frac{\pi}{4} d^2 \times \rho \times g}$$

$$\mathbf{h = \frac{4\sigma\cos\theta}{\rho \times g \times d}}$$



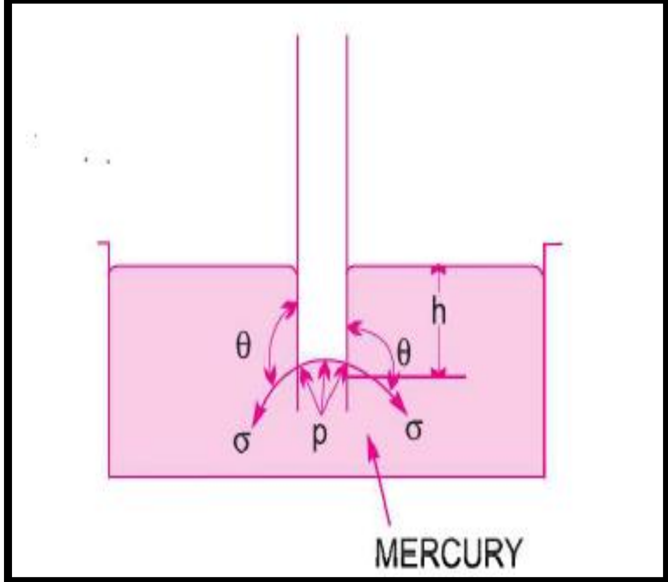
- The value of θ between water and clean glass tube is approximately equal to zero and hence $\cos\theta$ is equal to unity. Then rise of water is given by

$$\mathbf{h = \frac{4\sigma}{\rho \times g \times d}}$$

Expression for capillary Fall

- If the glass tube is dipped in mercury

- The level of mercury in the tube will be lower than the general level of the outside liquid as shown in Fig
- Then in equilibrium, two forces are acting on the mercury inside the tube
- First one is due to surface tension acting in down warddirection and equal to σ x $\pi\ d$ x$\cos\theta$



Expression for capillary Fall (Cont…)

- Second force is due to hydrostatic force acting upward and equal to intensity of Pressure at a depth 'h' x Area

$$= P \times \frac{\pi}{4} d^2 = (\rho g\, h) \times \frac{\pi}{4} d^2$$

Equating ,

$$\sigma \times \pi\, d \times \cos\theta = (\rho g\, h) \times \frac{\pi}{4} d^2$$

$$\mathbf{h} = \frac{4\sigma \cos\theta}{\rho g d}$$

- Value of θ for mercury and glass tube is 128^0

Problem:3

Find out the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is to be restricted to 2 mm. Consider surface tension of water in contact with air as 0.073575 N/m.

Solution. Given :

Capillary rise, $h = 2.0 \text{ mm} = 2.0 \times 10^{-3} \text{ m}$

Surface tension, $\sigma = 0.073575 \text{ N/m}$

Let dia. of tube $= d$

The angle θ for water $= 0^{\circ}$

The density for water, $\rho = 1000 \text{ kg/m}^3$

Problem:3 (Cont...)

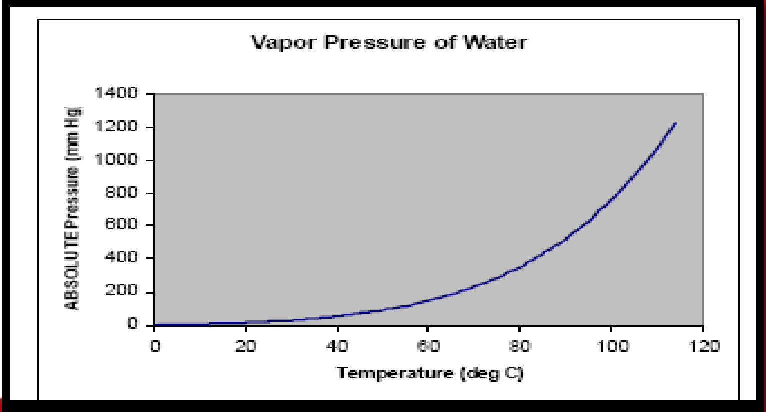
$$h = \frac{4\sigma}{\rho \times g \times d} \text{ or } 2.0 \times 10^{-3} = \frac{4 \times 0.073575}{1000 \times 9.81 \times d}$$

$$d = \frac{4 \times 0.073575}{1000 \times 9.81 \times 2 \times 10^{-3}} = 0.015 \text{ m} = \mathbf{1.5\ cm.\ Ans.}$$

Thus minimum diameter of the tube should be 1.5 cm.

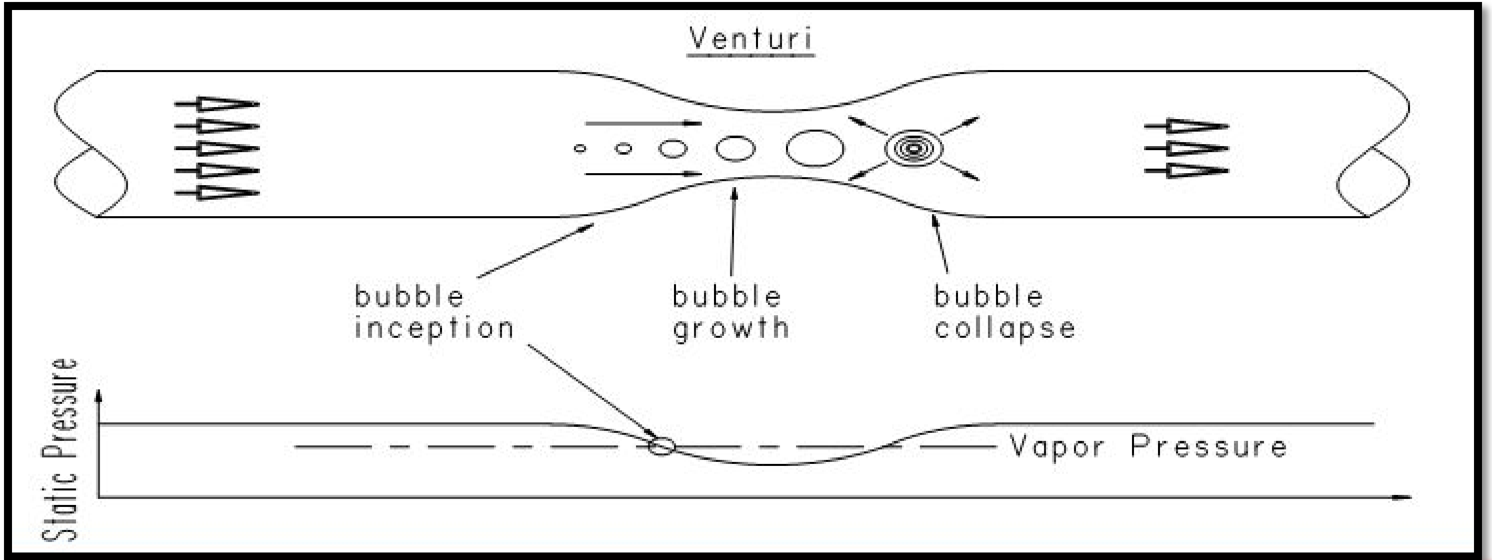
Vapor Pressure

- *Vapor pressure: the* pressure at which a liquid will boil
- Vapor pressure ↑ when temperature increases
- At atmospheric pressure, water will boil at 100 °C
- Water can boil at lower temperatures if the pressure is lower
    - When vapor pressure > the liquid's actual pressure
    - It will boil.



Cavitation

- It is the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below the vapour pressure and sudden collapsing of these vapour bubbles in a region of a higher pressure



Summary

- Compressibility is the reciprocal of the bulk modulus of elasticity
- Bulk modulus is defined as the ratio of compressive stress to volumetric strain
- Surface tension can be defined as the force per unit length holding a surface together
- Relation between surface tension and pressure for Liquid Droplet :**P** = $\frac{4\,\sigma}{d}$
- Relation between surface tension and pressure for Hollow Bubble: P= $\frac{8\sigma}{d}$
- Relation between surface tension and pressure for a Liquid Jet: **P=** $\frac{2\sigma}{\mathrm{d}}$
- Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid

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PRESSURE AND ITS MEASUREMENT: PART-1

Presented By:
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Lecture Details:
Unit-1(Fluid Statics) Pressure and its Measurement
FM & HM/Mechanical Engineering, I Semester .

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
- FLUID DYNAMICS
- CLOSED CONDUIT FLOW
- BOUNDARY LAYER THEORY AND APPLICATIONS
- BASICS OF TURBO MACHINERY
- HYDRAULIC TURBINES
- PERFORMANCE OF HYDRAULIC TURBINES
- CENTRIFUGAL PUMPS
- RECIPROCATING PUMPS

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PRESSURE AND ITS MEASUREMENT: PART-1

Contents

- Pressure
- Pascal's Law
- Hydrostatic Law
- Pressure Measurement Terms
- Summary

Pressure

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- Pressure is the force exerted per unit area
- Pressure is force applied to, or distributed over, a surface
- The pressure P of a force F distributed over an area A is defined as P = F/A
- Units – N/m^2, or Pascal (Pa)
- Atmospheric pressure (1 atm.) is equal to 101325 N/m^2
- **1 bar = 10^5 Pascal**
- 1 pound per square inch (1 psi) is equal to:

1 psi = 6944 Pa = 0.068 atm , 1atm = 14.7 psi

Pascal's Law

"It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions "

Forces acting on the fluid element are:

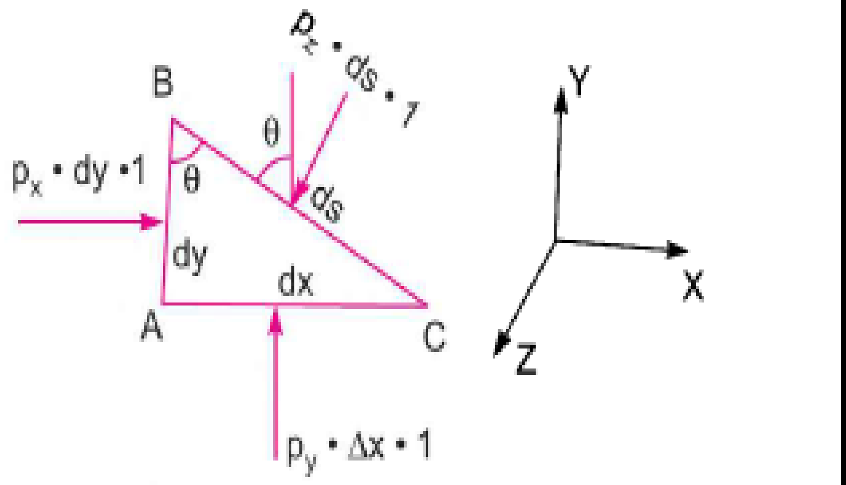
1. Pressure forces normal to the surfaces

2. Weight of element in the vertical direction

1. Pressure forces:

On Face AB = P_x *x Area of face AB*

= P_x *x dy x 1*



Forces on a fluid element.

Pascal's Law (Cont...)

Similarly ,

force on face AC = $P_y \times dx \times 1$

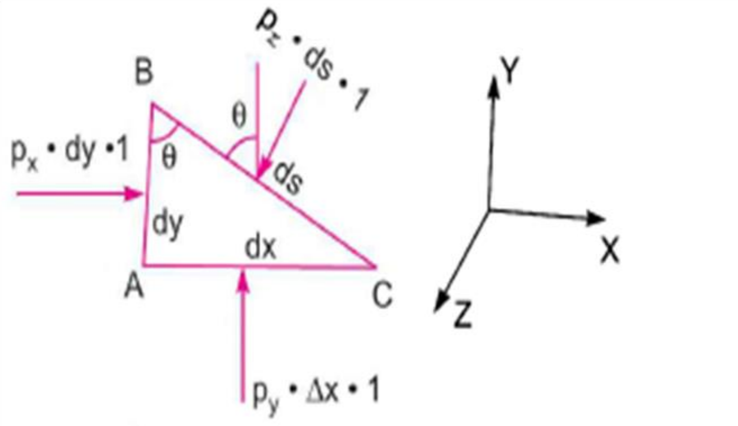
force on face BC = $P_z \times ds \times 1$

2. Weight of the element

= (mass of element)x g

= (volume x density) x g

$= \left(\frac{AB \times AC}{2} \times 1\right) \times \rho \times g$



Forces on a fluid element.

Pascal's Law (Cont...)

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Resolving forces in x-direction

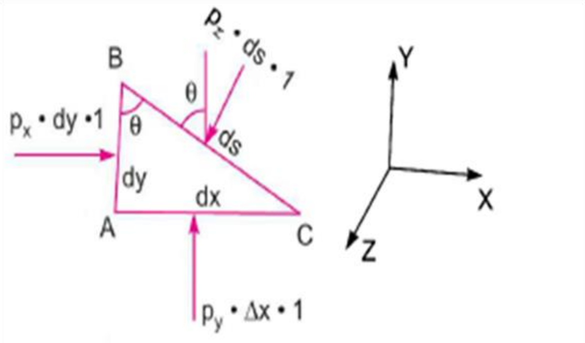
$$P_x \; x \; dy \; x \; 1 - (P_z \; x \; ds \;\; x \; 1) \sin(90 - \theta) = 0$$

$$P_x \; x \; dy \; x \; 1 - (P_z \; x \; ds \;\; x \; 1) \cos\theta \; = 0$$

But ds $\cos\theta$ *= AB=dy*

$$P_x \; x \; dy \; x \; 1 - \; P_z \; x \; dy \; x \; 1 = 0$$

$$P_x = \; P_z$$



Forces on a fluid element.

Similarly ,

Resolving the forces in y- direction

$$P_y \; x \; dx \; x \; 1 - (P_z \; x \; ds \;\; x \; 1) \cos (90 - \theta) - \frac{dx \; x \; dy}{2} \; x \; 1x \; \rho \; x \; g = 0$$

Pascal's Law (Cont...)

$$P_y\ x\ dx - P_z\ x\ ds\ \ \sin\theta - \frac{dx \text{ x } dy}{2} \text{ x } \rho \text{ x } g = 0$$

But ds sin θ= dx and also element is very small and hence weight is negligible

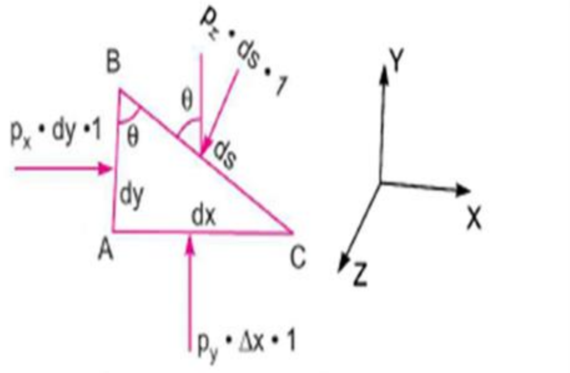
$$P_y\ dx - P_z\ dx = 0$$

$$P_y = P_z$$

Hence

$$\boldsymbol{P_x = P_y = P_z}$$

i.e pressure at any point is the same in all the directions

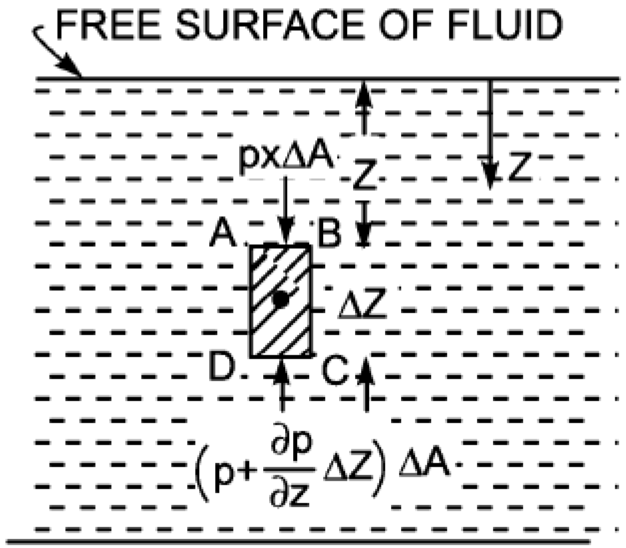


Forces on a fluid element.

Hydrostatic Law

"It states that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point"

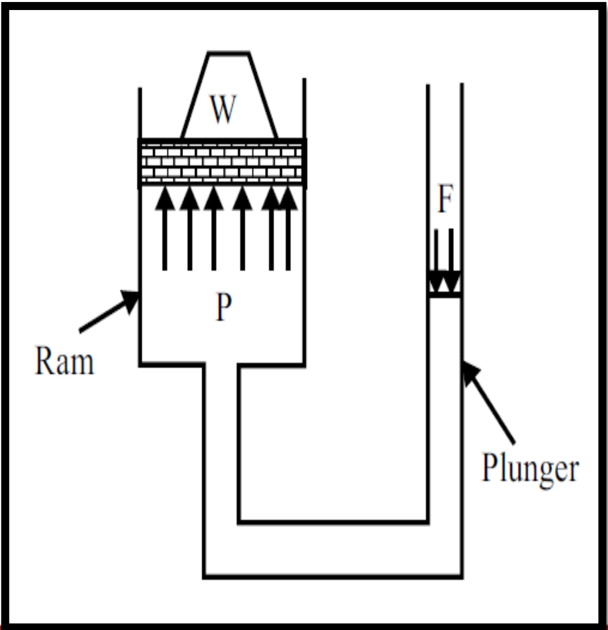
$$P = \rho g Z$$



Forces on a fluid element.

Problem:1

Find the weight that can be lifted by a hydraulic press when the force applied at the plunger is 350 N and has diameters of 250 mm and 40 mm of ram and plunger respectively



Solution: Given data:

$$F = 350\text{N}, d_{ram} = 0.25\text{m}, d_{plunger} = 0.04\text{m}$$

Area of the ram

$$A_{ram} = \frac{\pi d_{ram}^{2}}{4} = \frac{3.14 * 0.25^2}{4} = 0.049\,\text{m}^2$$

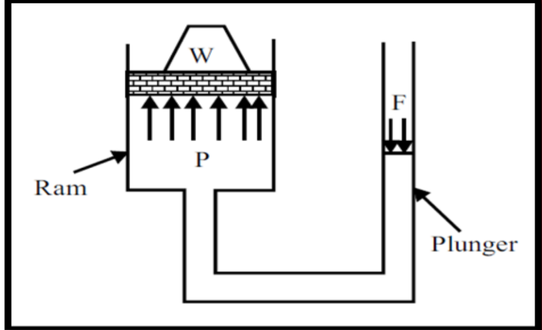
Area of the plunger

$$A_{plunger} = \frac{\pi d_{plunger}^{2}}{4} = \frac{3.14 * 0.04^2}{4} = 0.001256\,\text{m}^2$$

Problem:1(Cont...)

Normal pressure intensity due to the force applied

$$p = \frac{F}{A_{plunger}} = \frac{350}{0.001256} = 278.6\,\text{kPa}$$



As the pressure intensity due to the force in the plunger lifts the weight, so the total pressure acting on the weight in the ram must be equal or greater to the weight, and hence

$$p = \frac{W}{A_{ram}}$$

$$\Rightarrow W = p * A_{ram} = 278.6 * 0.049 = 13.65\text{kN}$$

Problem:2

An oil of specific gravity 0.8 is under pressure of 137.2 kPa, then determine pressure head expressed in terms of meters of oil.

Solution: Given data: $\rho_{water} = 1000\,\text{kg/m}^3, P_{gauge} = 137.2\,\text{kPa}, S = 0.8$

Pressure head

$$P_{gauge} = \rho_{oil} gh$$

$$S(\text{for liquids}) = \frac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$$

$$\Rightarrow P_{gauge} = S * \rho_{water} gh$$

$$\Rightarrow 137.2 * 1000 = 0.8 * 1000 * 9.81 * h$$

$$\Rightarrow h = 17.48\,\text{m of oil}$$

So pressure head in terms of oil is 17.48 m.

Problem:3

Calculate the pressure due to a column of 0.3 of (a) water, (b) an oil of sp. gr. 0.8, and (c) mercury of sp. gr. 13.6. Take density of water, $\rho = 1000\ kg/m^3$.

Solution. Given :

Height of liquid column, $Z = 0.3$ m.

The pressure at any point in a liquid is given by equation as

$$p = \rho g Z$$

Problem:3(Cont..)

(*a*) For water, $\rho = 1000 \text{ kg/m}^3$

$\therefore$ $p = \rho g Z = 1000 \times 9.81 \times 0.3 = 2943 \text{ N/m}^2$

$$= \frac{2943}{10^4} \text{ N/cm}^2 = \mathbf{0.2943 \text{ N/cm}^2}. \textbf{ Ans.}$$

(*b*) For oil of sp. gr. 0.8,

From equation , we know that the density of a fluid is equal to specific gravity of fluid multiplied by density of water.

$\therefore$ Density of oil, $\rho_0 = \text{Sp. gr. of oil} \times \text{Density of water}$ (ρ_0 = Density of oil)

$$= 0.8 \times \rho = 0.8 \times 1000 = 800 \text{ kg/m}^3$$

Problem:3(Cont..)

Now pressure, $p = \rho_0 \times g \times Z$

$$= 800 \times 9.81 \times 0.3 = 2354.4 \frac{\text{N}}{\text{m}^2} = \frac{2354.4}{10^4} \frac{\text{N}}{\text{cm}^2}.$$

$$= \mathbf{0.2354 \frac{N}{cm^2}}.\ \textbf{Ans.}$$

(*c*) For mercury, sp. gr. $= 13.6$

From equation (...) we know that the density of a fluid is equal to specific gravity of fluid multiplied by density of water

∴ Density of mercury, ρ_s = Specific gravity of mercury × Density of water

$$= 13.6 \times 1000 = 13600 \text{ kg/m}^3$$

Problem:3(Cont..)

$$\therefore \quad p = \rho_s \times g \times Z$$

$$= 13600 \times 9.81 \times 0.3 = 40025 \ \frac{\text{N}}{\text{m}^2}$$

$$= \frac{40025}{10^4} = \mathbf{4.002 \ \frac{N}{cm^2}}. \quad \textbf{Ans.}$$

Problem:4

An open tank contains water upto a depth of 2 m and above it an oil of sp. gr. 0.9 for a depth of 1 m. Find the pressure intensity (i) at the interface of the two liquids, and (ii) at the bottom of the tank.

Solution. Given :

Height of water, $Z_1 = 2$ m

Height of oil, $Z_2 = 1$ m

Sp. gr. of oil, $S_0 = 0.9$

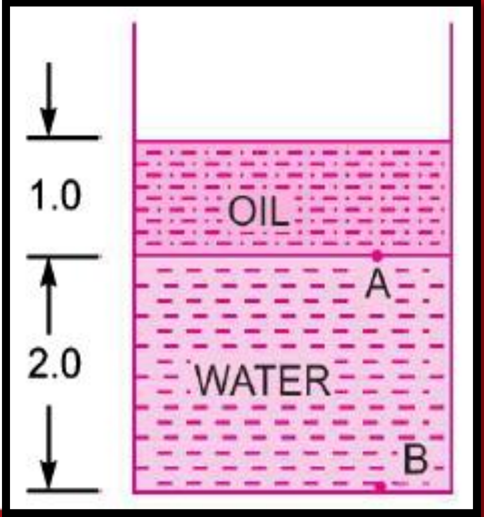
Density of water, $\rho_1 = 1000$ kg/m^3

Density of oil, ρ_2 = Sp. gr. of oil × Density of water

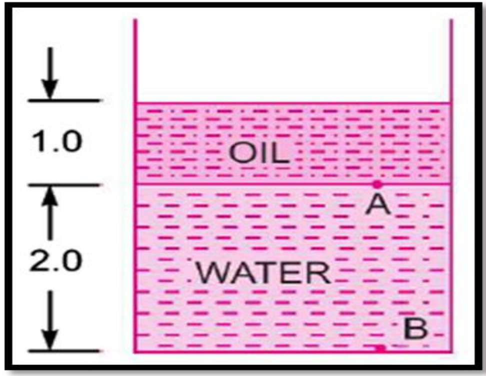
$= 0.9 \times 1000 = 900$ kg/m^3

Pressure intensity at any point is given by

$$p = \rho \times g \times Z.$$



Problem:4(Cont…)



(*i*) At interface, *i.e.*, at *A*

$$p = \rho_2 \times g \times 1.0$$
$$= 900 \times 9.81 \times 1.0$$
$$= 8829 \frac{\text{N}}{\text{m}^2} = \frac{8829}{10^4} = \mathbf{0.8829\ N/cm^2.\ Ans.}$$

(*ii*) At the bottom, *i.e.*, at *B*

$$p = \rho_2 \times gZ_2 + \rho_1 \times g \times Z_1 = 900 \times 9.81 \times 1.0 + 1000 \times 9.81 \times 2.0$$
$$= 8829 + 19620 = 28449\ \text{N/m}^2 = \frac{28449}{10^4}\ \text{N/cm}^2 = \mathbf{2.8449\ N/cm^2.\ Ans.}$$

Pressure Measurement Terms

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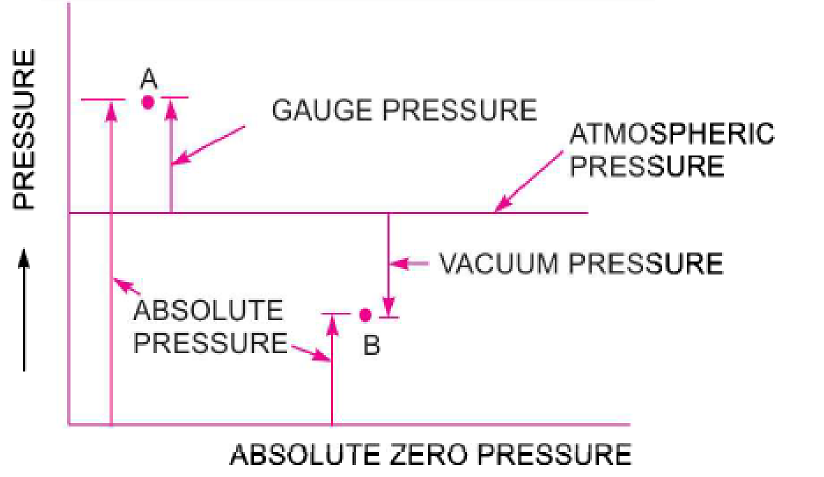
Absolute Pressure

- ➢ Measured above total vacuum or zero absolute. Zero absolute represents total lack of pressure

Absolute Pressure =

Atmospheric Pressure + Gauge Pressure

$$P_{ab} = P_{atm} + P_{gauge}$$



Relationship between pressures.

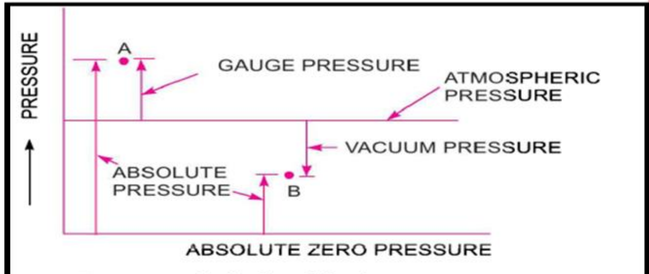
Pressure Measurement Terms(Cont…)

Atmospheric Pressure

- The pressure exerted by the earth’s atmosphere. Atmospheric pressure at sea level is 14.696 psi (1 psi = 6944 Pa = 0.068 atm)
- The value of atmospheric pressure decreases with increasing altitude

Barometric Pressure

Atmospheric pressure is also known as **barometric pressure** because barometers are used to measure it



Relationship between pressures.

Pressure Measurement Terms(Cont…)

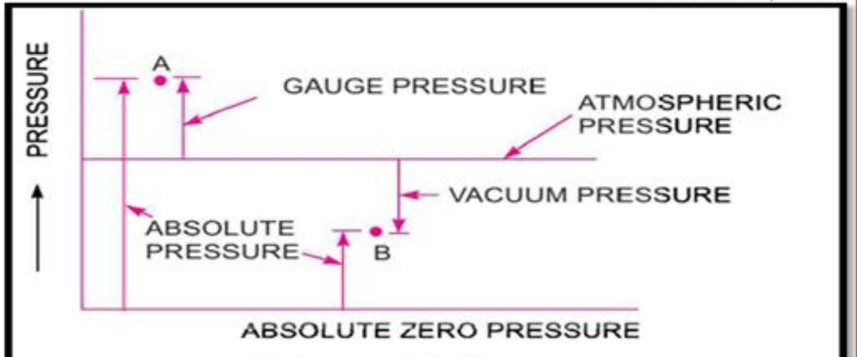
Vacuum Pressure

- Pressure below the atmospheric pressure

Vacuum Pressure =

Atmospheric Pressure – Absolute pressure

$$P_{\text{vacuum}} = P_{atm} - P_{\text{ab}}$$



Relationship between pressures.

Gauge Pressure

- The pressure above atmospheric pressure
- Can be converted to absolute by adding actual atmospheric pressure value ($P_{gauge} = P_{ab} - P_{atm}$)

Problem:5

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What are the gauge pressure and absolute pressure at a point 3 m below the free surface of a liquid having a density of 1.53×10^3 kg/m^3 if the atmospheric pressure is equivalent to 750 mm of mercury ? The specific gravity of mercury is 13.6 and density of water = 1000 kg/m^3.

Solution. Given :

Depth of liquid, $Z_1 = 3$ m

Density of liquid, $\rho_1 = 1.53 \times 10^3$ kg/m^3

Atmospheric pressure head, $Z_0 = 750$ mm of Hg

$$= \frac{750}{1000} = 0.75 \text{ m of Hg}$$

Problem:5(Cont…)

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$\therefore$ Atmospheric pressure, $p_{atm} = \rho_0 \times g \times Z_0$

where ρ_0 = Density of Hg = Sp. gr. of mercury × Density of water = 13.6×1000 kg/m^3

and Z_0 = Pressure head in terms of mercury.

$$\therefore \quad p_{atm} = (13.6 \times 1000) \times 9.81 \times 0.75 \text{ N/m}^2 \qquad (\because \; Z_0 = 0.75)$$

$$= 100062 \text{ N/m}^2$$

Pressure at a point, which is at a depth of 3 m from the free surface of the liquid is given by,

$$p = \rho_1 \times g \times Z_1$$

$$= (1.53 \times 1000) \times 9.81 \times 3 = 45028 \text{ N/m}^2$$

Problem:5(Cont…)

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$\therefore$ Gauge pressure, $p =$ **45028 N/m^2. Ans.**

Now absolute pressure $=$ Gauge pressure + Atmospheric pressure

$= 45028 + 100062 =$ **145090 N/m^2. Ans.**

Summary

- Pressure is measured as ratio of force per unit area
- From Pascal's law pressure at a point in a static fluid is equal in all directions
- *From Hydrostatic Law, rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point*
- Pressure measurement terms like absolute pressure, atmospheric pressure, barometric pressure ,vacuum pressure and gauge pressure are important for pressure measuring instruments

PRESSURE AND ITS MEASUREMENT: PART-2

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Lecture Details:
Unit-1(Fluid Statics) Pressure and its Measurement
FM & HM/Mechanical Engineering, I Semester .

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
- FLUID DYNAMICS
- CLOSED CONDUIT FLOW
- BOUNDARY LAYER THEORY AND APPLICATIONS
- BASICS OF TURBO MACHINERY
- HYDRAULIC TURBINES
- PERFORMANCE OF HYDRAULIC TURBINES
- CENTRIFUGAL PUMPS
- RECIPROCATING PUMPS

PRESSURE AND ITS MEASUREMENT: PART-2

Contents

- Measurement of Pressure
- Classification
- Piezometer
- U-Tube Manometer
- Summary

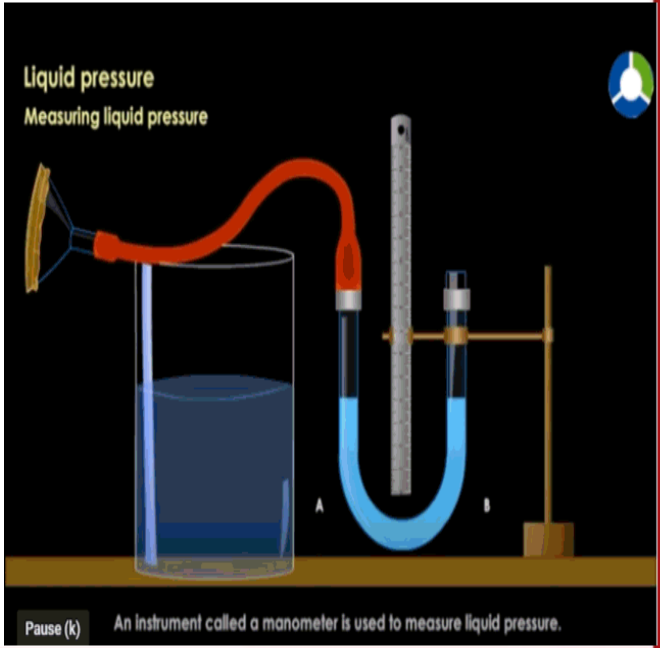
Measurement of Pressure

The pressure of a fluid is measured by the following devices

1. Manometers
2. Mechanical Gauges

Manometers:

- Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid



Measurement of Pressure(Cont...)

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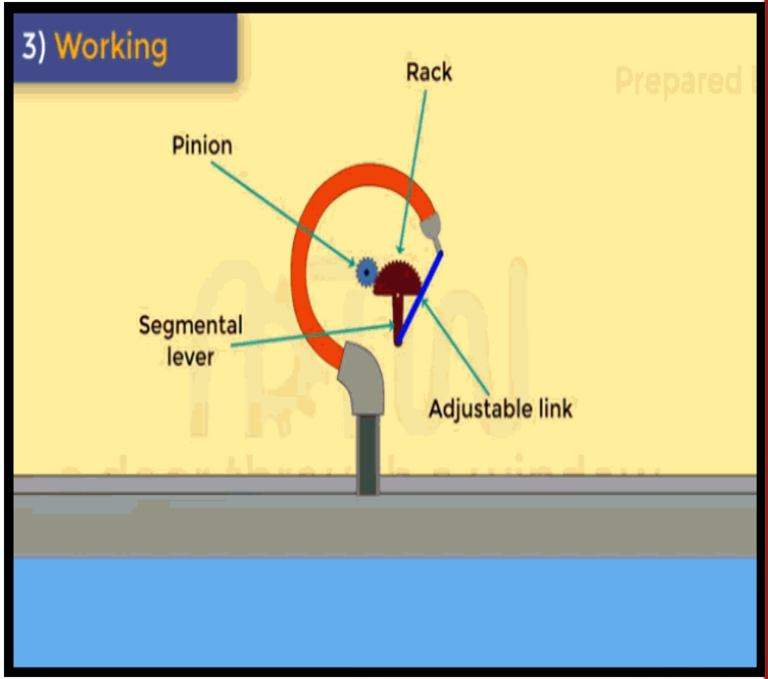
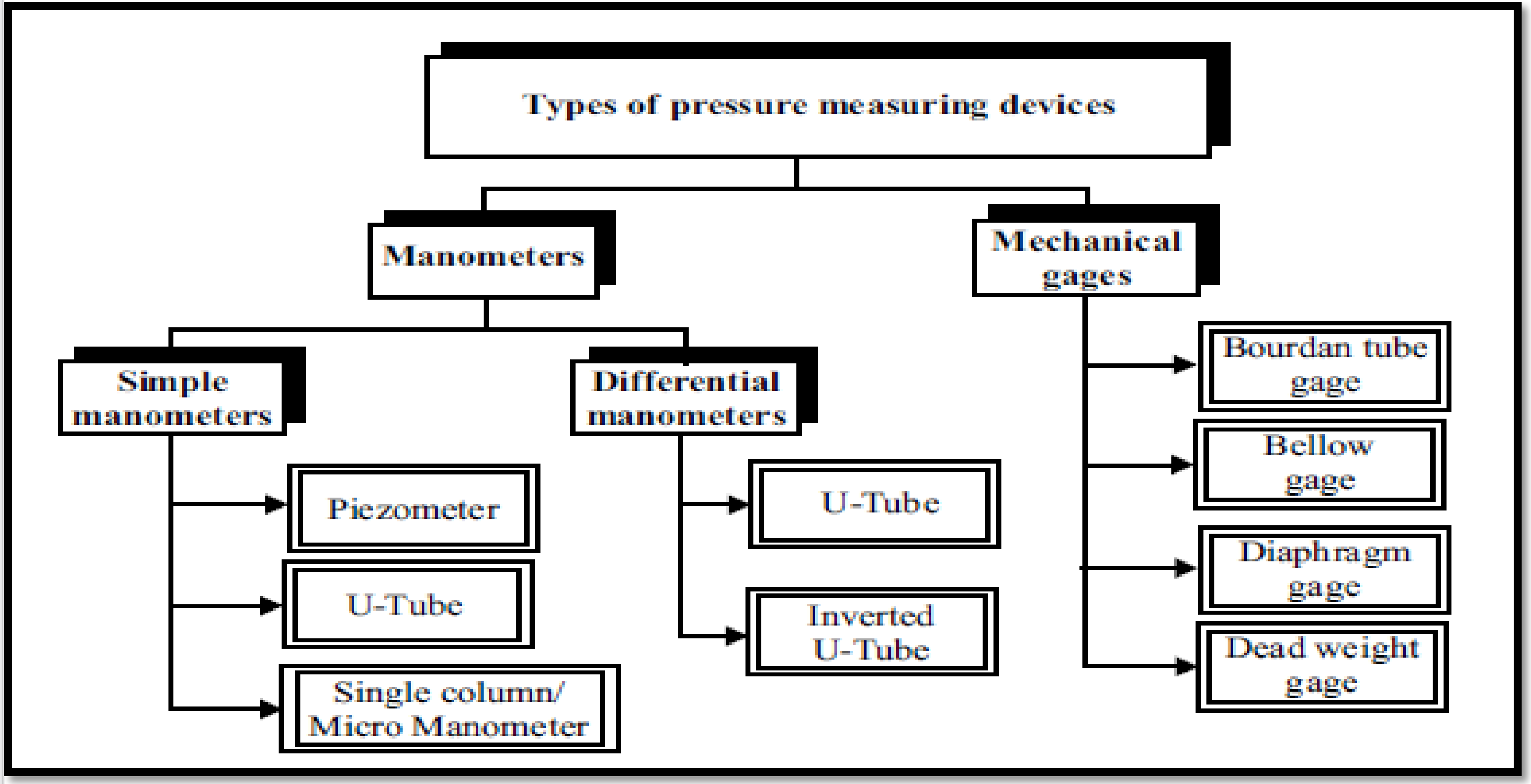
Mechanical Gauges:

- Mechanical Gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight



Classification



Piezometer

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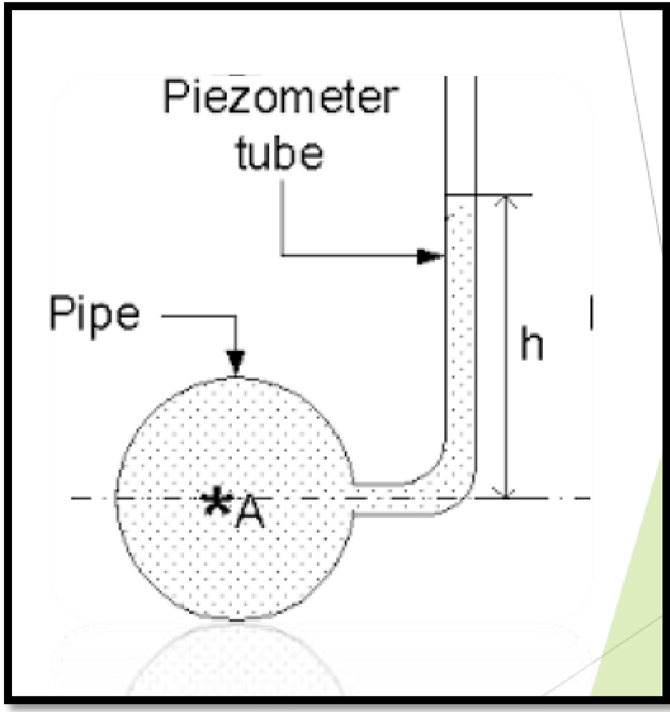
- A piezometer is the simplest form of the manometer
- It measures gauge pressure only
- The pressure at any point in the liquid is indicated by the height of the liquid in the tube above that point
- Which can read on the calibrated scale on glass tube

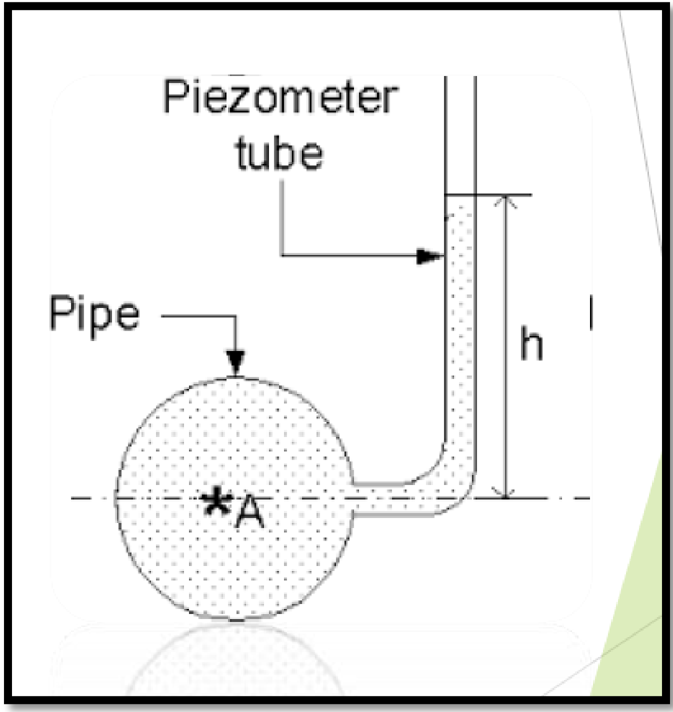


Piezometer(Cont...)

- The pressure at point A is given by

P= $\rho g h = w h$

$$h = \frac{\boldsymbol{P}}{\boldsymbol{\rho g}} \Longrightarrow \textit{Piezometer head}$$

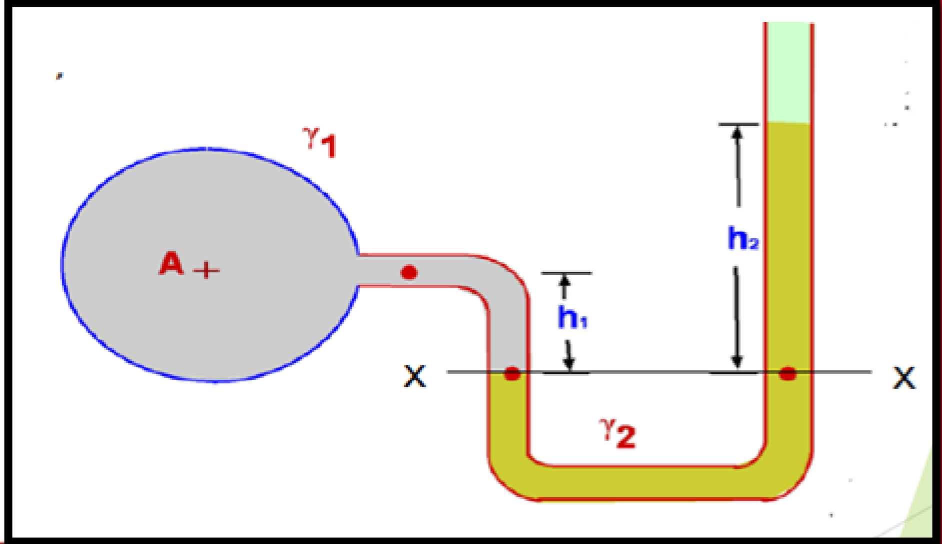


U-Tube Manometer

- It can be measure large pressure or vacuum pressure and gas pressure
- Pressure at XX in left column = Pressure at XX in right column

$$P_A + \rho_1 g h_1 = \rho_2 g h_2$$

$$P_A = \rho_2 g h_2 - \rho_1 g h_1$$



U-Tube Manometer(Cont…)

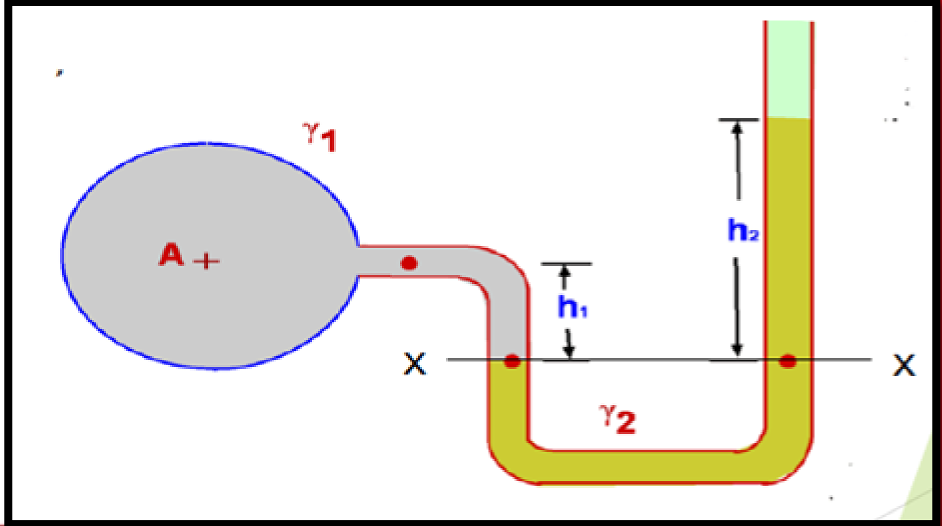
Now , $P_A = \rho$ gh

h =head in terms of water column

$$\rho gh = \rho_2 g h_2 - \rho_1 g h_1$$

$$h = \frac{\rho_2}{\rho} h_2 - \frac{\rho_1}{\rho} h_1$$

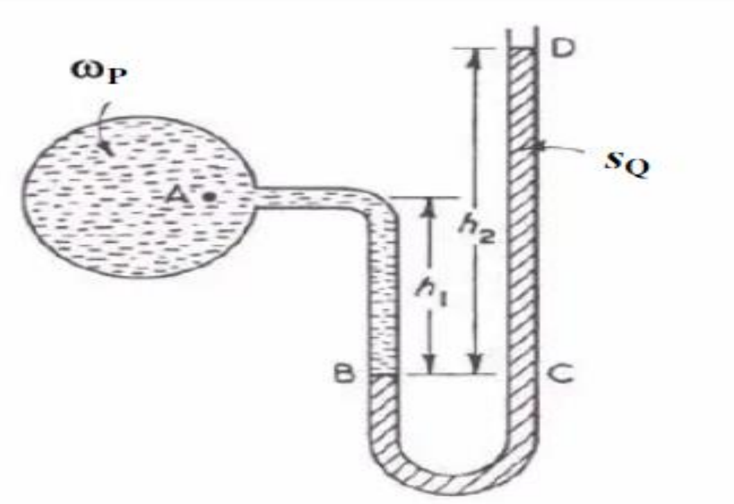
$$\mathbf{h = \boldsymbol{s_2\, h_2 - s_1\, h_1}}$$



Problem:1

A U-tube manometer similar to that shown in Figure is used to measure the gauge pressure of water (mass density $\rho = 1000\ kg/m^3$). If the density of mercury is $13.6 \times 10^3\ kg/m^3$, what will be the gauge pressure at A if $h_1 = 0.45\ m$ and D is $0.7\ m$ above BC.

Solution :



Considering

$$\rho_Q = 13.6 \times 10^3\ kg/m^3$$

$$\rho_P = 1.0 \times 10^3\ kg/m^3$$

$$h_1 = 0.45\ m$$

$$h_2 = 0.7\ m$$

Problem:1(Cont..)

the pressure at left-hand limb;

$$p_B = \text{Pressure, } p_A \text{ at A} + \text{Pressure due to depth, } h_1 \text{ of fluid P}$$

$$= p_A + \omega_P h_1$$

$$= p_A + \rho_P g h_1$$

the pressure at right-hand limb;

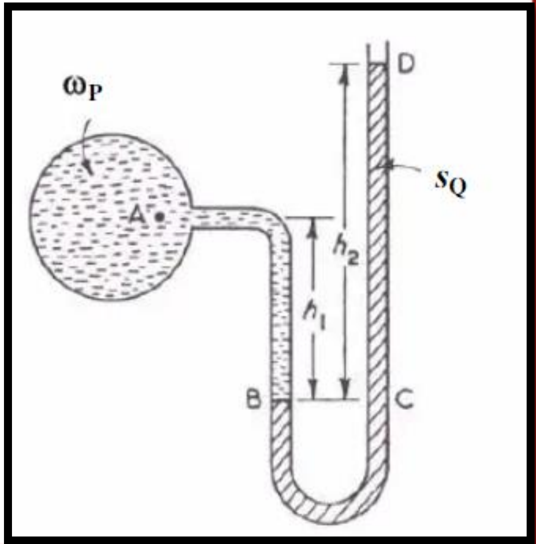
$$p_C = \text{Pressure } p_D \text{ at D} + \text{Pressure due to depth } h_2 \text{ of liquid Q}$$

$$p_C = 0 + \omega_Q h_2$$

$$= 0 + \rho_Q g h_2$$

Since $p_B = p_C$

$$p_A + \rho_P g h_1 = \rho_Q g h_2$$



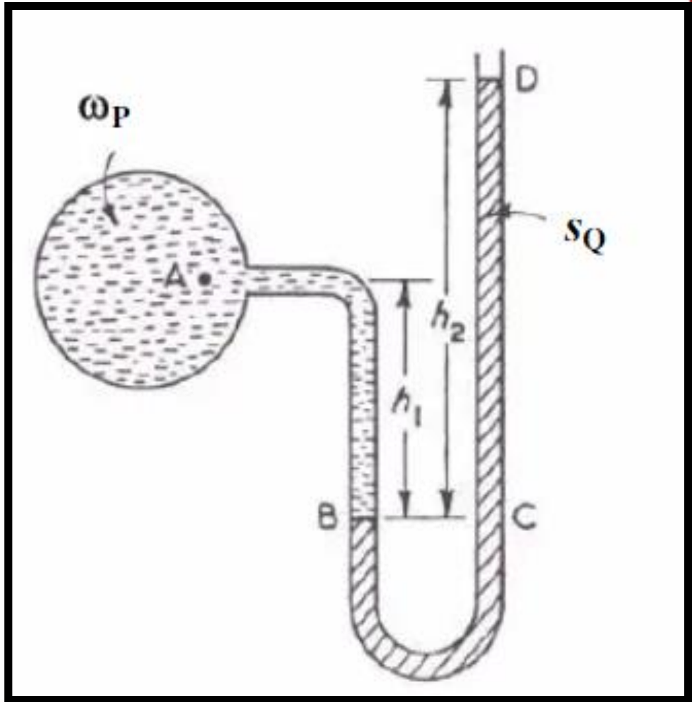
Problem:1(Cont..)

$$p_A = \rho_Q g h_2 - \rho_P g h_1$$

$$= 13.6\times10^3 \times 9.81\times0.7 - 1.0\times10^3 \times 9.81\times0.45$$

$$= 88976.7\ N/m^2$$

$$= 88.97\times10^3\ N/m^2$$



Problem:2

A U-tube manometer similar to that shown in Figure is used to measure the gauge pressure of a fluid P of density $\rho = 1000\ kg/m^3$. If the density of the liquid Q is $13.6 \times 10^3\ kg/m^3$, what will be the gauge pressure at A if $h_1 = 0.15$ m and $h_2 = 0.25\ m$ above BC. Take into consideration $p_{atm} = 101.3\ kN/m^2$.

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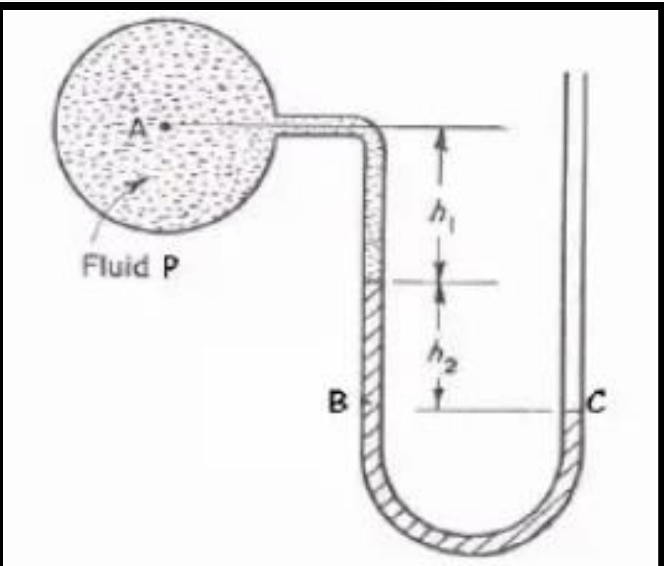


Solution to :

Putting ,

$$\rho_Q = 13.6 \times 10^3$$
$$\rho_P = 1000\ kg/m^3$$
$$h_1 = 0.15\ m$$
$$h_2 = 0.25\ m$$

Problem:2(Cont…)

pressure at left-hand limb;

p_B = Pressure p_A at A + Pressure due to depth h_1 of fluid P + Pressure due to depth h_2 of liquid Q

$$= p_A + \omega_P h_1 + \omega_Q h_2$$

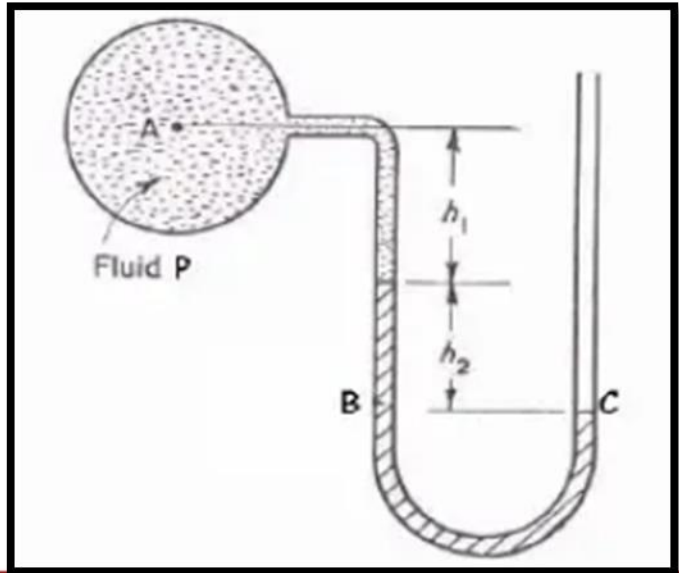
$$= p_A + \rho_P g h_1 + \rho_Q g h_2$$

pressure at right-hand limb;

p_C = Pressure p_D at D

p_D = Atmospheric pressure

$p_C = p_{atm}$



Problem:2(Cont…)

Since $p_B = p_C$,

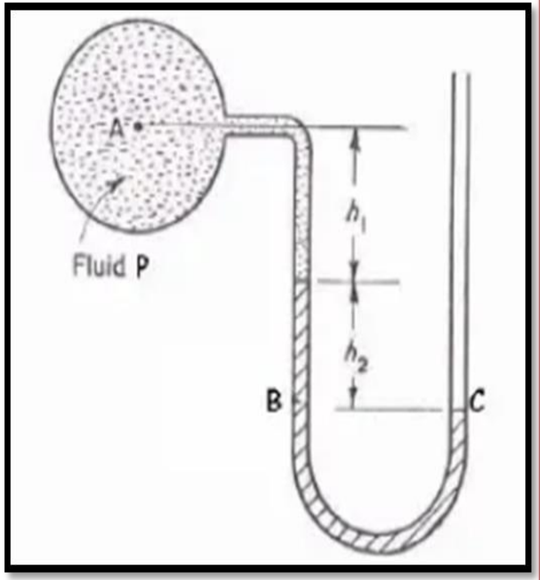
$$p_A + \rho_P g h_1 + \rho_Q g h_2 = p_D$$

$$p_A = p_D - (\rho_P g h_1 + \rho_Q g h_2)$$

$$= 101.3 - (13.6 \times 103 \times 9.81 \times 0.15 + 1000 \times 9.81 \times 0.25)$$

$$= 70835.1\ N/m^2$$

$$= 70.84\ kN/m^2$$



Problem:3

Fig. . . shows a conical vessel having its outlet at A to which a U-tube manometer is connected. The reading of the manometer given in the figure shows when the vessel is empty. Find the reading of the manometer when the vessel is completely filled with water.

Solution. Vessel is empty. Given :

Difference of mercury level $h_2 = 20$ cm

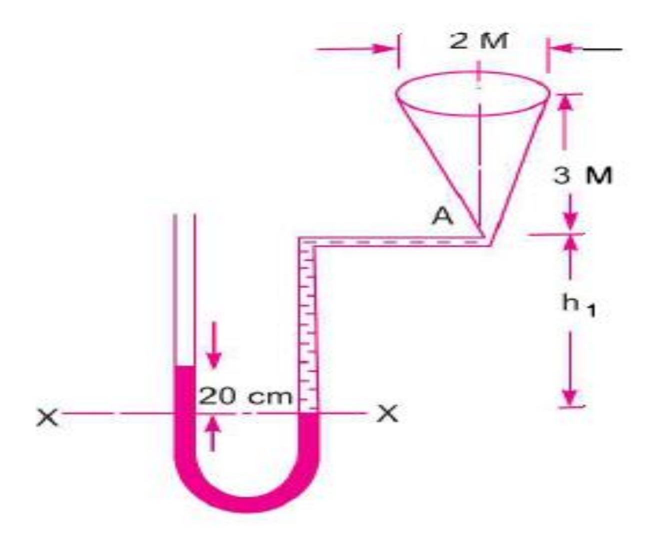
Let h_1 = Height of water above X-X

Sp. gr. of mercury, $S_2 = 13.6$

Sp. gr. of water, $S_1 = 1.0$

Density of mercury, $\rho_2 = 13.6 \times 1000$

Density of water, $\rho_1 = 1000$



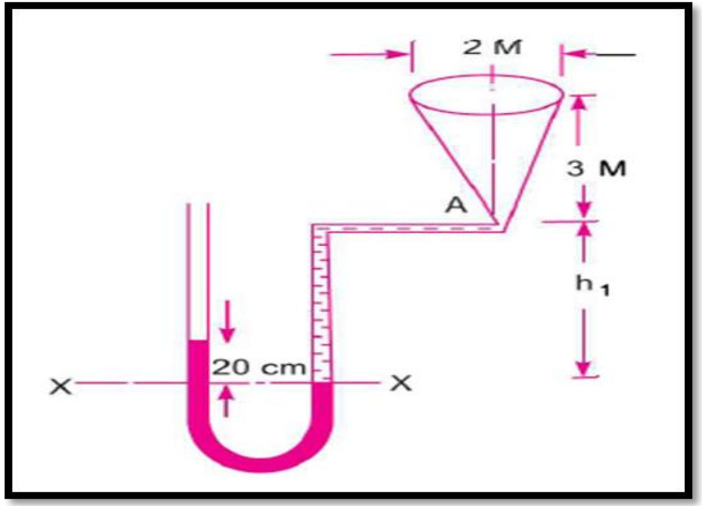
Problem:3(Cont…)

Equating the pressure above datum line *X-X*, we have

$$\rho_2 \times g \times h_2 = \rho_1 \times g \times h_1$$

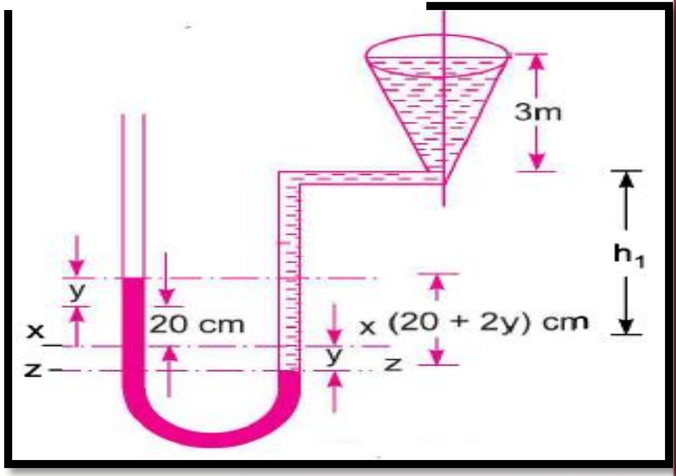
or $$13.6 \times 1000 \times 9.81 \times 0.2 = 1000 \times 9.81 \times h_1$$

$$h_1 = 2.72 \text{ m of water.}$$



Problem:3(Cont…)

Vessel is full of water. When vessel is full of water, the pressure in the right limb will increase and mercury level in the right limb will go down. Let the distance through which mercury goes down in the right limb be, y cm as shown in Fig. . The mercury will rise in the left by a distance of y cm. Now the datum line is Z-Z. Equating the pressure above the datum line Z-Z.



Pressure in left limb = Pressure in right limb

$$13.6 \times 1000 \times 9.81 \times (0.2 + 2y/100)$$
$$= 1000 \times 9.81 \times (3 + h_1 + y/100)$$

Problem:3(Cont...)

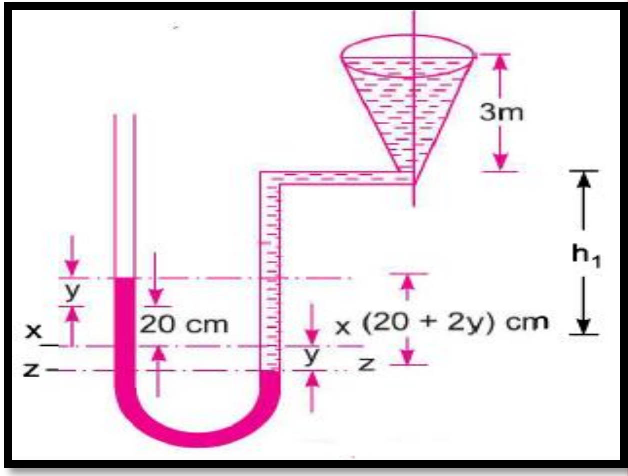
$$13.6 \times (0.2 + 2y/100) = (3 + 2.72 + y/100) \quad (\because \quad h_1 = 2.72 \text{ m})$$

$$2.72 + 27.2y/100 = 3 + 2.72 + y/100$$

$$(27.2y - y)/100 = 3.0$$

$$26.2y = 3 \times 100 = 300$$

$$y = \frac{300}{26.2} = 11.45 \text{ cm}$$



The difference of mercury level in two limbs

$$= (20 + 2y) \text{ cm of mercury}$$

$$= 20 + 2 \times 11.45 = 20 + 22.90$$

$$= 42.90 \text{ cm of mercury}$$

$\therefore$ Reading of manometer = **42.90 cm. Ans.**

Summary

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- Pressure of a fluid is measured by Manometers and Mechanical Gauges
- A **piezometer** is the simplest form of the manometer
- *Piezometer head* is given by **h=** $\frac{P}{\rho g}$
- **U-Tube Manometer** can be measure large pressure or vacuum pressure and gas pressure
- Head in terms of water column in U-tube manometer is given by

h= $s_2\ h_2$ - $s_1\ h_1$

PRESSURE AND ITS MEASUREMENT: PART-3

Presented By:
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Lecture Details:
Unit-1(Fluid Statics) Pressure and its Measurement
FM & HM/Mechanical Engineering, I Semester .

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
- FLUID DYNAMICS
- CLOSED CONDUIT FLOW
- BOUNDARY LAYER THEORY AND APPLICATIONS
- BASICS OF TURBO MACHINERY
- HYDRAULIC TURBINES
- PERFORMANCE OF HYDRAULIC TURBINES
- CENTRIFUGAL PUMPS
- RECIPROCATING PUMPS

PRESSURE AND ITS MEASUREMENT: PART-3

Contents

- Single column Manometer
- U-tube differential manometer
- Inverted U-tube differential manometer
- Summary

Single column Manometer

Single column manometer is divided into mainly two types

A. Vertical single column manometer

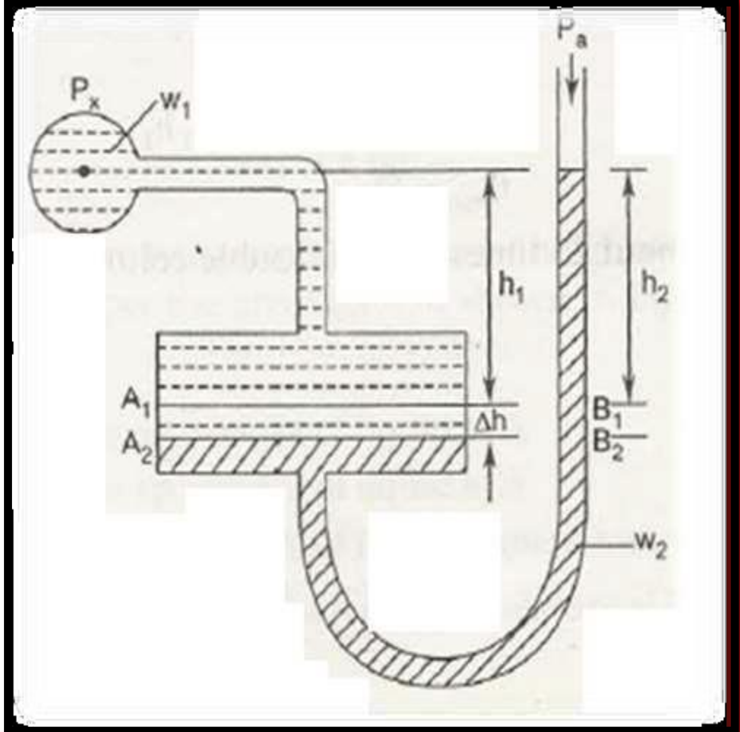
B. Inclined single column manometer

(A) Vertical Single Column Manometer

- One of the limbs in double column manometer is converted into a reservoir having large cross sectional area (about 100 times) with respect to the other limb
- Volume of heavy liquid fall in reservoir = Volume of heavy liquid rise in right column

$$A \times \Delta h = a \times h_2$$

$$\Delta h = \frac{a \times h_2}{A}$$



(A) Vertical Single Column Manometer(Cont…)

Pressure in left column = Pressure in right column

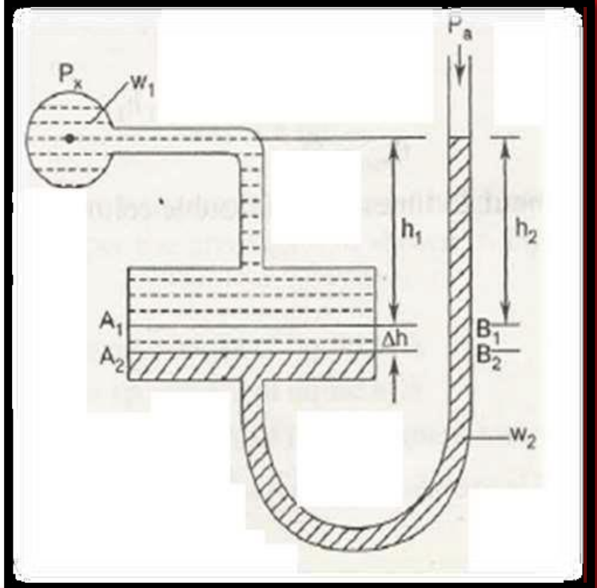
P+ ρ_1 g h_1 + ρ_1 gΔh= ρ_2 g h_2 +ρ_2 g Δh

P= ρ_2 g h_2 +ρ_2 g Δh −ρ_1 g h_1 − ρ_1 gΔh

P=Δh[ρ_2 g - ρ_1 g] + ρ_2 g h_2 - ρ_1 g h_1

But $\Delta h = \frac{a \times h_2}{A}$

P= $\frac{a \times h_2}{A}$ [ρ_2 g - ρ_1 g] + ρ_2 g h_2 - ρ_1 g h_1 ------eq(1)



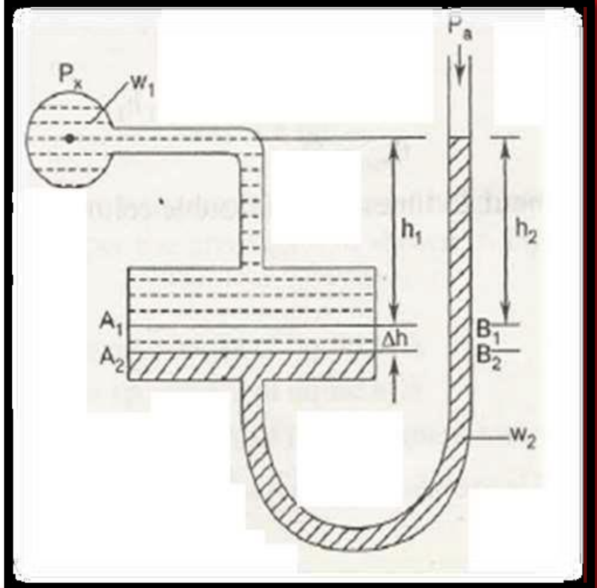
(A) Vertical Single Column Manometer(Cont…)

- " A "is very large as compared to "a" , so " $\frac{a}{A}$ "

  becomes very small , then neglecting $\frac{a \times h_2}{A}$

{ P= $\frac{a \times h_2}{A}$ $[\rho_2 g - \rho_1 g] + \rho_2 g h_2 - \rho_1 g h_1$ ------eq(1) }

P = $\boldsymbol{\rho_2 g h_2 - \rho_1 g h_1}$



Problem:1

A single column manometer is connected to a pipe containing a liquid of sp. gr. 0.9 as shown in Fig. . Find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube for the manometer reading shown in Fig. . The specific gravity of mercury is 13.6.

Solution. Given :

Sp. gr. of liquid in pipe, $S_1 = 0.9$

$\therefore$ Density $\rho_1 = 900 \text{ kg/m}^3$

Sp. gr. of heavy liquid, $S_2 = 13.6$

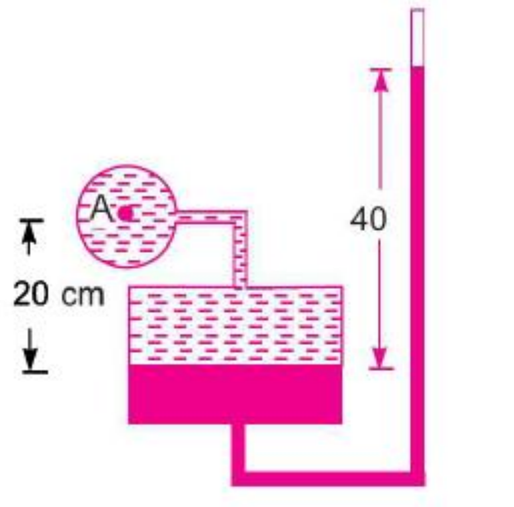
Density, $\rho_2 = 13.6 \times 1000$

$$\frac{\text{Area of reservoir}}{\text{Area of right limb}} = \frac{A}{a} = 100$$

Height of liquid, $h_1 = 20 \text{ cm} = 0.2 \text{ m}$

Rise of mercury in right limb,

$h_2 = 40 \text{ cm} = 0.4 \text{ m}$

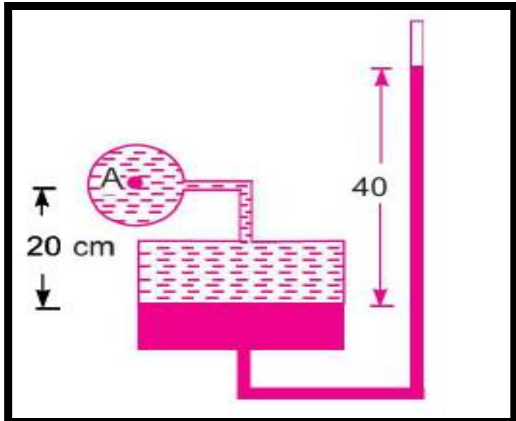


Problem:1(Cont…)

Let p_A = Pressure in pipe

Using equation , we get

$$p_A = \frac{a}{A} h_2[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$



$$= \frac{1}{100} \times 0.4[13.6 \times 1000 \times 9.81 - 900 \times 9.81] + 0.4 \times 13.6 \times 1000 \times 9.81 - 0.2 \times 900 \times 9.81$$

$$= \frac{0.4}{100} [133416 - 8829] + 53366.4 - 1765.8$$

$= 533.664 + 53366.4 - 1765.8 \text{ N/m}^2 = 52134 \text{ N/m}^2 =$ **5.21 N/cm^2. Ans.**

(B) Inclined single column manometer

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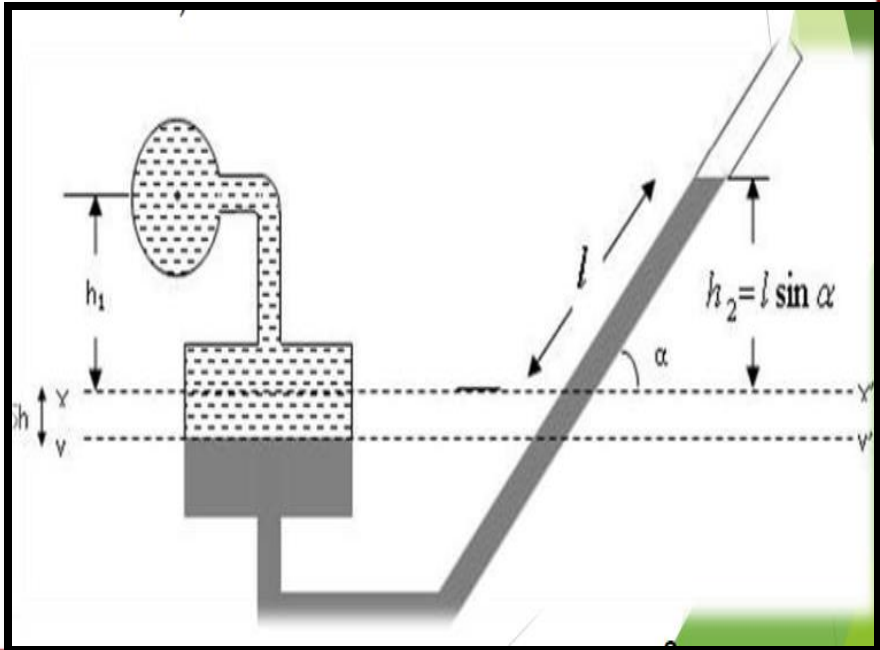
- It is modified form of vertical column manometer
- This is useful for measuring small pressure values
- Here vertical height will be $h_2 = l \sin\alpha$
- It will be substitute into the eq(1) of vertical single column manometer
- [P= $\frac{a \times h_2}{A}$ [ρ_2 g - ρ_1 g] + ρ_2 g h_2 - ρ_1 g h_1 ------eq(1)]



(B) Inclined single column manometer(Cont…)

Hence

$$P = \frac{a\, l \sin\alpha}{A}\left[\rho_2\, g - \rho_1\, g\right] + \rho_2\, g\, l \sin\alpha - \rho_1\, g\, h_1$$

Since , a<< A , neglecting first term

$$P = \rho_2\, g\, h_2 - \rho_1\, g\, h_1$$

But P= $\rho g h$

$$\rho g h = \rho_2\, g\, h_2 - \rho_1\, g\, h_1$$

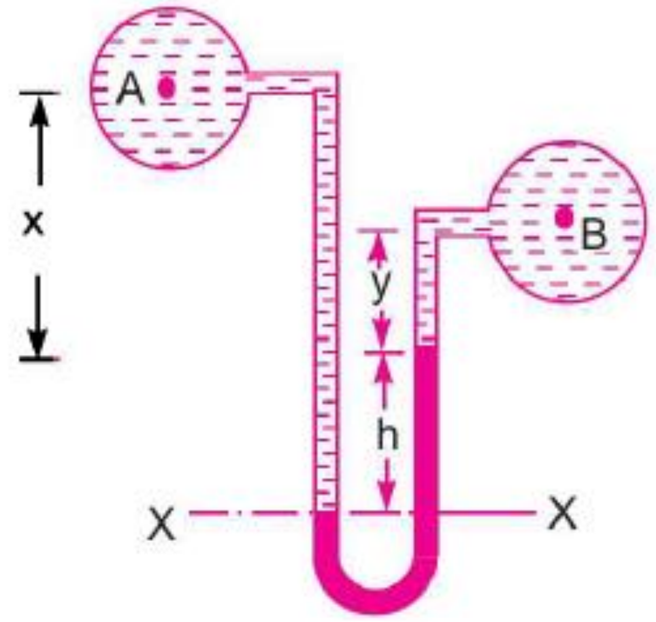
$$\mathbf{h = s_2\, l \sin\theta - s_1\, h_1}$$

U-tube Differential Manometer

Case 1: U-tube upright differential manometer connected between two pipes at different levels and carrying different fluids

Let h= Difference of mercury level in the U –tube

y= Distance of the center of B, from the mercury level in the in the right limb



(a)Two pipes at different levels

U-tube Differential Manometer(Cont…)

x= Distance of the center of A, from the mercury level in the right limb

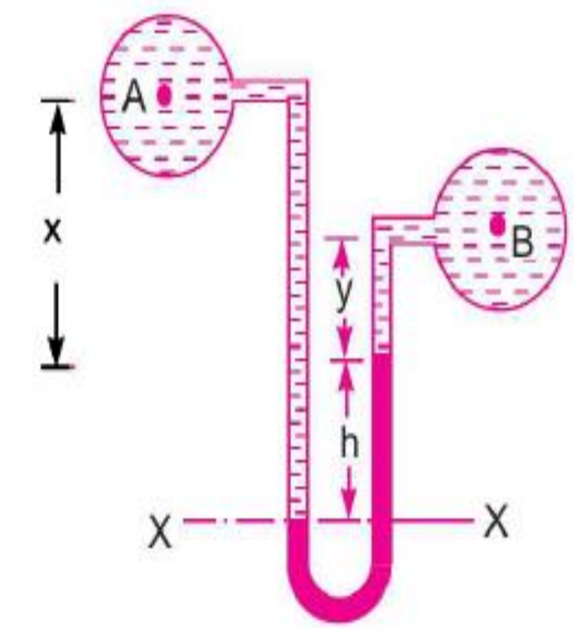
ρ_1= Density of liquid at A

ρ_2= Density of liquid at B

ρ_g= Density of heavy liquid or mercury

- Taking datum line at X-X

Pressure above X-X in the left limb = $P_A + \rho_1$g(x + h)



(a)Two pipes at different levels

U-tube differential manometer (Cont…)

Pressure above X-X in the right limb = $P_B + \rho_2 gy + \rho_g gh$

Equating the two pressures ,

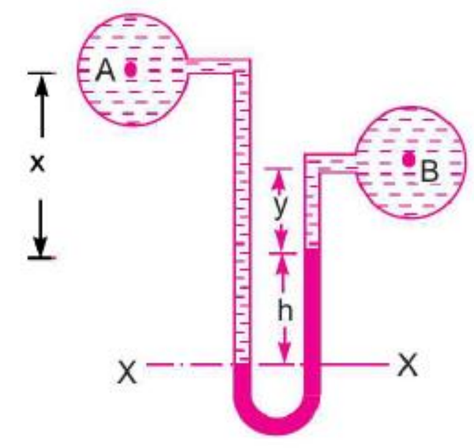
$$P_A + \rho_1 g(x+h) = P_B + \rho_2 gy + \rho_g gh$$

$$P_A - P_B = \rho_g gh + \rho_2 gy - \rho_1 g(x+h)$$

$$P_A - P_B = \rho_g gh + \rho_2 gy - \rho_1 gh - \rho_1 g\,x$$

Difference of pressure at A and B

$= h\,g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx$

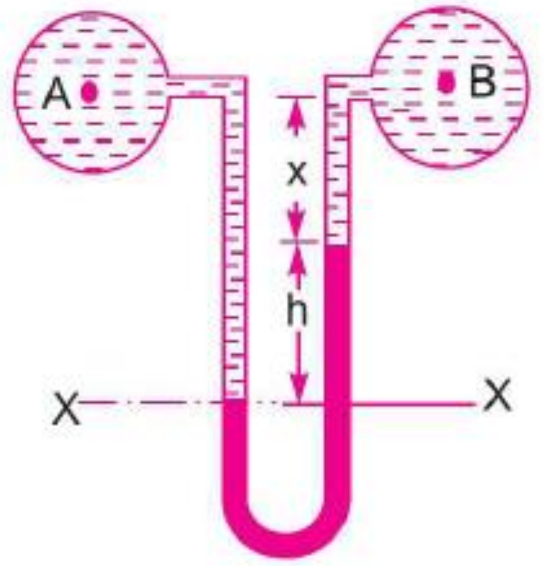


(a)Two pipes at different levels

U-tube differential manometer(Cont…)

Case 2: U- tube upright differential manometer connected at two points in a pipe at same level

- In fig(b) the two points A and B are at the same level and contains the Same liquid of density ρ_1



(b) A and B are at the same level

U-tube differential manometer(Cont…)

Pressure above X-X in right limb =

$P_B\ + \rho_1 gx + \rho_g gh$

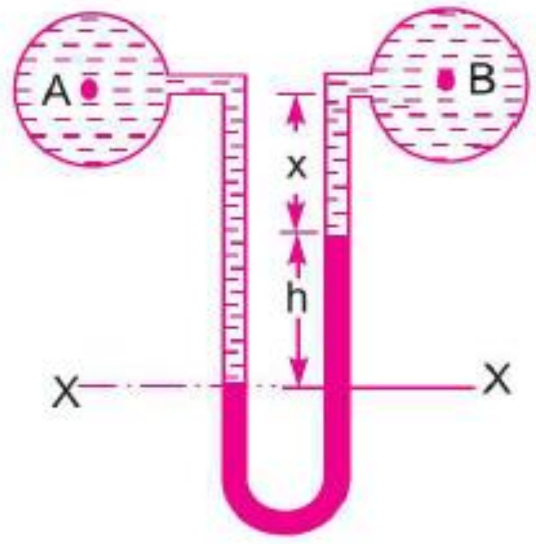
Pressure above X-X in left limb = $P_A\ + \rho_1 g(x + h)$

Equating the two pressures

$$P_B\ + \rho_1 gx + \rho_g gh = P_A\ + \rho_1 g(x + h)$$

$$P_A - P_B = \rho_g gh + \rho_1 gx - \rho_1 g(x+h)$$

$$P_A - P_B = \mathbf{gh(\rho_g - \rho_1)}$$

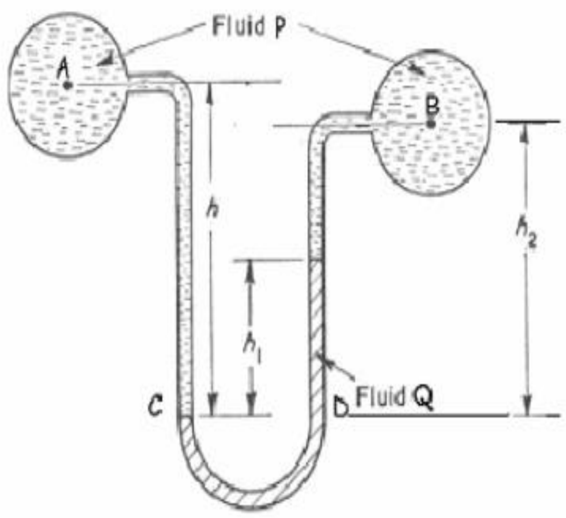


(b) A and B are at the same level

Problem:2

A U tube manometer measures the pressure difference between two points A and B in a liquid. The U tube contains mercury. Calculate the difference in pressure if h =1.5 m, h_2 = 0.75 m and h_1 = 0.5 m. The liquid at A and B is water ($\omega = 9.81 \times 10^3\ N/m^2$) and the specific gravity of mercury is 13.6.

Solution



Since C and D are at the same level in the same liquid at rest

Pressure p_P at C = Pressure p_Q at D

For the left hand limb

$$p_C = p_A + \omega h$$

For the right hand limb

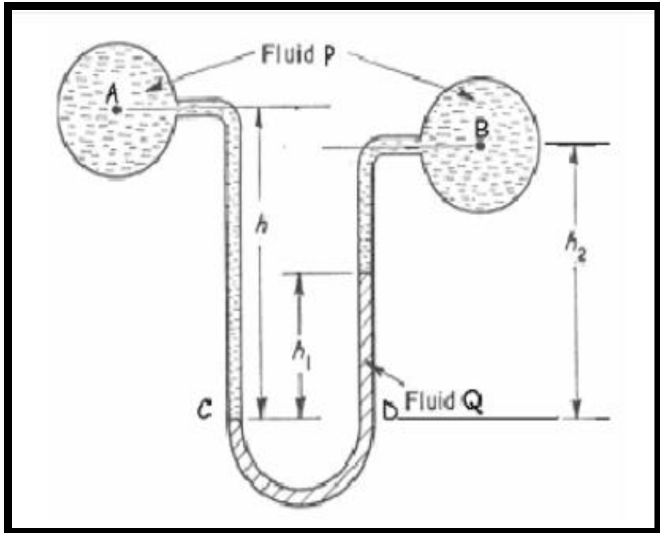
$$p_D = p_B + \omega(h_2 - h_1) + s\omega h_1$$

$$= p_B + \omega h_2 - \omega h_1 + s\omega h_1$$

Problem:2(Cont…)

since $p_C = p_D$

$$p_A + \omega h = p_B + \omega h_2 - \omega h_1 + s\omega h_1$$



Pressure difference $p_A - p_B$

$$= \omega h_2 - \omega h_1 + s\omega h_1 - \omega h$$

$$= \omega h_2 - \omega h + s\omega h_1 - \omega h_1$$

$$= \omega(h_2 - h) + \omega h_1 (s-1)$$

$$= 9.81\times10^3(0.75-1.5) + 9.81\times10^3(0.5)(13.6-1)$$

$$= 54445.5\ N/m^2 \qquad = \underline{54.44\ kN/m^2}$$

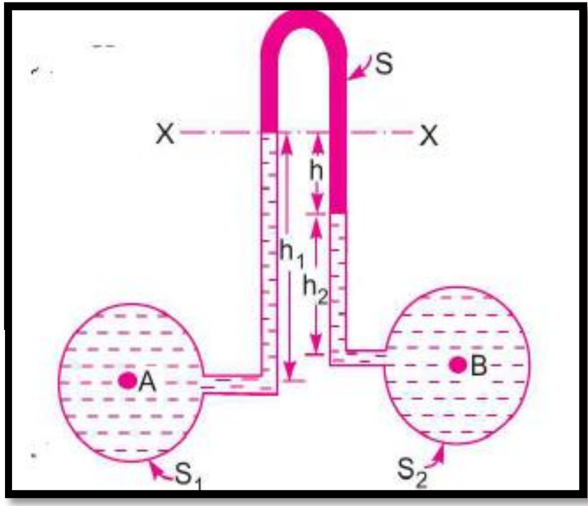
Inverted U-tube differential manometer

Taking X-X as datum line. Then pressure in the left limb below X-X

$$= p_A - \rho_1 \times g \times h_1.$$

Pressure in the right limb below X-X

$$= p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$



Equating the two pressure

$$p_A - \rho_1 \times g \times h_1 = p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

$$p_A - p_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h.$$

Problem:3

Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp. gr. 0.8 is connected. The pressure head in the pipe A is 2 m of water, find the pressure in the pipe B for the manometer readings as shown in Fig.

Solution. Given :

Pressure head at $A = \dfrac{p_A}{\rho g} = 2$ m of water

$\therefore$ $p_A = \rho \times g \times 2 = 1000 \times 9.81 \times 2 = 19620\ \text{N/m}^2$

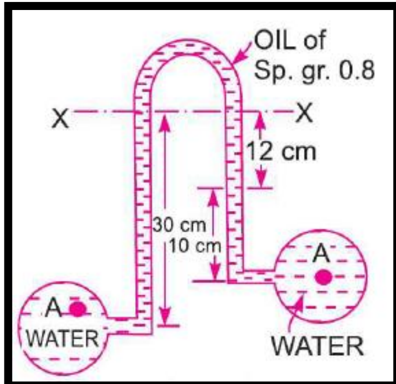


Fig. shows the arrangement. Taking *X-X* as datum line.

Pressure below *X-X* in the left limb $= p_A - \rho_1 \times g \times h_1$

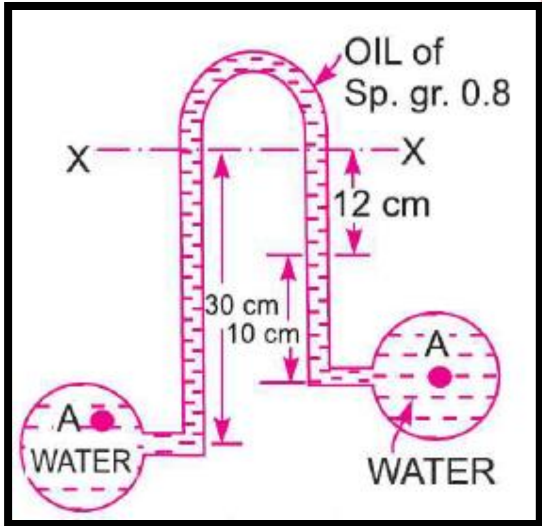
Problem:3(Cont...)

$$= 19620 - 1000 \times 9.81 \times 0.3 = 16677 \text{ N/m}^2.$$

Pressure below *X-X* in the right limb

$$= p_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12$$

$$= p_B - 981 - 941.76 = p_B - 1922.76$$



Equating the two pressure, we get

$$16677 = p_B - 1922.76$$

$$p_B = 16677 + 1922.76 = 18599.76 \text{ N/m}^2$$

$$p_B = \mathbf{1.8599\ N/cm^2.\ Ans.}$$

Summary

In this is lecture covered things like:

- ❑ Inclined single column manometer are used to measure small pressure differences
- ❑ U-tube differential manometers are used to measure the pressure difference between two pipes
- ❑ Solved some problems on single column manometer and U-tube differential manometers

FLUID KINEMATICS: PART-1

Presented By:
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Lecture Details:
Unit-2 , Fluid Kinematics
FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
- FLUID DYNAMICS
- CLOSED CONDUIT FLOW
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FLUID KINEMATICS: PART-1

Contents

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- What is Fluid Kinematics ?
- Methods of describing Fluid Motion
- Langrangian Method
- Eulerian Method
- Types of Fluid Flow
- Steady and Unsteady flows
- One, Two and Three Dimensional Flows
- Laminar and Turbulent flow
- Summary

What is Fluid Kinematics ?

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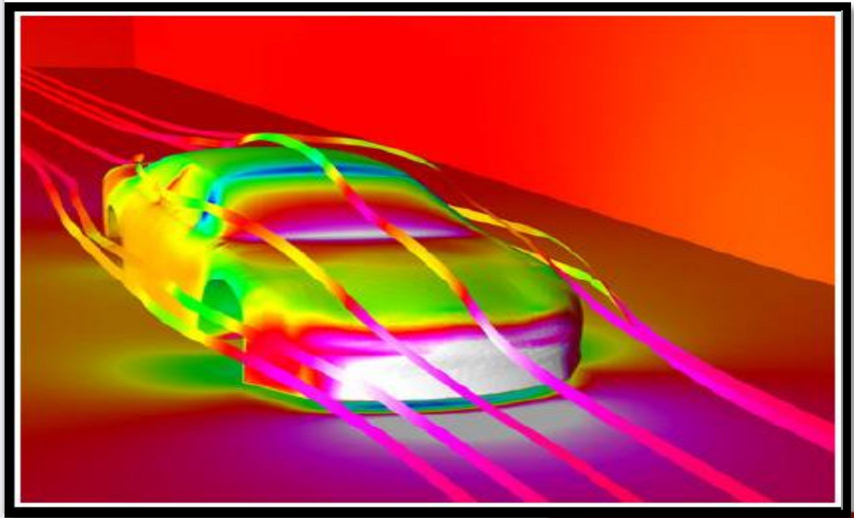
- Fluid kinematics deals with the motion of fluids without considering the forces and moments that cause the motion



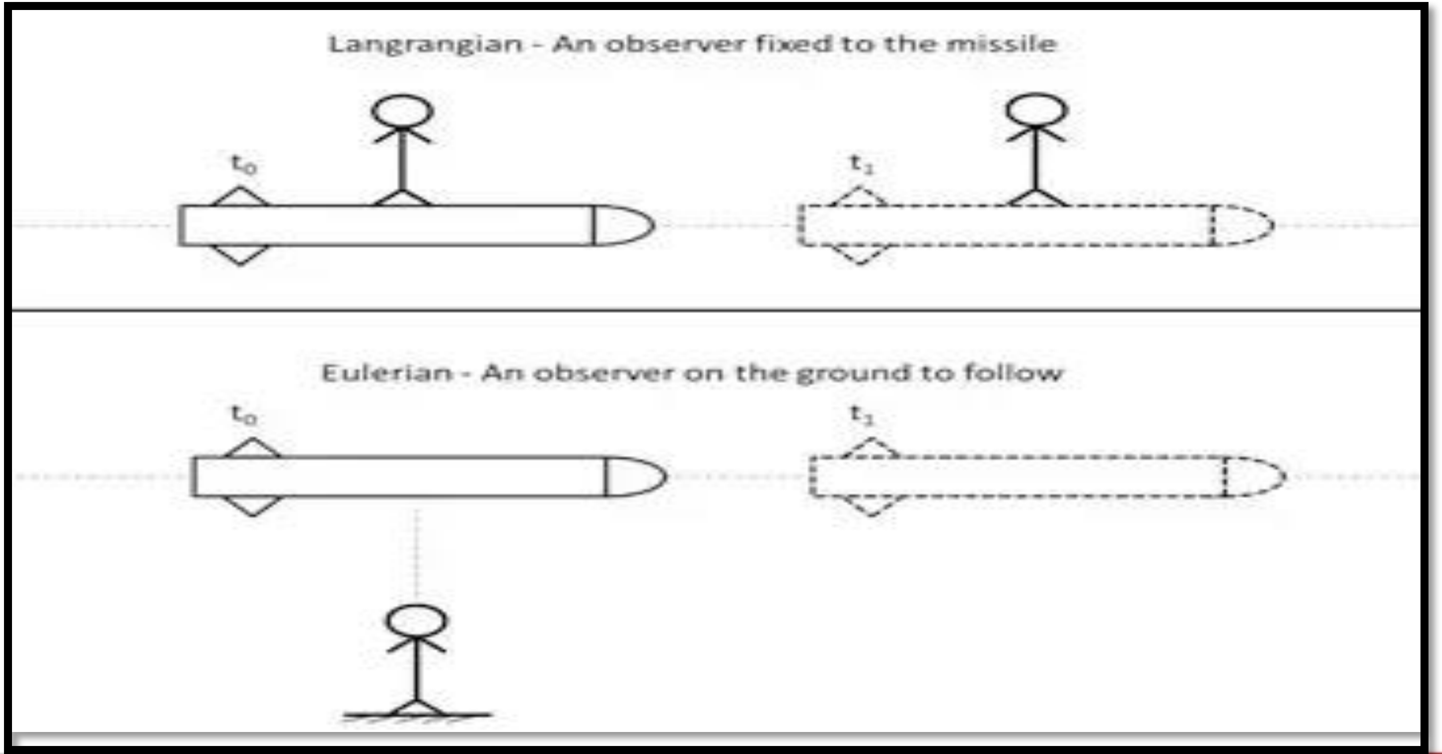
Methods of Describing Fluid Motion

Fluid motion is described by two methods

i) Langrangian Method

ii) Eulerian Method

Methods of Describing Fluid Motion (Cont…)

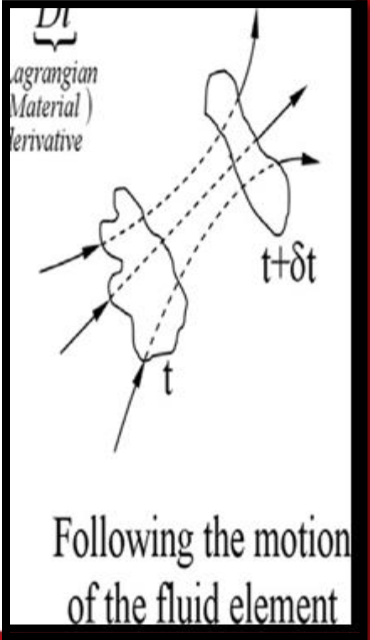


i) Langrangian Method

In the **Lagrangian** description of **fluid** **flow**, individual **fluid** particles are "marked," and their positions, velocities, acceleration, temperature and pressure etc. are described as a **function** of time

Examples:

- Track the location of a migrating bird
- Travel with the flow and observe what happens around you

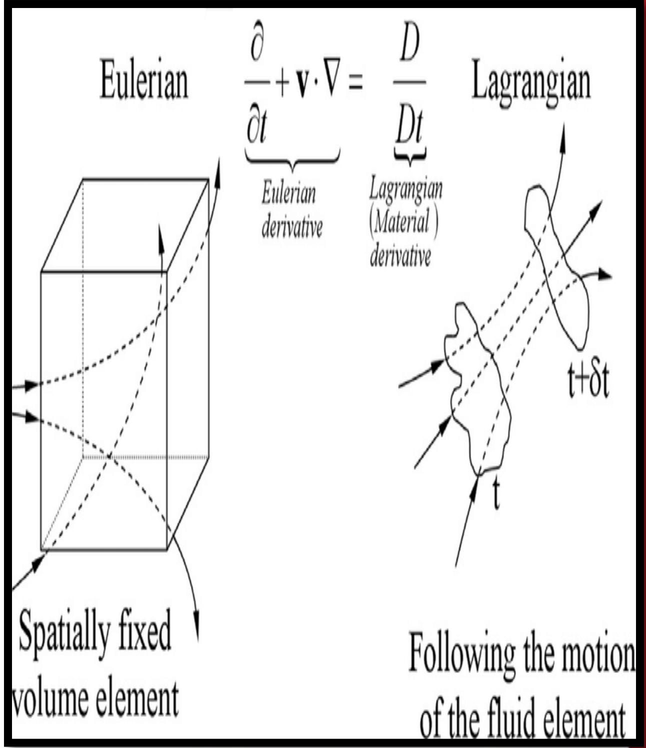


i) Langrangian Method(Cont…)

- This method requires us to track the position and velocity of each individual fluid particle
- If the number of objects is small, such as billiard balls on a pool table(as shown in side fig), individual objects can be tracked
- However, if a fluid lump changes its shape, size and state as its moves with time, it is difficult to trace the lump in Lagrangian description

ii) Eulerian Method

- To describe the fluid flow, a flow domain of a finite volume or control volume is defined, through which fluid flows in and out of the control volume
- Instead of tracking individual fluid particles, we define **field variables such as velocity, pressure** as functions of space and time, within the control volume

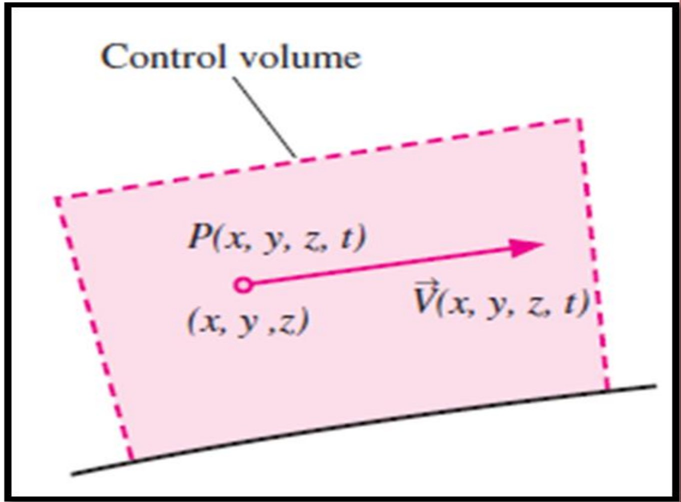


ii) Eulerian Method(Cont...)

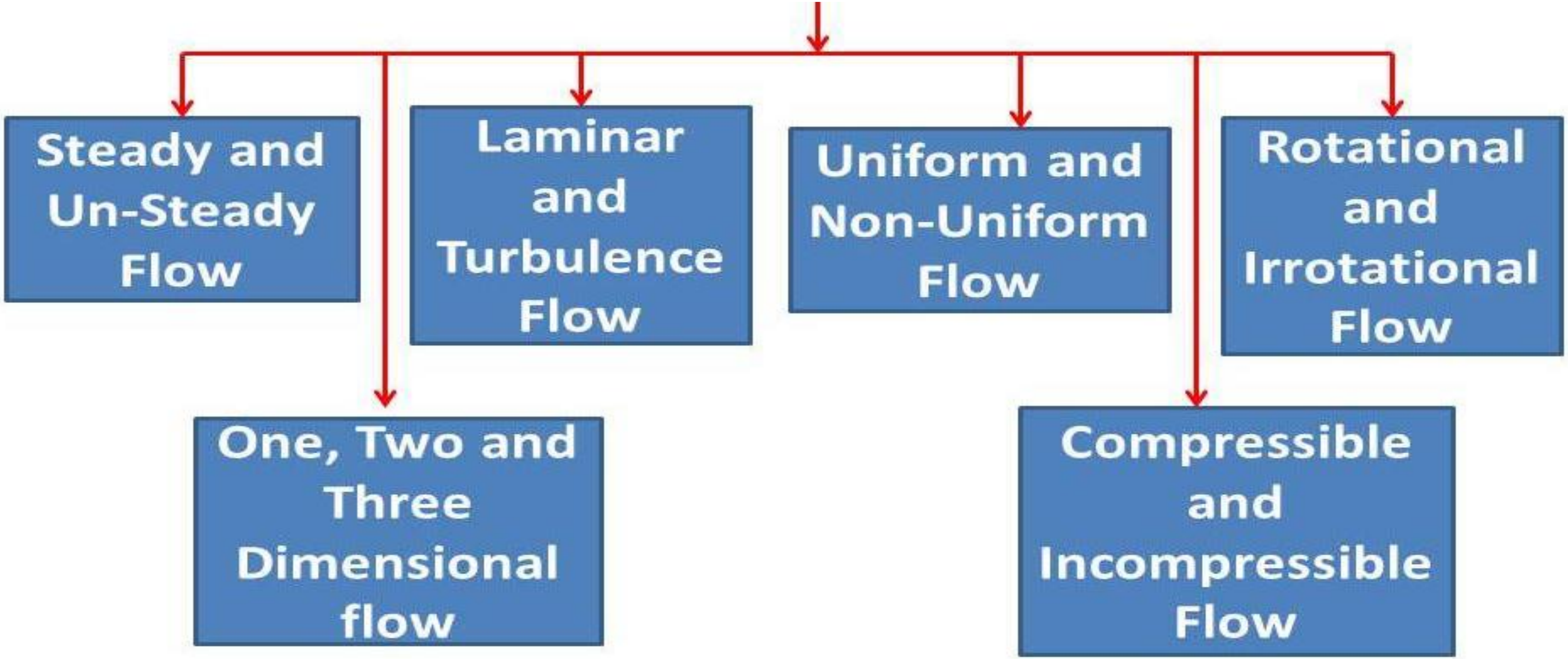
- Eulerian description is often more convenient for fluid mechanics applications
- Experimental measurements are generally more suited with Eulerian approach

Examples:

- Counting the birds passing a particular location
- Sit and observe a fixed area from a fixed point



Types of Fluid Flow



Steady and Unsteady flows

Steady flow:

It is the type of flow in which the fluid characteristics like velocity, pressure, density , etc., at a point do not change with time

Mathematically:

$$\left(\frac{\partial V}{\partial t}\right)_{x_0,y,z_0} = 0,$$

$$\left(\frac{\partial p}{\partial t}\right)_{x_0,y,z_0} = 0,$$

$$\left(\frac{\partial \rho}{\partial t}\right)_{x_0,y,z_0} = 0,$$

Where (x_0, y, z_0) fixed point in the field

Steady and Unsteady flows(Cont…)

Unsteady flow :

It is the type of flow in which the fluid characteristics like velocity, pressure, density , etc., at a point changes with time

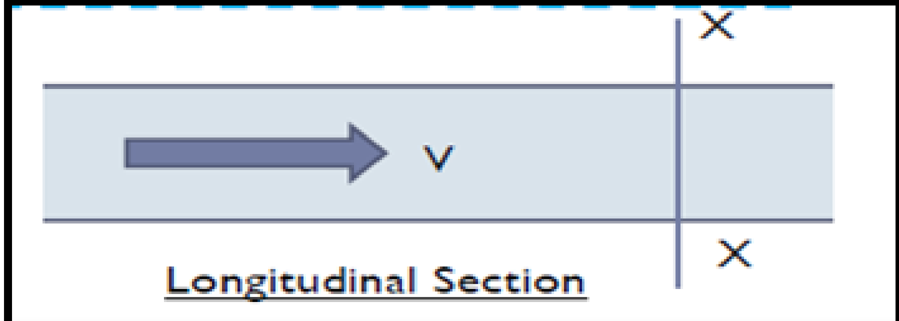
Mathematically:

$$\left(\frac{\partial V}{\partial t}\right)_{x_0,y,z_0} \neq 0,$$

$$\left(\frac{\partial p}{\partial t}\right)_{x_0,y,z_0} \neq 0,$$

$$\left(\frac{\partial \rho}{\partial t}\right)_{x_0,y,z_0} \neq 0,$$

Where (x_0, y, z_0) fixed point in the field



$$\frac{\partial V}{\partial t} \neq 0; \Rightarrow V = \text{variable}$$

One, Two and Three Dimensional Flows

- Although in general all fluids flow three-dimensionally
- Pressures and velocities and other flow properties varying in all directions, in many cases the greatest changes only occur in two directions or even only in one direction
- In these cases changes in the other direction can be effectively ignored, for making analysis much more simple

One, Two and Three Dimensional Flows(Cont…)

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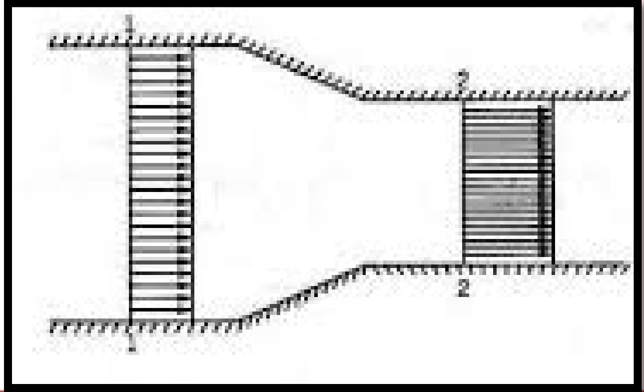
One-dimensional flow :

- *In which the flow parameter such as velocity is a function of time and one space co-ordinate only, say x*
- For a steady one-dimensional flow, the velocity is a function of one -space co-ordinate only
- The variation of velocities in other two mutually perpendicular directions is assumed negligible



Mathematically:

$$u = f(x),\ v = 0,\ w = 0$$

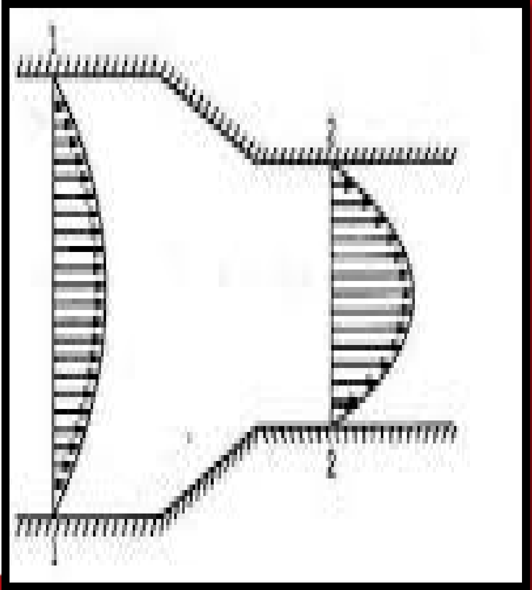
Where u, v and w are velocity components in x,y and z directions respectively

One, Two and Three Dimensional Flows(Cont…)

<u>**Two-dimensional flow:**</u>

- In which the velocity is a function of time and two rectangular space co-ordinates say x and y
- For a steady two dimensional flow the velocity is a function of two space co-ordinates only
- The variation of velocity in the third direction is negligible

Mathematically: $u = f_1(x,y)$, $v = f_2(x,y)$, and $w = 0$



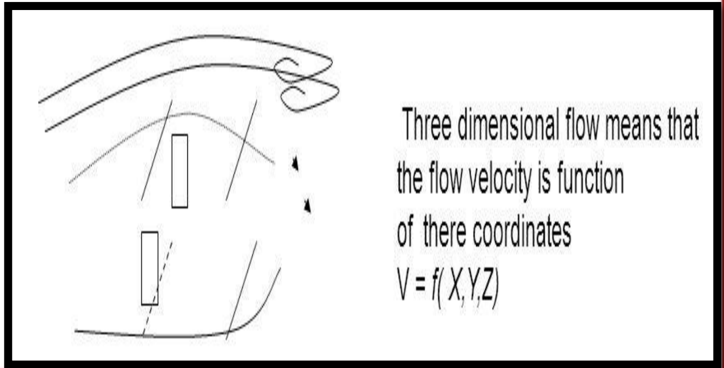
One, Two and Three Dimensional Flows(Cont…)

<u>Three – dimensional flow :</u>

- In which the velocity is a function of time and three mutually perpendicular directions
- But for a steady three – dimensional flow the fluid parameters are functions of three space co-ordinates (x, y and z) only

Mathematically:

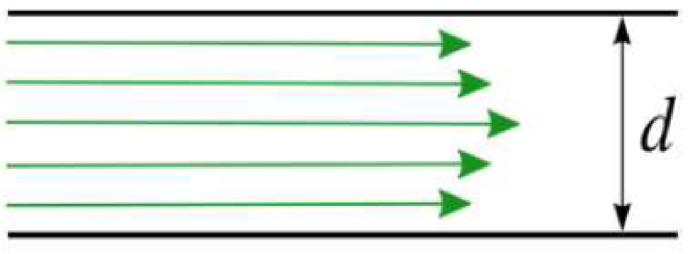
u = f_1(x,y,z), v= f_2(x,y,z), and w = f_3(x,y,z)



Laminar and Turbulent flow

Laminar flow :

- In which the fluid particles move along well- defined paths or stream line and all the stream- lines are straight and parallel
- Particles move in laminas or layers gliding smoothly over the adjacent layer
- This type of flow is also called stream-line flow or viscous flow

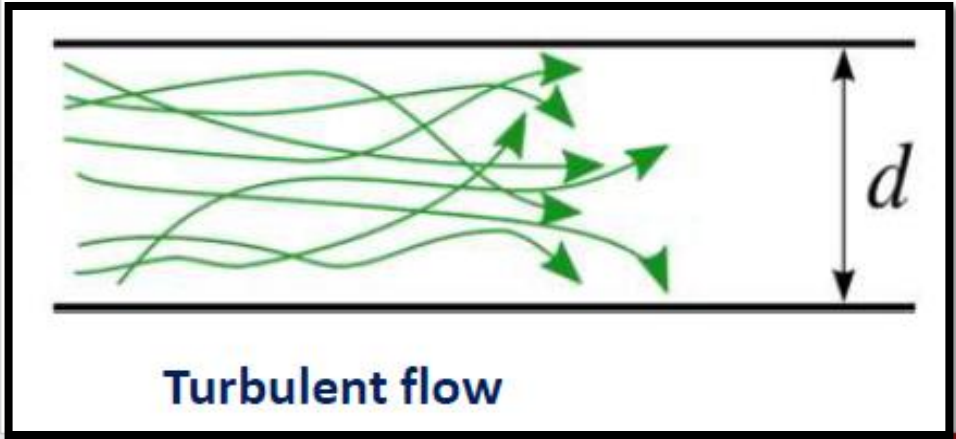


Laminar flow

Laminar and Turbulent flow(Cont…)

<u>Turbulent flow</u> :

- In which the fluid particles move in zig - zag way
- Due to this, the eddies formation takes place which are responsible for high energy loss



Laminar and Turbulent flow(Cont…)

- For pipe flow , the type of flow is determined by a non-dimensional number is called Reynold’s number(R_e)

$$R_e = \frac{VD}{v}$$

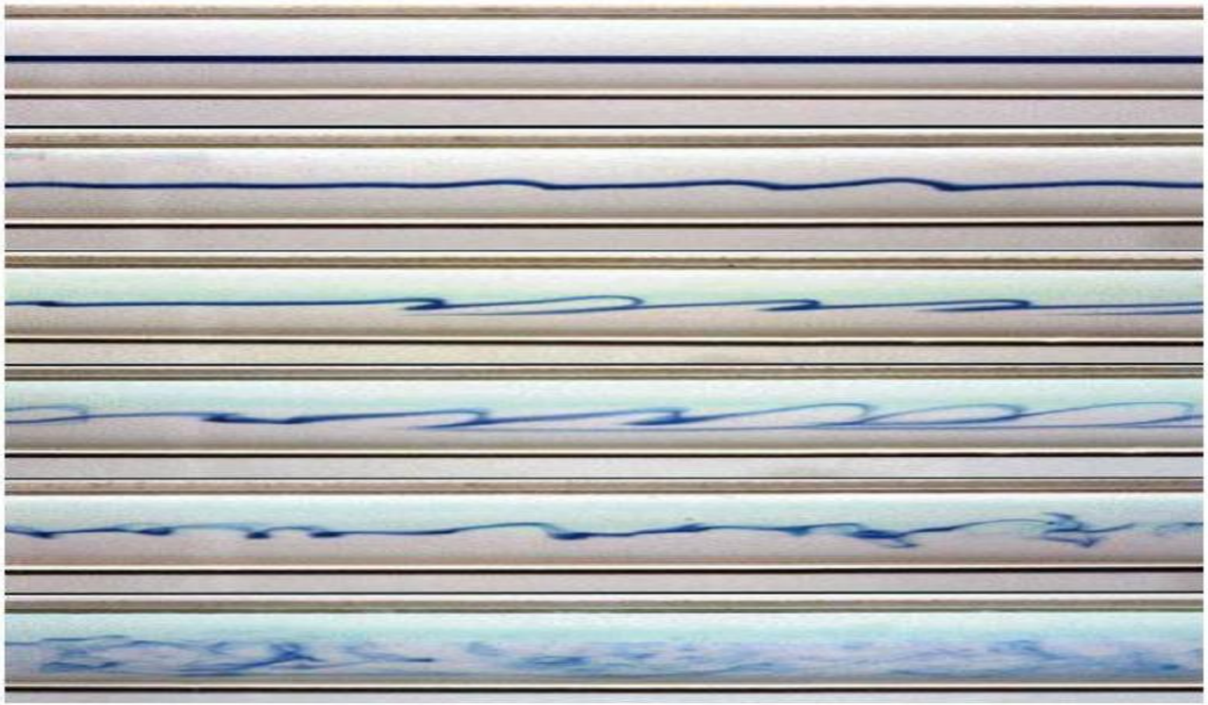
Where : D – Diameter of pipe, V- Mean velocity of flow in pipe, v – Kinematic viscosity of fluid

If $R_e < 2000$, then the flow is laminar flow

If $R_e > 4000$, then the flow is turbulent flow

If $2000 < R_e < 4000$, then the flow is transient flow

Laminar and Turbulent flow(Cont…)



Transition of flow from Laminar to turbulent

Summary

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- Fluid Motion is defined by two methods: i) **Langrangian Method ii) Eulerian Method**
- If flow properties do not change with time it is called steady flow
- If flow properties changes with time it is called unsteady flow
- For a steady one-dimensional flow, the velocity is a function of one –space –co-ordinate only
- For a steady two dimensional flow the velocity is a function of two space co-ordinates only
- But for a steady three – dimensional flow the fluid parameters are functions of three space co-ordinates (x, y and z) only
- In laminar flow all the stream- lines are straight and parallel
- In turbulent flow the fluid particles move in zig - zag way

FLUID KINEMATICS: PART-2

Presented By:

Shaik Nayeem

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Mechanical Engineering

GIET(A)

Lecture Details:

Unit-2 , Fluid Kinematics

FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
  - FLUID KINEMATICS
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FLUID KINEMATICS:
PART-2

Contents

- Uniform and Non-uniform flow
- Compressible and Incompressible flows
- Rotational & Irrotational Flow
- Visualization of flow Pattern
- Stream line
- Path Line
- Streak line
- Stream Tubes
- Comparison
- Summary

Uniform and Non-uniform flow

Uniform flow:

The flow in which velocity at a given time does not change with respect to space (i.e., length of direction of flow) is called as uniform flow

Mathematically:

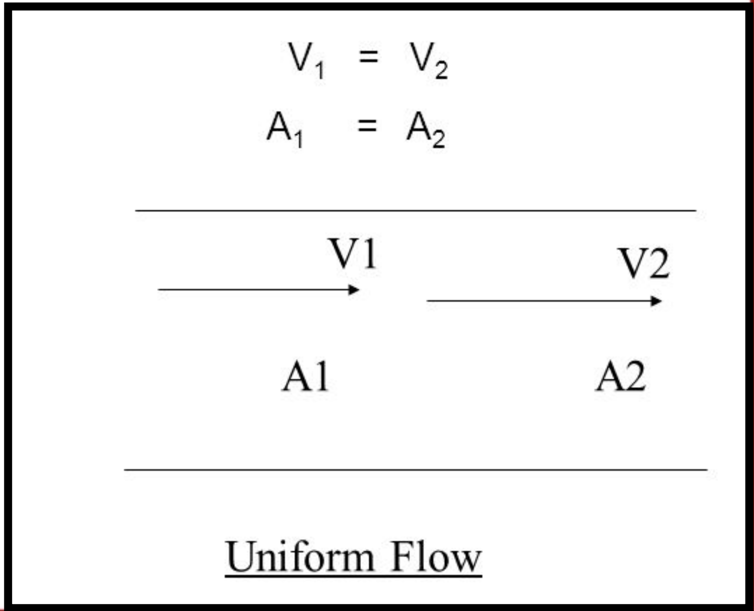
$$\left(\frac{\partial V}{\partial s}\right)_{t=constant} = 0,$$

Where

∂V - Change of velocity

∂s – Length of flow in the direction of S

e.g. Constant discharge though a constant diameter pipe



Uniform and Non-uniform flow(Cont..)

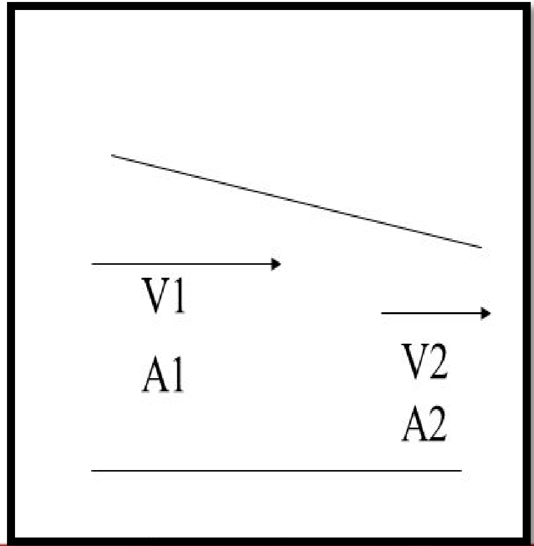
Non – Uniform flow :

The flow in which velocity at a given time changes with respect to space (length of direction of flow) is called as non – uniform flow

Mathematically:

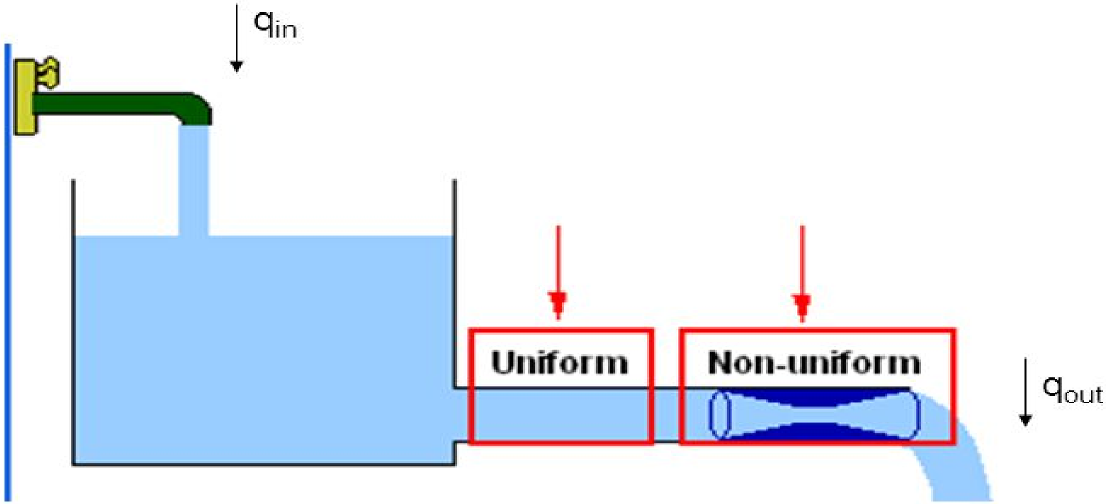
$$\left(\frac{\partial V}{\partial s}\right)_{t=constant} \neq 0,$$

e.g., Constant discharge through variable diameter pipe



Uniform and Non-uniform flow(Cont..)

- Flow could be steady if $q_{in} = q_{out}$
- In the non-uniform flow, the fluid acceleration is ***not*** equal to zero



Compressible and Incompressible flows

Compressible Flow:

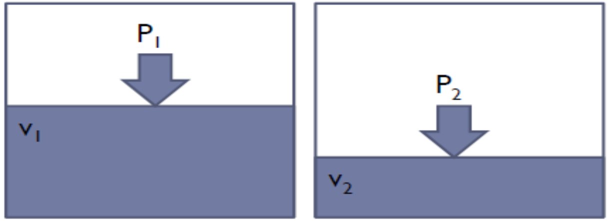
In which density of the fluid changes from point to point

or

Density(ρ) is not constant for the fluid

Mathematically:

$$\rho \neq \textbf{Contant}$$



Compressible fluid

Compressible and Incompressible flows(Cont…)

Incompressible flow

- In which the density is constant for the fluid flow
- Liquids are generally incompressible while gases are compressible

Mathematically:

$$\boldsymbol{\rho} = \textbf{Contant}$$

Rotational & Irrotational Flow

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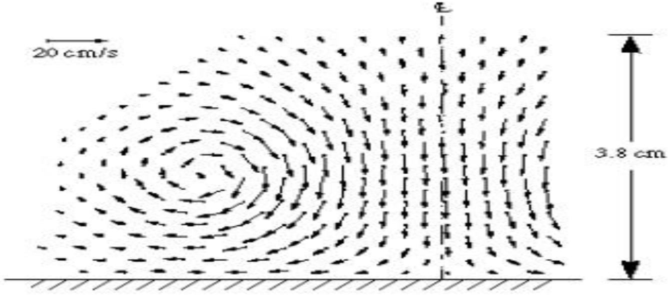
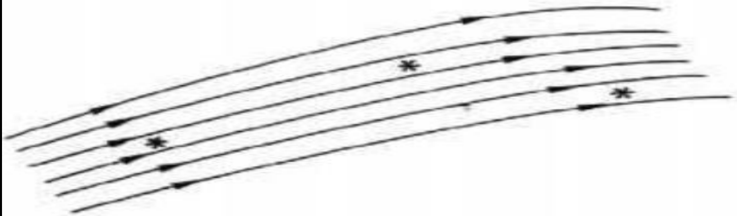
<u>Rotational Flows :-</u>

The flow in which fluid particle while flowing along stream lines, also rotate about their own axis is called as rotational flow



<u>Irrotational Flows:-</u>

The flow in which the fluid particle while flowing along stream lines, do not rotate about their axis is called as irrotational flow



Visualization of flow Pattern

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- In order to visualize the flow pattern it is useful to define some other properties of the flow
- They are :

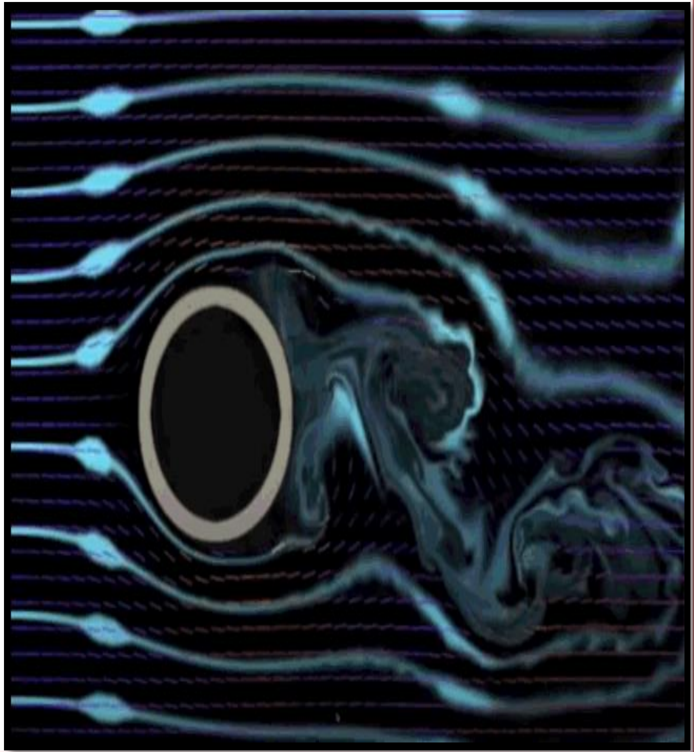
  1. **Stream lines**
  2. **Path lines**
  3. **Streak lines**

Stream line

- Stream lines are a family of curves that are instantaneously tangent to the velocity vector of the flow

or

- The flow in which every particle that passes a particular point moves along exactly the same path, as followed by the particles which passed the point earlier



Stream line(Cont...)

Stream line(Cont...)

Characteristics of Streamline :

- Streamlines can not cross each other
- Streamline can't be a folding line, but a smooth curve
- Streamline cluster density reflects the magnitude of velocity (Dense streamlines mean large velocity; while sparse streamlines mean small velocity)

Stream line(Cont…)

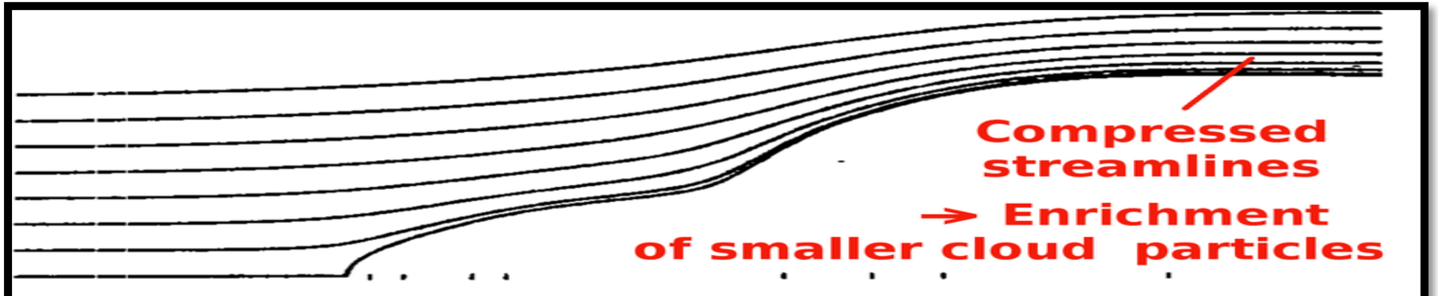
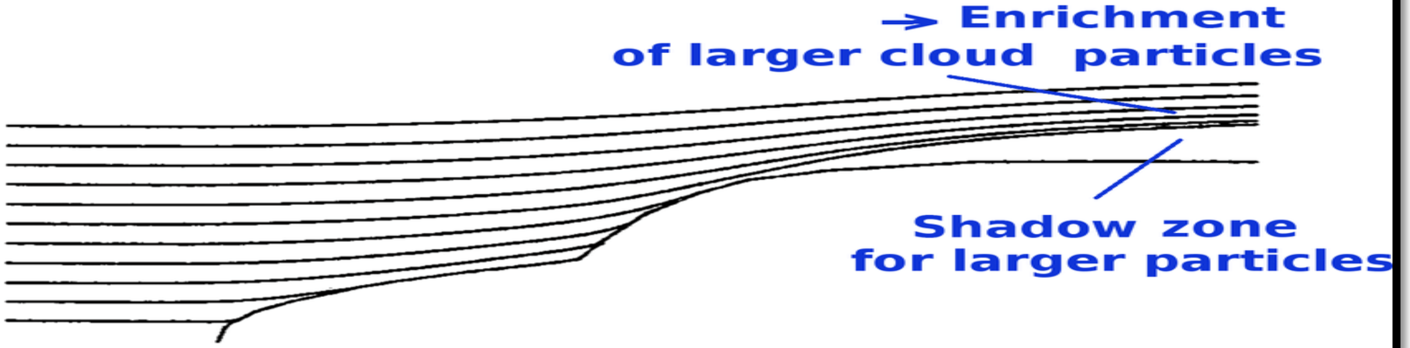


FIG. 4. Streamlines around the simulated aircraft fusalage of F-27.

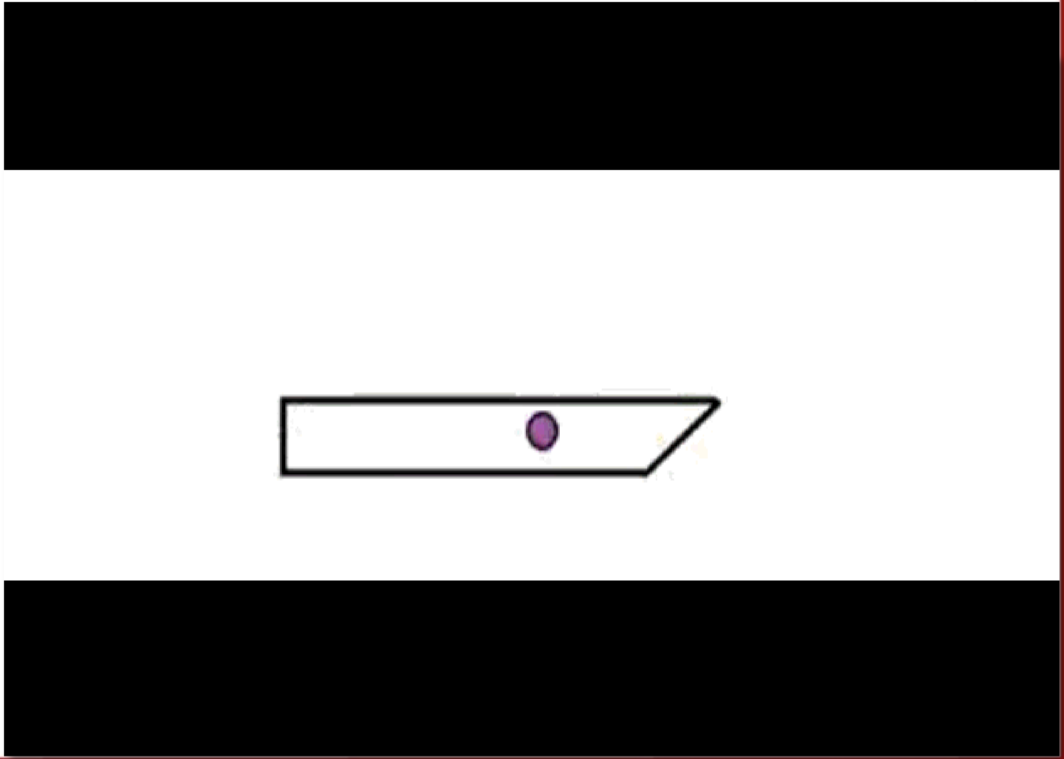


Trajectories around the F-27 for water drops of diameter 100 μm travelling at 90 m s^{-1}, $\eta = 1.75 \times 10^{-5}$ kg s^{-1}.

Path Line

- A Path line is the actual path travelled by an individual fluid particle over some time period

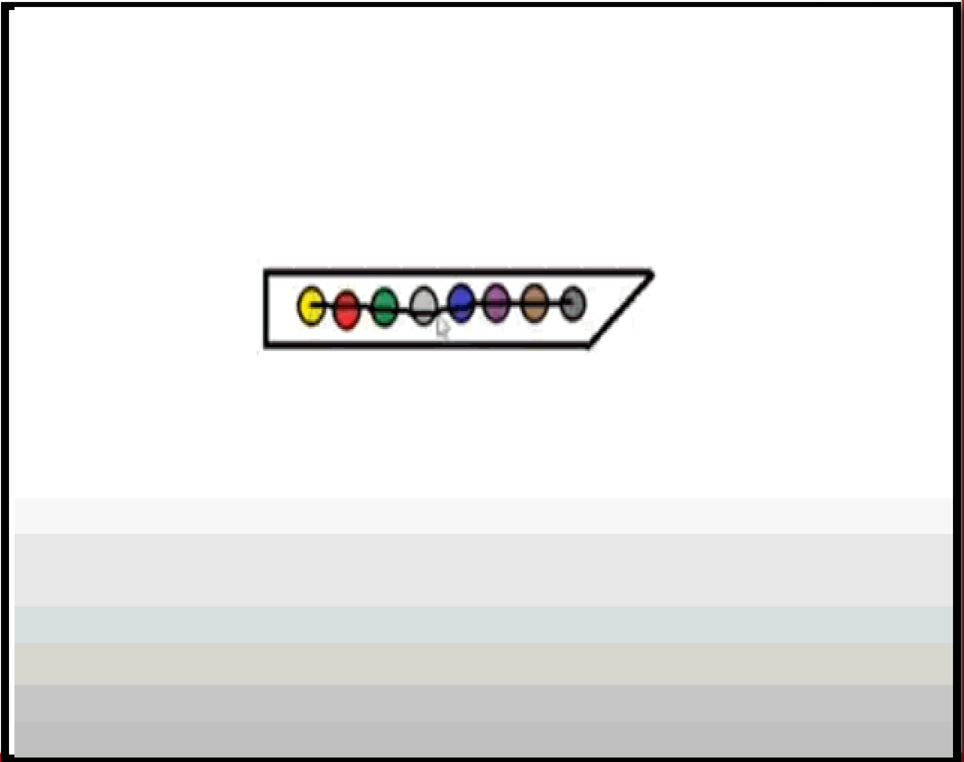
- And the path of a particle same as Streamline for Steady Flow



Streak line

- It is an instantaneous picture of the position of all particles in flow that have passed through a given point

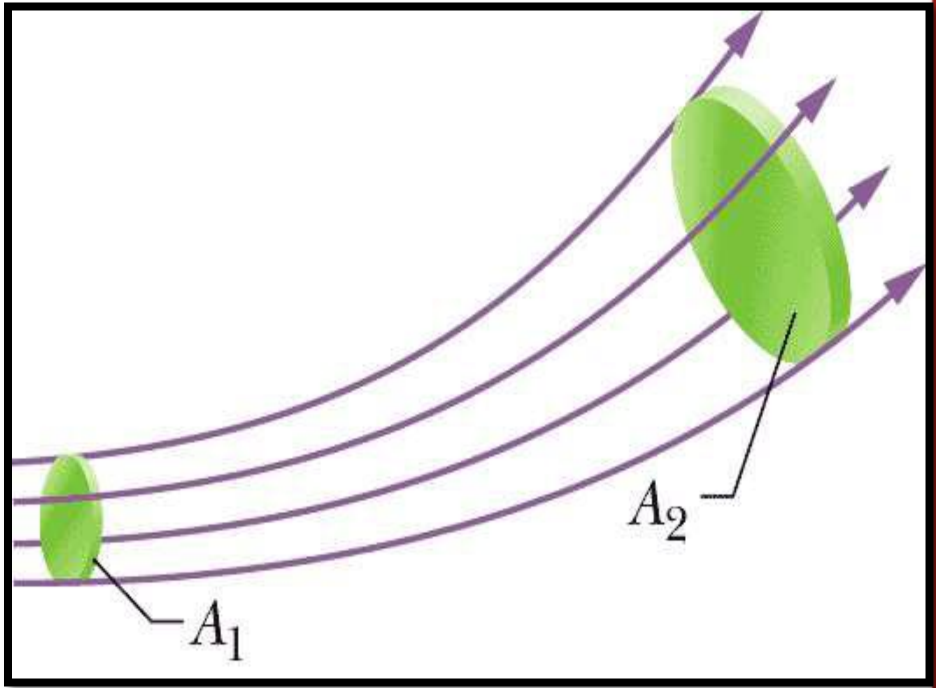
- Easy to generate in experiments like dye in a water flow, or smoke in an airflow



Stream Tubes

Stream tube :

- Is an imaginary tube whose boundary consists of streamlines
- The volume flow rate must be the same for all cross sections of the stream tube



Comparison

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Stream Lines	Streak Lines	Path Lines
1) Stream Lines are Imaginary, can be mathematically expressed	1) Streak lines are real lines executed by the stream or chain of fluid particles emanating from their source.	1) The path lines are the actual path traced by every single particle of the flow.
2) The tangent to any point of stream line gives the direction of flow of fluid in that direction.	2) Two streak lines can intersect with each other.	2) Two path lines can also intersect each other.
3) Two stream lines can never intersect each other	3) In stream line flow the stream lines and streak lines do coincide with each other.	3) In streamline flow the path lines and stream lines do coincide with each other.
4) when the fluid particles exactly follows the stream lines then the flow is called as streamline flow		

Summary

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- The velocity at a given time does not change with respect to space is called as uniform flow
- The velocity at a given time changes with respect to space is called as non – uniform flow
- In compressible flow density of the fluid changes from point to point
- In incompressible flow the density is constant
- The flow in which fluid particle while flowing along stream lines, also rotate about their own axis is called as rotational flow
- The flow in which the fluid particle while flowing along stream lines, do not rotate about their axis is called as irrotational flow
- A **streamline** is a path traced out by a massless particle as it moves with the flow
- A Path line is the actual path travelled by an individual fluid particle over some time period

FLUID KINEMATICS: PART-3

Presented By:

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Lecture Details:

Unit-2 , Fluid Kinematics

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FLUID KINEMATICS: PART-3

Contents

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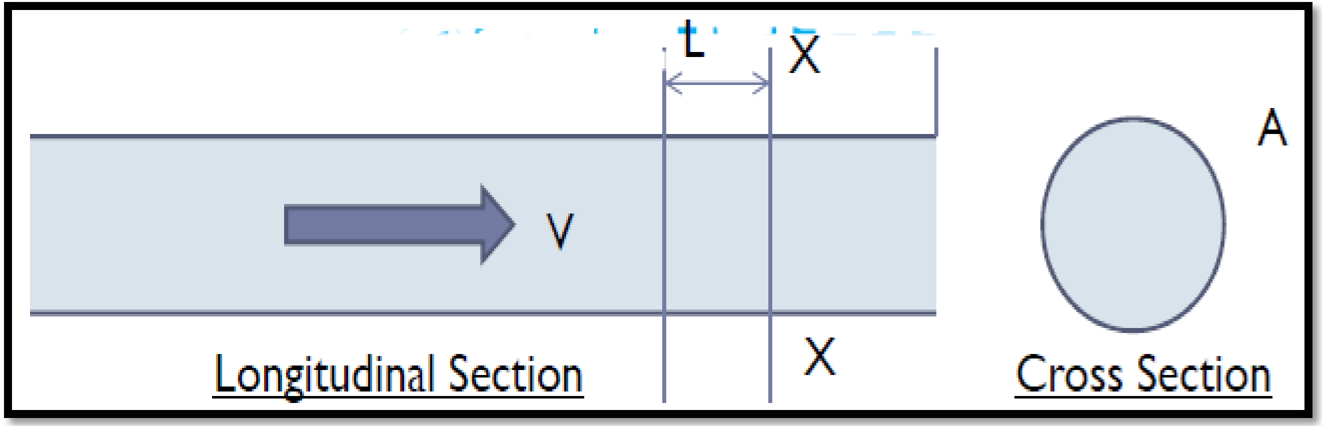
- Fluid Flow
- Continuity
- Continuity Equation
- Summary

Fluid Flow



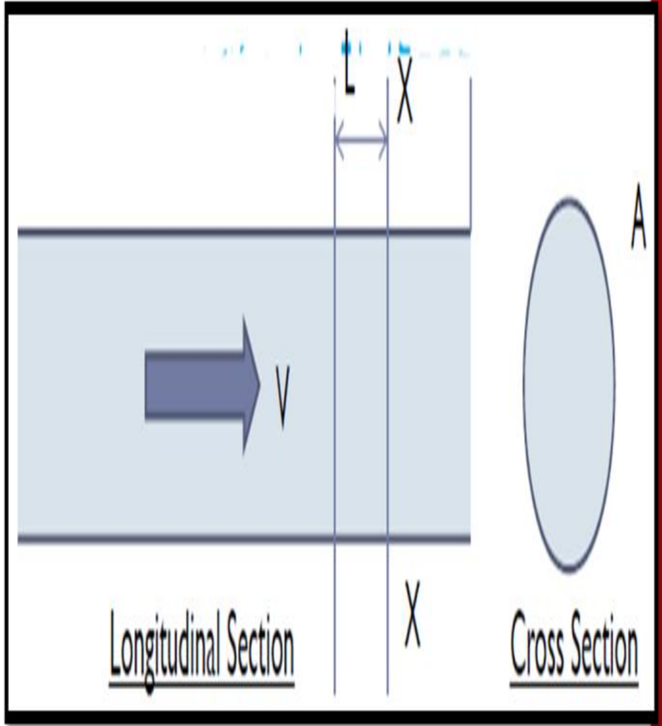
- Let's consider a pipe in which a fluid is flowing with mean velocity, V, in unit time, t, volume of fluid (AL) passes through section X-X,

Fluid Flow(Cont…)

1. Volume flow rate: Q = $\frac{volume\ of\ fluid}{time} = \frac{AL}{t}$

2. Mass flow rate: M = $\frac{Mass\ of\ fluid}{time} = \frac{\rho(AL)}{t}$

3. Weight flow rate: G = $\frac{Weight\ of\ fluid}{time} = \frac{\rho g(AL)}{t} = \frac{\omega(AL)}{t}$



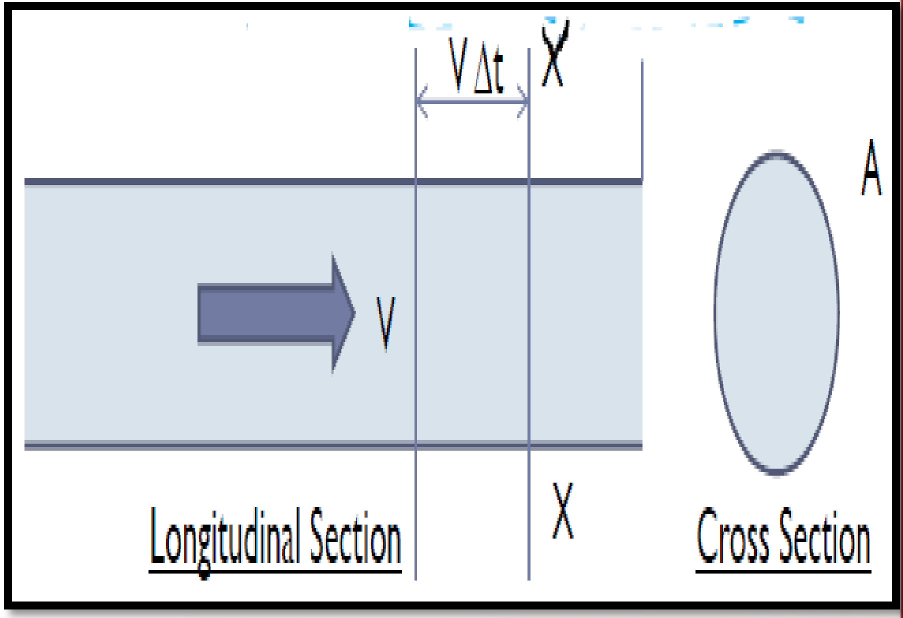
Fluid Flow(Cont…)

➢ Let's consider a fluid flowing with mean velocity, V, in a pipe of uniform cross-section

➢ Thus volume of fluid that passes through section X-X in unit time Δt , becomes (Here L= $\mathrm{V}\Delta t$) as

Volume of fluid $=$LA

$$=(\mathrm{V}\Delta t)\ \mathrm{A}$$



Fluid Flow(Cont...)

Volume flow rate: Q = $\frac{volume\ of\ fluid}{time}$

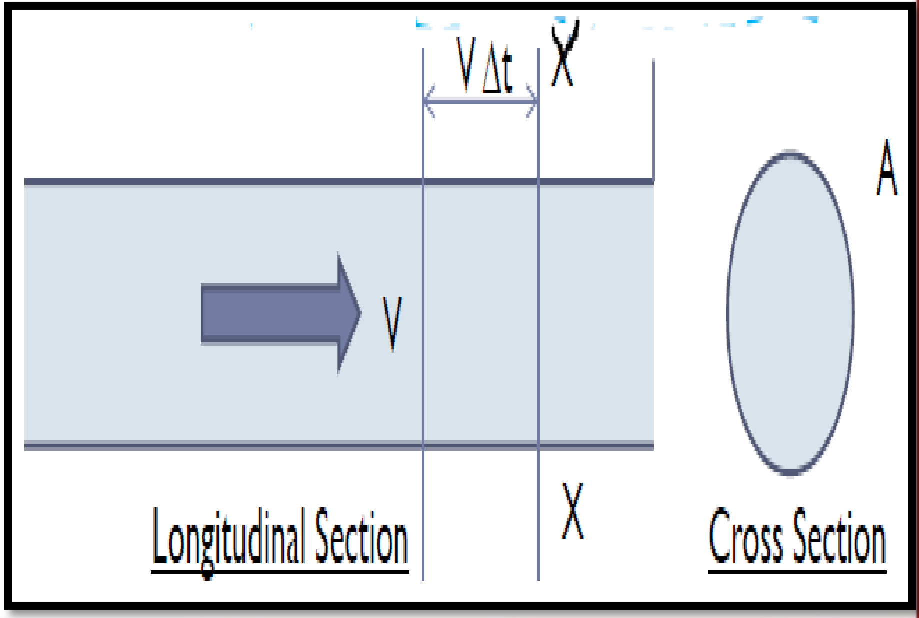
$$= \frac{(\Delta t V\ )\ A}{\Delta t}$$

Q = AV

Similarly

$$M = \frac{\rho(AL)}{\Delta t} = \rho AV$$

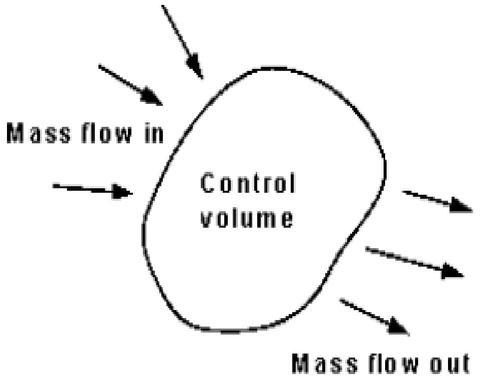
$$G = \frac{\omega(AL)}{\Delta t} = \omega AV$$



Continuity

- Matter cannot be created or destroyed - (it is simply changed in to a different form of matter)
- This principle is known as the *conservation of mass* and we use it in the analysis of flowing fluids
- The principle is applied to fixed volumes, known as control volumes as shown in figure
- For any control volume the principle of conservation of mass says

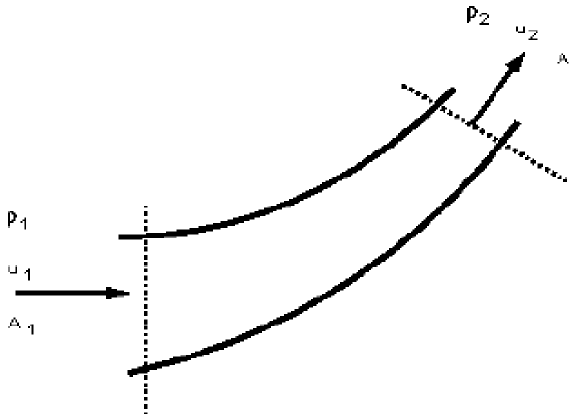
Mass entering per unit time -Mass leaving per unit time = Increase of mass in the control volume per unit time



An arbitrarily shaped control volume.

Continuity Equation

- For steady flow there is no increase in the mass within the control volume
- So, **Mass entering per unit time = Mass leaving per unit time**
- Lets consider a stream tube
- ρ_1, v_1 and A_1 are mass density, velocity and cross sectional area at section1
- Similarly ρ_2, v_2 and A_2 are mass density, velocity and cross – sectional area at section 2



A stream tube

$$M_1 = \rho_1 A_1 V_1$$
$$M_2 = \rho_2 A_2 V_2$$

Continuity Equation(Cont...)

According to mass conservation

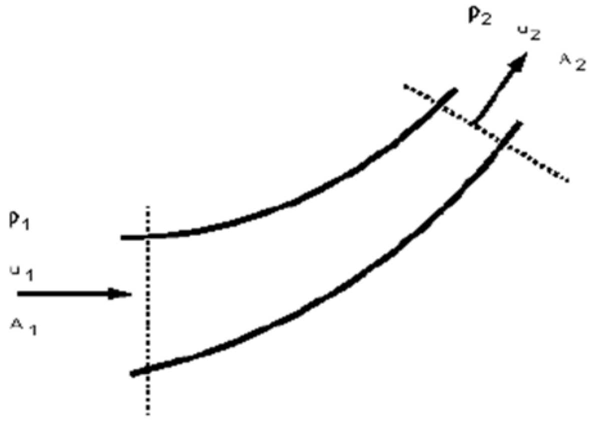
$$M_1 - M_2 = \frac{dM_{CV}}{dt}$$

$$\rho_1\, A_1\, V_1 - \rho_2\, A_2\, V_2 = \frac{dM_{CV}}{dt}$$

For steady flow condition $\frac{dM_{CV}}{dt} = 0$

$$\rho_1\, A_1\, V_1 - \rho_2\, A_2\, V_2 = 0$$

$$\rho_1\, A_1\, V_1 = \rho_2\, A_2\, V_2$$



A stream tube

$$M_1 = \rho_1 A_1 V_1$$
$$M_2 = \rho_2 A_2 V_2$$

Continuity Equation(Cont…)

$$M = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

- Hence, for stead flow condition, mass flow rate at section 1= mass flow rate at section 2. i.e., mass flow rate is constant
- Assuming incompressible fluid, $\rho_1 = \rho_2 = \rho$

$$A_1 V_1 = A_2 V_2$$

$$Q_1 = Q_2$$

- Therefore, according to mass conservation for steady flow of incompressible fluids volume flow rate remains same from section to section. ($Q_1 = Q_2 = Q_3 = Q_4$ etc)

Problem:1

The diameters of a pipe at the sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5 m/s. Determine also the velocity at section 2.

Solution. Given :

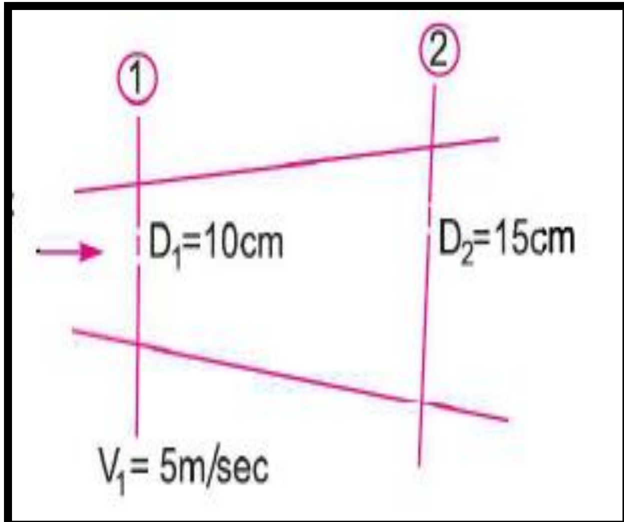
At section 1, $D_1 = 10 \text{ cm} = 0.1 \text{ m}$

$$A_1 = \frac{\pi}{4}(D_1^2) = \frac{\pi}{4}(.1)^2 = 0.007854 \text{ m}^2$$

$V_1 = 5 \text{ m/s.}$

At section 2, $D_2 = 15 \text{ cm} = 0.15 \text{ m}$

$$A_2 = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$$



Problem:1(Cont…)

(*i*) Discharge through pipe is given by equation (

or $$Q = A_1 \times V_1$$

$$= 0.007854 \times 5 = \mathbf{0.03927\ m^3/s.\ Ans.}$$

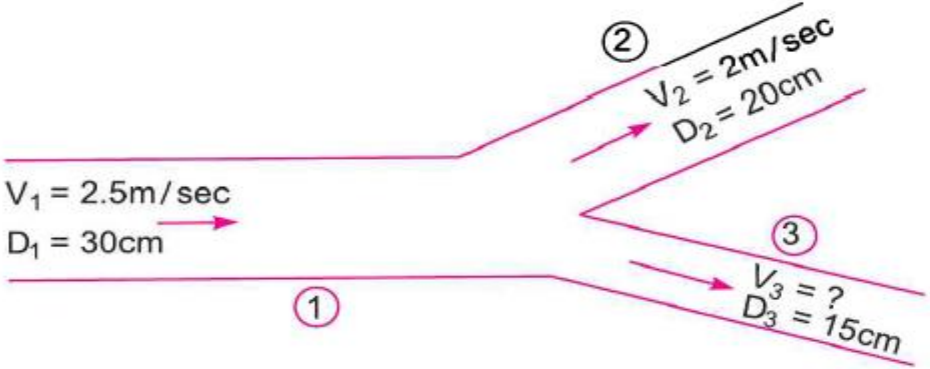
, we have $A_1V_1 = A_2V_2$

(*ii*) $\therefore$ $$V_2 = \frac{A_1V_1}{A_2} = \frac{0.007854}{0.01767} \times 5.0 = \mathbf{2.22\ m/s.\ Ans.}$$

Problem:2

A 30 cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.

Solution. Given :



Problem:2(Cont...)

$$D_1 = 30 \text{ cm} = 0.30 \text{ m}$$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times .3^2 = 0.07068 \text{ m}^2$$

$$V_1 = 2.5 \text{ m/s}$$

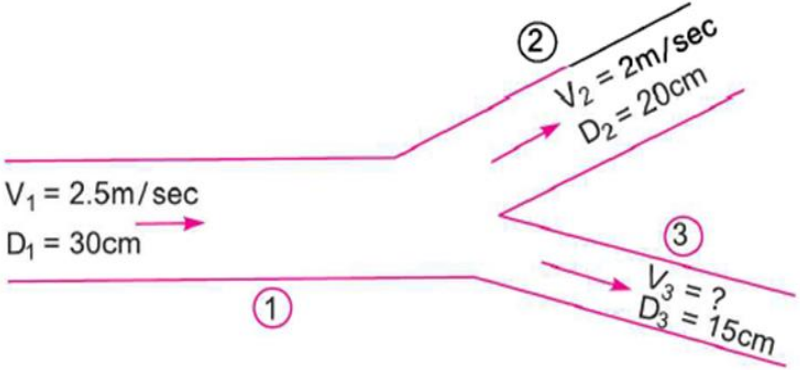
$$D_2 = 20 \text{ cm} = 0.20 \text{ m}$$

$$A_2 = \frac{\pi}{4} (.2)^2 = \frac{\pi}{4} \times .4 = 0.0314 \text{ m}^2,$$

$$V_2 = 2 \text{ m/s}$$

$$D_3 = 15 \text{ cm} = 0.15 \text{ m}$$

$$A_3 = \frac{\pi}{4} (.15)^2 = \frac{\pi}{4} \times 0.225 = 0.01767 \text{ m}^2$$



Problem:2(Cont…)

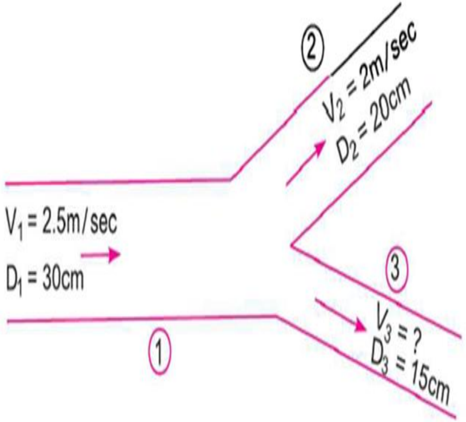
Find (*i*) Discharge in pipe 1 or Q_1

(*ii*) Velocity in pipe of dia. 15 cm or V_3

Let Q_1, Q_2 and Q_3 are discharges in pipe 1, 2 and 3 respectively.

Then according to continuity equation

$$Q_1 = Q_2 + Q_3$$



(*i*) **The discharge Q_1** in pipe 1 is given by

$$Q_1 = A_1V_1 = 0.07068 \times 2.5 \text{ m}^3/\text{s} = \mathbf{0.1767 \text{ m}^3/\text{s}. \text{ Ans.}}$$

(*ii*) **Value of V_3**

$$Q_2 = A_2V_2 = 0.0314 \times 2.0 = 0.0628 \text{ m}^3/\text{s}$$

Problem:2(Cont...)

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Substituting the values of Q_1 and Q_2 in equation (

$$0.1767 = 0.0628 + Q_3$$

$$\therefore \quad Q_3 = 0.1767 - 0.0628 = 0.1139 \text{ m}^3/\text{s}$$

But $\quad Q_3 = A_3 \times V_3 = 0.01767 \times V_3 \quad \text{or} \quad 0.1139 = 0.01767 \times V_3$

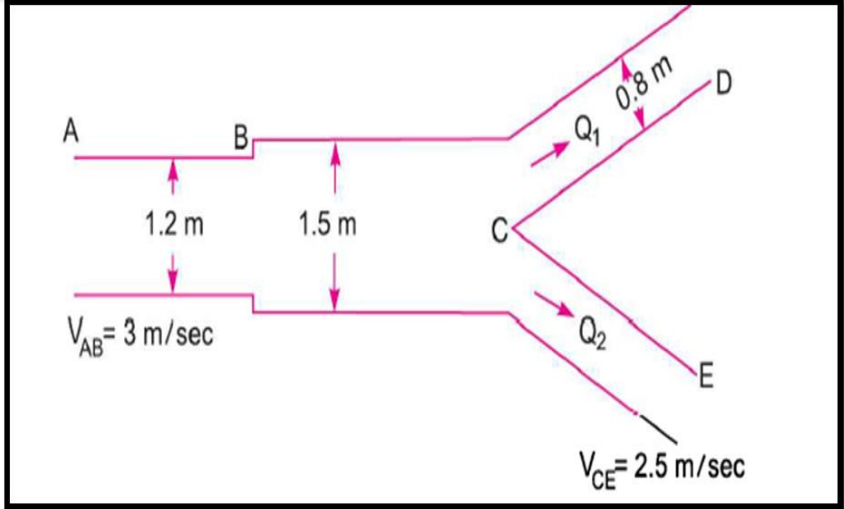
$$\therefore \quad V_3 = \frac{0.1139}{0.01767} = \mathbf{6.44\ m/s.\ Ans.}$$

Problem:3

Water flows through a pipe AB 1.2 m diameter at 3 m/s and then passes through a pipe BC 1.5 m diameter. At C, the pipe branches. Branch CD is 0.8 m in diameter and carries one-third of the flow in AB. The flow velocity in branch CE is 2.5 m/s. Find the volume rate of flow in AB, the velocity in BC, the velocity in CD and the diameter of CE.

Solution. Given :

Diameter of pipe *AB*,	$D_{AB} = 1.2$ m
Velocity of flow through *AB*,	$V_{AB} = 3.0$ m/s
Dia. of pipe *BC*,	$D_{BC} = 1.5$ m
Dia. of branched pipe *CD*,	$D_{CD} = 0.8$ m
Velocity of flow in pipe *CE*,	$V_{CE} = 2.5$ m/s
Let the flow rate in pipe	$AB = Q$ m^3/s
Velocity of flow in pipe	$BC = V_{BC}$ m/s
Velocity of flow in pipe	$CD = V_{CD}$ m/s

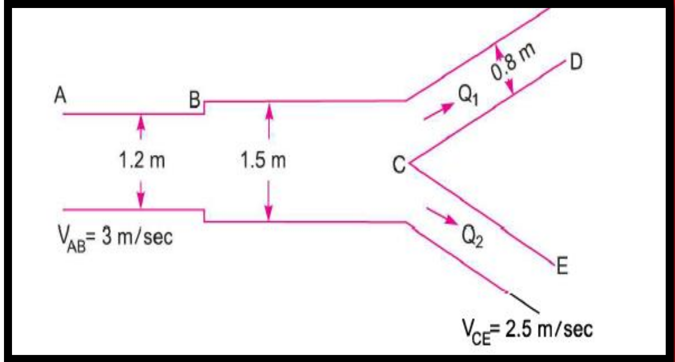


Problem:3(Cont...)

Diameter of pipe $CE = D_{CE}$

Then flow rate through $CD = Q/3$

and flow rate through $CE = Q - Q/3 = \dfrac{2Q}{3}$



(*i*) Now volume flow rate through $AB = Q = V_{AB} \times$ Area of AB

$$= 3.0 \times \frac{\pi}{4}(D_{AB})^2 = 3.0 \times \frac{\pi}{4}(1.2)^2 = \mathbf{3.393\ m^3/s.\ Ans.}$$

Problem:3(Cont…)

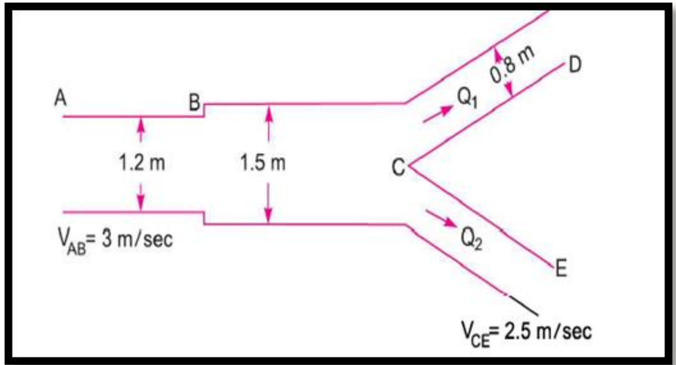
(*ii*) Applying continuity equation to pipe *AB* and pipe *BC*,

$$V_{AB} \times \text{Area of pipe } AB = V_{BC} \times \text{Area of pipe } BC$$

or $$3.0 \times \frac{\pi}{4}(D_{AB})^2 = V_{BC} \times \frac{\pi}{4}(D_{BC})^2$$

or $$3.0 \times (1.2)^2 = V_{BC} \times (1.5)^2 \qquad \left[\text{Divide by } \frac{\pi}{4}\right]$$

or $$V_{BC} = \frac{3 \times 1.2^2}{1.5^2} = \mathbf{1.92\ m/s.\ Ans.}$$



Problem:3(Cont…)

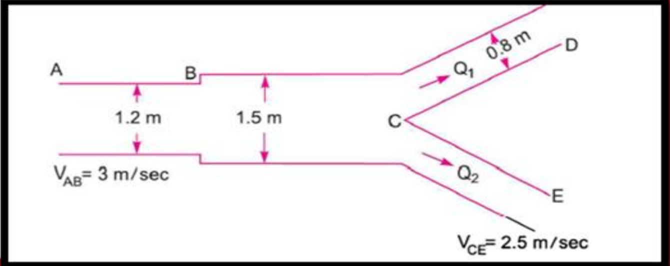
(*iii*) The flow rate through pipe

$$CD = Q_1 = \frac{Q}{3} = \frac{3.393}{3} = 1.131 \text{ m}^3/\text{s}$$

$$\therefore \qquad Q_1 = V_{CD} \times \text{Area of pipe } CD \times \frac{\pi}{4}(D_{CD})^2$$

$$1.131 = V_{CD} \times \frac{\pi}{4} \times 0.8^2 = 0.5026\ V_{CD}$$

$$\therefore \qquad V_{CD} = \frac{1.131}{0.5026} = \mathbf{2.25\ m/s.\ Ans.}$$



Problem:3(Cont…)

(*iv*) Flow rate through *CE*,

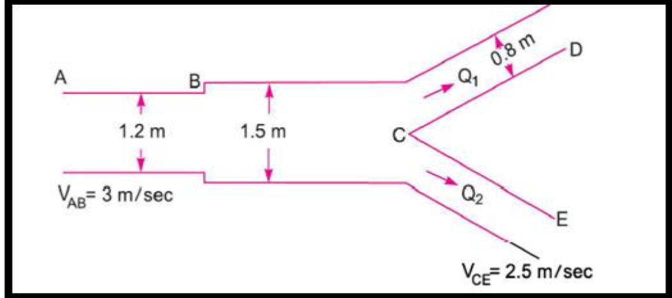
$$Q_2 = Q - Q_1 = 3.393 - 1.131 = 2.262 \text{ m}^3/\text{s}$$

$\therefore$ $$Q_2 = V_{CE} \times \text{Area of pipe } CE = V_{CE}\frac{\pi}{4}(D_{CE})^2$$

or $$2.263 = 2.5 \times \frac{\pi}{4} \times (D_{CE})^2$$

or $$D_{CE} = \sqrt{\frac{2.263 \times 4}{2.5 \times \pi}} = \sqrt{1.152} = 1.0735 \text{ m}$$

$\therefore$ Diameter of pipe CE = **1.0735 m. Ans.**



Summary

- Volume flow rate: Q = $\frac{volume\ of\ fluid}{time}$
- Matter cannot be created or destroyed - (it is simply changed in to a different form of matter)
- By continuity equation: For steady flow there is no increase in the mass within the control volume(i.e., **Mass entering per unit time = Mass leaving per unit time)**

FLUID DYNAMICS: PART-2

Presented By:
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Lecture Details:
Unit-2 , Fluid Dynamics
FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
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FLUID DYNAMICS: PART-2

Contents

- Assumptions of Bernoulli's equation
- Bernoulli's Equation from Euler's Equation
- Summary

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Assumptions of Bernoulli's equation

i. The fluid is ideal, i.e., viscosity is zero

ii. The flow is steady

iii. The flow is incompressible

iv. The flow is irrotational

Bernoulli's Equation from Euler's Equation

- Bernoulli's equation is obtained by integrating the Euler's equation of motion($\frac{dp}{\rho}$ +g dz $+ vdv$ = 0)

$$\int \frac{dp}{\rho} + \int \mathrm{g}\, dz + \int vdv = \text{constant}$$

- In above equation 'g' is constant and if flow is incompressible then $\rho\ is\ also\ constant$

$$\frac{1}{\rho} \int dp + \mathrm{g} \int dz + \int vdv = \text{constant}$$

Bernoulli's Equation from Euler's Equation(Cont…)

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$$\frac{p}{\rho} + g\,z + \frac{v^2}{2} = \text{constant}$$

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

$$\boldsymbol{\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}}$$

Above equation is known as **Bernoull's equation**

Bernoulli's Equation from Euler's Equation(Cont…)

Where ,

$\frac{p}{\rho g}$ - Pressure energy per unit weight of fluid or pressure head

$\frac{v^2}{2g}$ –Kinetic energy per unit weight or kinetic head

Z- Potential energy per unit weight or potential head

- Total head = Pressure head + Kinetic head + Potential head or
- Total energy per unit weight = Pressure energy per unit weight + Kinetic energy per unit weight + Potential energy per unit weight

Problem:1

Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5 m above the datum line.

Solution. Given :

Diameter of pipe $= 5 \text{ cm} = 0.5 \text{ m}$

Pressure, $p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

Velocity, $v = 2.0 \text{ m/s}$

Datum head, $z = 5 \text{ m}$

Total head = pressure head + kinetic head + datum head

Problem:1(Cont…)

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Pressure head $= \dfrac{p}{\rho g} = \dfrac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m}$ $\left\{ \rho \text{ for water} = 1000 \dfrac{\text{kg}}{\text{m}^3} \right\}$

Kinetic head $= \dfrac{v^2}{2g} = \dfrac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$

$\therefore$ Total head $= \dfrac{p}{\rho g} + \dfrac{v^2}{2g} + z = 30 + 0.204 + 5 = \mathbf{35.204 \text{ m.}}$ **Ans.**

Problem:2

A pipe, through which water is flowing, is having diameters, 20 cm and 10 cm at the cross-sections 1 and 2 respectively. The velocity of water at section 1 is given 4.0 m/s. Find the velocity head at sections 1 and 2 and also rate of discharge.

Solution. Given :

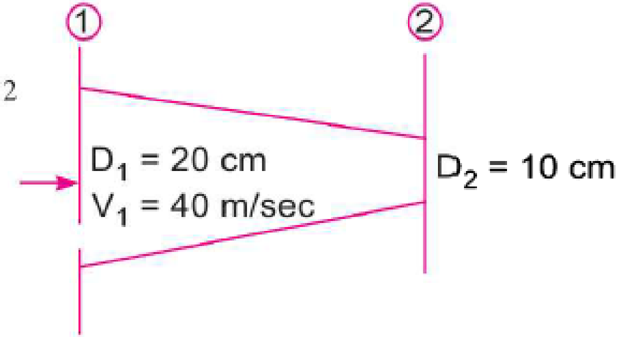
$$D_1 = 20 \text{ cm} = 0.2 \text{ m}$$

$\therefore$ Area,
$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}^2$$

$$V_1 = 4.0 \text{ m/s}$$

$$D_2 = 0.1 \text{ m}$$

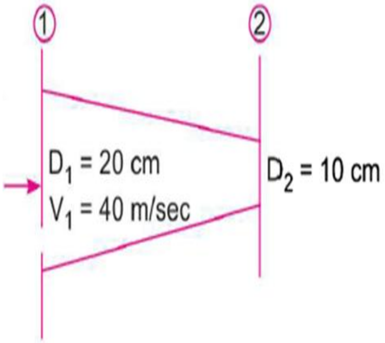
$\therefore$
$$A_2 = \frac{\pi}{4} (.1)^2 = .00785 \text{ m}^2$$



Problem:2(Cont…)

4

(*i*) Velocity head at section 1



$$= \frac{V_1^2}{2g} = \frac{4.0 \times 4.0}{2 \times 9.81} = \mathbf{0.815\ m.\ Ans.}$$

(*ii*) Velocity head at section 2 = $V_2^2/2g$

To find V_2, apply continuity equation at 1 and 2

$$\therefore \qquad A_1V_1 = A_2V_2 \quad \text{or} \quad V_2 = \frac{A_1V_1}{A_2} = \frac{.0314}{.00785} \times 4.0 = 16.0 \text{ m/s}$$

$$\therefore \qquad \text{Velocity head at section 2} = \frac{V_2^2}{2g} = \frac{16.0 \times 16.0}{2 \times 9.81} = \mathbf{83.047\ m.\ Ans.}$$

Problem:2(Cont…)

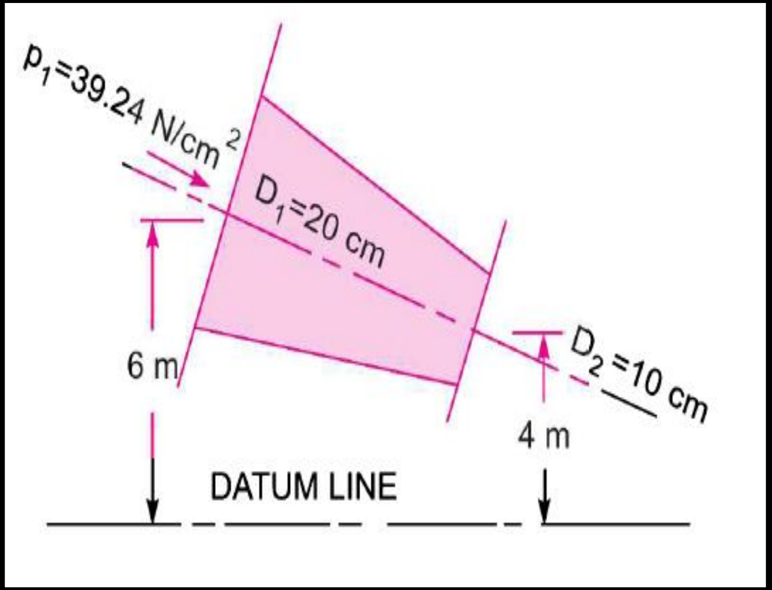
(*iii*) Rate of discharge $= A_1V_1$ or A_2V_2

$= 0.0314 \times 4.0 = 0.1256 \text{ m}^3/\text{s}$

$=$ **125.6 litres/s. Ans.** $\{\because 1 \text{ m}^3 = 1000 \text{ litres}\}$

Problem:3

The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 litres/s. The section 1 is 6 m above datum and section 2 is 4 m above datum. If the pressure at section 1 is 39.24 N/cm^2, find the intensity of pressure at section 2.



Problem:3(Cont…)

Solution. Given :

At section 1, $D_1 = 20 \text{ cm} = 0.2 \text{ m}$

$$A_1 = \frac{\pi}{4}(.2)^2 = .0314 \text{ m}^2$$

$$p_1 = 39.24 \text{ N/cm}^2 = 39.24 \times 10^4 \text{ N/m}^2$$

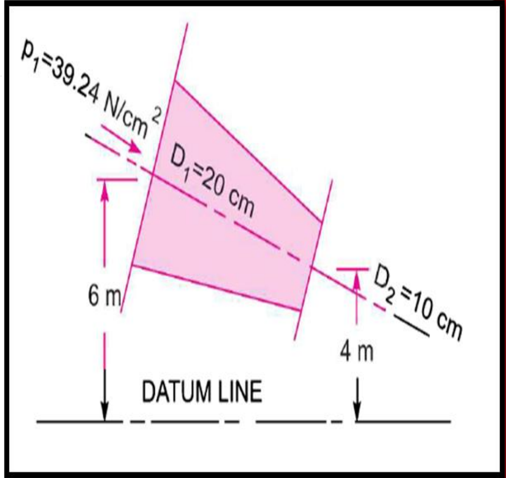
$$z_1 = 6.0 \text{ m}$$

At section 2, $D_2 = 0.10 \text{ m}$

$$A_2 = \frac{\pi}{4}(0.1)^2 = .00785 \text{ m}^2$$

$$z_2 = 4 \text{ m}$$

$$p_2 = ?$$



Problem:3(Cont…)

Rate of flow, $Q = 35 \text{ lit/s} = \dfrac{35}{1000} = .035 \text{ m}^3\text{/s}$

Now $Q = A_1 V_1 = A_2 V_2$

$\therefore$ $V_1 = \dfrac{Q}{A_1} = \dfrac{.035}{.0314} = 1.114 \text{ m/s}$

and $V_2 = \dfrac{Q}{A_2} = \dfrac{.035}{.00785} = 4.456 \text{ m/s}$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Problem:3(Cont…)

or $$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{p_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

or $$40 + 0.063 + 6.0 = \frac{p_2}{9810} + 1.012 + 4.0$$

or $$46.063 = \frac{p_2}{9810} + 5.012$$

$\therefore$ $$\frac{p_2}{9810} = 46.063 - 5.012 = 41.051$$

$\therefore$ $$p_2 = 41.051 \times 9810 \text{ N/m}^2$$

$$= \frac{41.051 \times 9810}{10^4} \text{ N/cm}^2 = \mathbf{40.27\ N/cm^2.\ Ans.}$$

Problem:4

The water is flowing through a taper pipe of length 100 m having diameters 600 mm at the upper end and 300 mm at the lower end, at the rate of 50 litres/s. The pipe has a slope of 1 in 30. Find the pressure at the lower end if the pressure at the higher level is 19.62 N/cm².

Solution. Given :

Length of pipe, $L = 100$ m

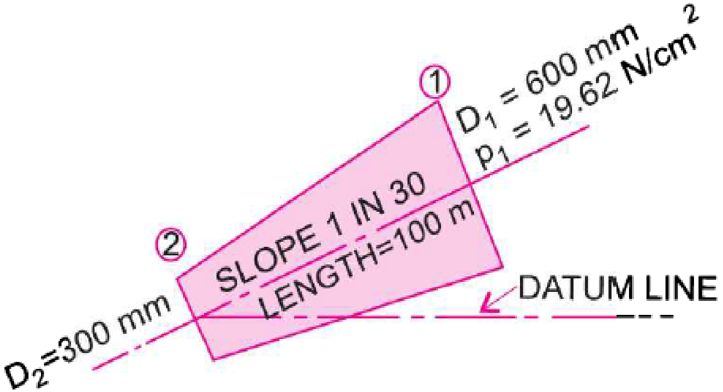
Dia. at the upper end, $D_1 = 600$ mm $= 0.6$ m

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (.6)^2$

$= 0.2827$ m^2

p_1 = pressure at upper end

$= 19.62$ N/cm^2

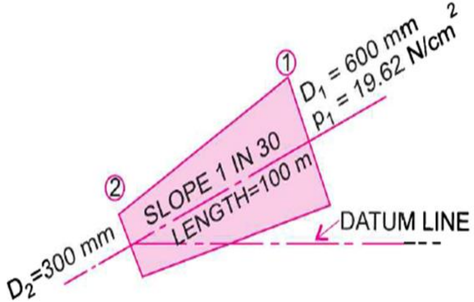


Problem:4(Cont…)

$$= 19.62 \times 10^4 \text{ N/m}^2$$

Dia. at lower end, $D_2 = 300 \text{ mm} = 0.3 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} (.3)^2 = 0.07068 \text{ m}$



$$Q = \text{rate of flow} = 50 \text{ litres/s} = \frac{50}{1000} = 0.05 \text{ m}^3\text{/s}$$

Let the datum line passes through the centre of the lower end.

Then $z_2 = 0$

As slope is 1 in 30 means $z_1 = \frac{1}{30} \times 100 = \frac{10}{3}$ m

Problem:4(Cont…)

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Also we know $$Q = A_1V_1 = A_2V_2$$

$$\therefore \quad V_1 = \frac{Q}{A} = \frac{0.05}{.2827} = 0.1768 \text{ m/sec} = 0.177 \text{ m/s}$$

and $$V_2 = \frac{Q}{A_2} = \frac{0.5}{.07068} = 0.7074 \text{ m/sec} = 0.707 \text{ m/s}$$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Problem:4(Cont…)

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or $$\frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{.177^2}{2 \times 9.81} + \frac{10}{3} = \frac{p_2}{\rho g} + \frac{.707^2}{2 \times 9.81} + 0$$

or $$20 + 0.001596 + 3.334 = \frac{p_2}{\rho g} + 0.0254$$

or $$23.335 - 0.0254 = \frac{p_2}{1000 \times 9.81}$$

or $$p_2 = 23.3 \times 9810 \text{ N/m}^2 = 228573 \text{ N/m}^2 = \mathbf{22.857\ N/cm^2}.$$ **Ans.**

Summary

- Bernoulli's equation is obtained by integrating the Euler's equation of motion

$$\int \frac{dp}{\rho} + \int g\, dz + \int v dv = \text{constant}$$

- "$\boldsymbol{\frac{p}{\rho g} + \frac{v^2}{2g} + z}$ **= constant** is known as **Bernoull's equation**
- Total head (total energy per unit weight) = Pressure head(Pressure energy per unit weight of fluid) + Kinetic head(Kinetic energy per unit weight)+Potential head(Potential energy per unit weight)

FLUID DYNAMICS: PART-3

Presented By:
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Lecture Details:
Unit-2 , Fluid Dynamics
FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

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FLUID DYNAMICS: PART-3

Contents

Bernoulli's Equation for Real Fluid

- The Bernoulli's equation was derived on the assumption that fluid is inviscid (non-viscous) and therefore frictionless
- But all the real fluids are viscous and hence offer resistance to flow

$$\frac{p_1}{\rho g} + \frac{{V_1}^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{{V_2}^2}{2g} + z_2 + h_L$$

Where h_L is loss of energy between point 1 and 2

Problem:1

A pipe of diameter 400 mm carries water at a velocity of 25 m/s. The pressures at the points A and B are given as 29.43 N/cm² and 22.563 N/cm² respectively while the datum head at A and B are 28 m and 30 m. Find the loss of head between A and B.

Solution. Given :

Dia. of pipe, $D = 400 \text{ mm} = 0.4 \text{ m}$

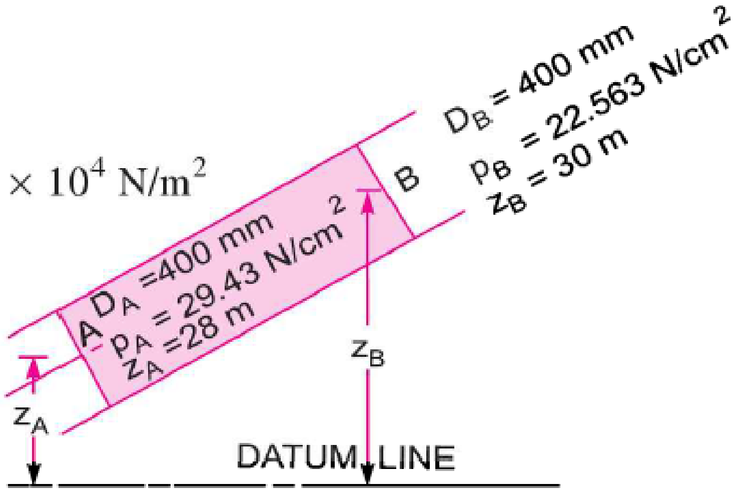
Velocity, $V = 25 \text{ m/s}$

At point *A*, $p_A = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

$z_A = 28 \text{ m}$

$v_A = v = 25 \text{ m/s}$

$\therefore$ Total energy at *A*, $E_A = \dfrac{p_A}{\rho g} + \dfrac{v_A^2}{2g} + z_A$

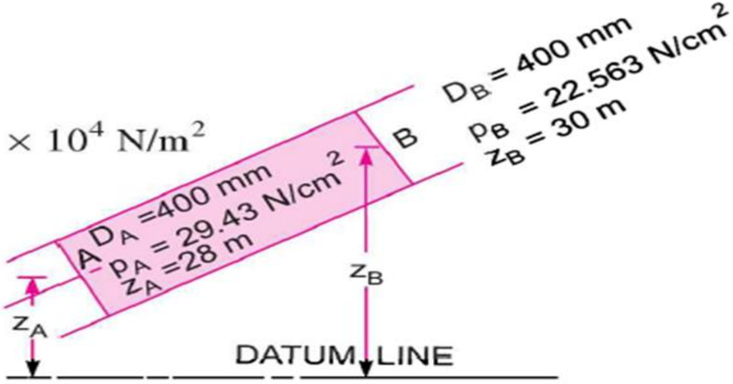


Problem:1(Cont…)

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 28$$

$$= 30 + 31.85 + 28 = 89.85 \text{ m}$$

$\times\ 10^4$ N/m^2



At point *B*,

$$p_B = 22.563 \text{ N/cm}^2 = 22.563 \times 10^4 \text{ N/m}^2$$

$$z_B = 30 \text{ m}$$

$$v_B = v = v_A = 25 \text{ m/s}$$

Problem:1(Cont…)

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$\therefore$ Total energy at B, $$E_B = \frac{p_B}{\rho g} + \frac{v_B^2}{2g} + z_B$$

$$= \frac{22.563 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 30 = 23 + 31.85 + 30 = 84.85 \text{ m}$$

$\therefore$ Loss of energy $= E_A - E_B = 89.85 - 84.85 = \mathbf{5.0\ m.\ Ans.}$

The Momentum Equation

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- It is based on the law of conservation of momentum or on the momentum principle, which states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction
- The force acting on a fluid mass 'm' is given by the Newton's second law of motion

$$F = m \times a$$

- Where "a" is the acceleration acting in the same direction as force F

But $\quad a = \frac{dv}{dt}$

The Momentum Equation(Cont…)

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$$F = m\frac{dv}{dt}$$

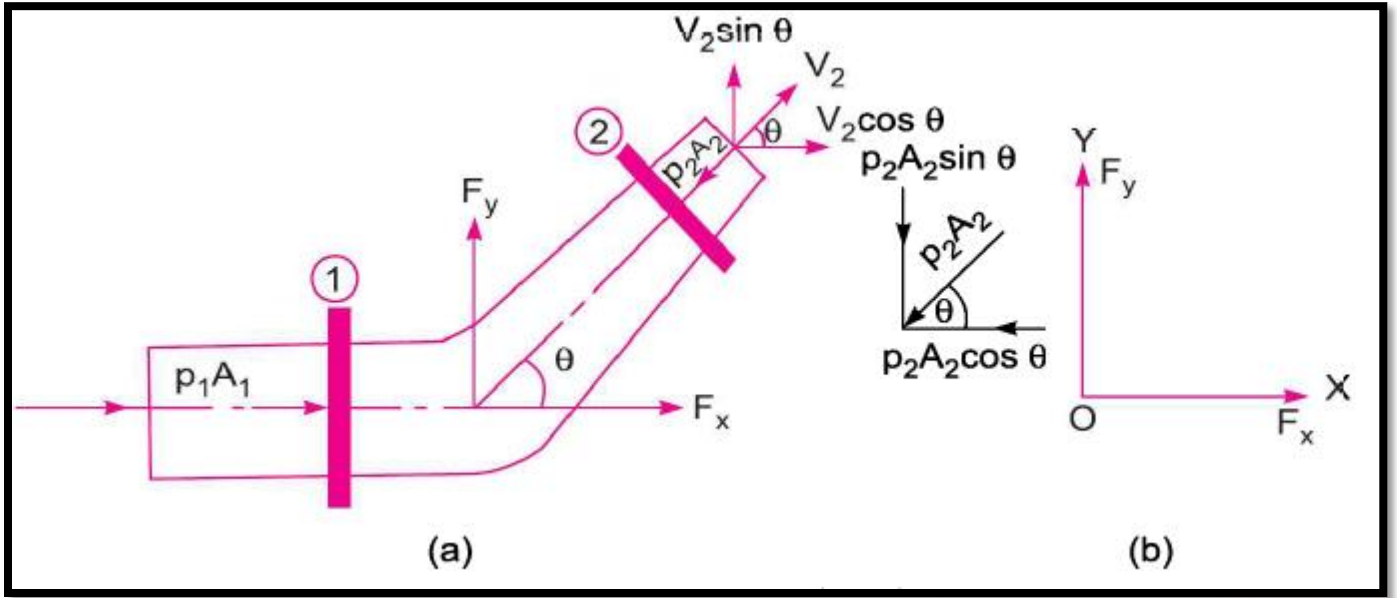
$$= \frac{d(mv)}{dt} \quad \{\text{ m is constant and can be taken inside the differential }\}$$

$$F = \frac{d(mv)}{dt} \text{ -------------(1)}$$

- Equation 1 is known as the momentum principle
- Equation 1 can be written as F.dt = d(mv) -----(2)
- Equation 2 is known as the **impulse- momentum equation** and states that the impulse of a force F acting on a fluid of mass m in a short interval of time dt is equal to the change of momentum d(mv) in the direction of force

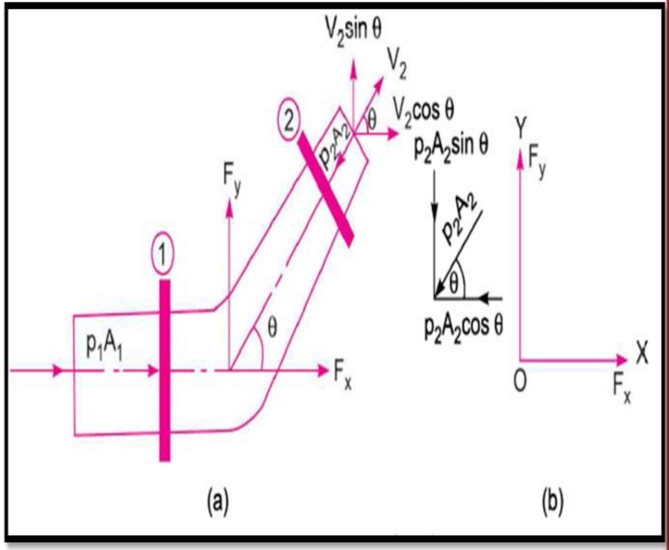
Force Exerted by a Flowing Fluid on a Pipe Bend

- The impulse- momentum equation(2) is used to determine the resultant force exerted by a flowing fluid on a pipe bend



Force Exerted by a Flowing Fluid on a Pipe Bend(Cont…)

- Consider two sections (1) and (2) as shown in the fig



Let V_1= velocity of flow at section(1)

p_1= pressure intensity at section (1)

A_1= area of cross- section of pipe at section (1)

and

V_2, p_2 , A_2 = corresponding values of velocity , pressure and area at section(2)

Force Exerted by a Flowing Fluid on a Pipe Bend(Cont…)

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- Let F_x and F_y be the components of the forces exerted by the flowing fluid on the bend in x and y – directions respectively
- Then the force exerted by the bend on the fluid in the directions of x and y will be equal to F_x and F_y but in the opposite directions
- Net force acting on fluid in the direction of x = Rate of change of momentum in x –direction

Force Exerted by a Flowing Fluid on a Pipe Bend(Cont…)

p_1A_1 - p_2A_2 cosθ - F_x = (mass per sec) (change of velocity)

=ρQ(Final velocity in the direction of x

– Initial velocity in the direction of x)

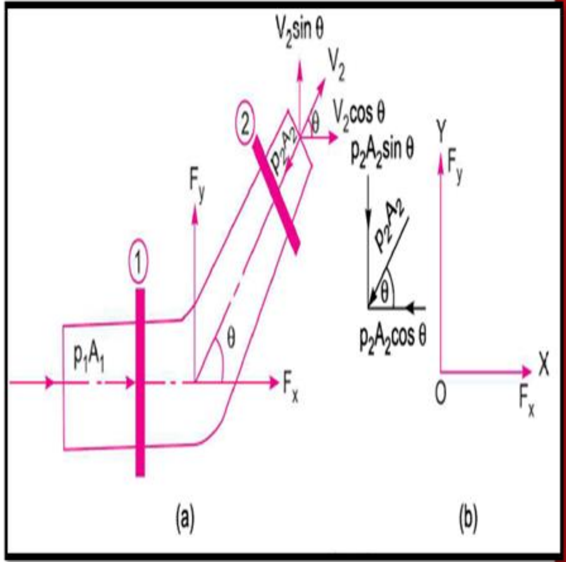
$$p_1A_1 - p_2A_2 \cos\theta - F_x = \rho Q(V_2 \cos\theta - V_1)$$

$$F_x = \rho Q(V_1 - V_2 \cos\theta) + p_1A_1 - p_2A_2 \cos\theta$$

Similarly the momentum equation in y-direction gives

$$0 - p_2A_2 \sin\theta - F_y = \rho Q(V_2 \sin\theta - 0)$$

$$F_y = \rho Q(-V_2 \sin\theta) - p_2A_2 \sin\theta$$



Force Exerted by a Flowing Fluid on a Pipe Bend(Cont…)

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- Now the resultant force(F_R) acting on the bend

$$= \sqrt{{F_x}^2 + {F_y}^2}$$

- And the angle made by the resultant force with horizontal direction is given by

$$\tan\theta = \frac{F_y}{F_x}$$

Problem:2

A 45° reducing bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 600 mm and 300 mm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet to bend is 8.829 N/cm² and rate of flow of water is 600 litres/s.

Solution. Given :

Angle of bend, $\theta = 45°$

Dia. at inlet, $D_1 = 600 \text{ mm} = 0.6 \text{ m}$

∴ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.6)^2$

$= 0.2827 \text{ m}^2$

Dia. at outlet, $D_2 = 300 \text{ mm} = 0.30 \text{ m}$

∴ Area, $A_2 = \frac{\pi}{4} (.3)^2 = 0.07068 \text{ m}^2$

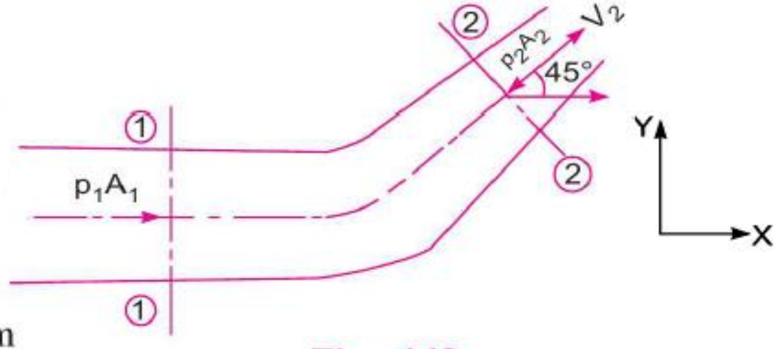


Fig. 6.19

Problem:2(Cont...)

Pressure at inlet, $p_1 = 8.829\ \text{N/cm}^2 = 8.829 \times 10^4\ \text{N/m}^2$

$$Q = 600\ \text{lit/s} = 0.6\ \text{m}^3/\text{s}$$

$$V_1 = \frac{Q}{A_1} = \frac{0.6}{.2827} = 2.122\ \text{m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.6}{.07068} = 8.488\ \text{m/s.}$$

$$F_x = \rho Q(V_1 - V_2\ \cos\theta) + p_1 A_1 - p_2 A_2 \cos\theta$$

$$F_y = \rho Q(-V_2\ \sin\theta) - p_2 A_2 \sin\theta$$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

But $z_1 = z_2$

Problem:2(Cont…)

$$\therefore \quad \frac{p_1}{\rho g}+\frac{V_1^2}{2g}=\frac{p_2}{\rho g}+\frac{V_2^2}{2g} \quad \text{or} \quad \frac{8.829\times 10^4}{1000\times 9.81}+\frac{2.122^2}{2\times 9.81}=\frac{p_2}{\rho g}+\frac{8.488^2}{2\times 9.81}$$

$$9 + .2295 = p_2/\rho g + 3.672$$

$$\therefore \quad \frac{p_2}{\rho g} = 9.2295 - 3.672 = 5.5575 \text{ m of water}$$

$$\therefore \quad p_2 = 5.5575 \times 1000 \times 9.81 \text{ N/m}^2 = 5.45 \times 10^4 \text{ N/m}^2$$

Problem:2(Cont...)

Forces on the bend in x- and y-directions are given by equations (6.18) and (6.20) as

$$F_x = \rho Q\,[V_1 - V_2 \cos\theta] + p_1A_1 - p_2A_2 \cos\theta$$

$$= 1000 \times 0.6\,[2.122 - 8.488 \cos 45^\circ]$$

$$+ 8.829 \times 10^4 \times .2827 - 5.45 \times 10^4 \times .07068 \times \cos 45^\circ$$

$$= -2327.9 + 24959.6 - 2720.3 = 24959.6 - 5048.2$$

$$= 19911.4 \text{ N}$$

and

$$F_y = \rho Q\,[-V_2 \sin\theta] - p_2A_2 \sin\theta$$

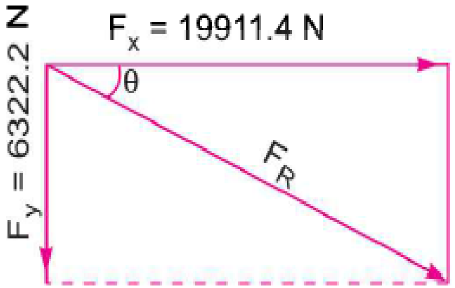
$$= 1000 \times 0.6\,[-8.488 \sin 45^\circ] - 5.45 \times 10^4 \times .07068 \times \sin 45^\circ$$

$$= -3601.1 - 2721.1 = -6322.2 \text{ N}$$

Problem:2(Cont…)

–ve sign means F_y is acting in the downward direction

$\therefore$ Resultant force,
$$F_R = \sqrt{F_x^2 + F_y^2}$$
$$= \sqrt{(19911.4)^2 + (-6322.2)^2}$$
$$= \mathbf{20890.9\ N.\ Ans.}$$



The angle made by resultant force with x-axis is given by equation (6.22) or

$$\tan\theta = \frac{F_y}{F_x} = \frac{6322.2}{19911.4} = 0.3175$$

$$\therefore \qquad \theta = \tan^{-1} .3175 = \mathbf{17^\circ\ 36'.\ Ans.}$$

Summary

- All the real fluids are viscous and hence offer resistance to flow

$$\frac{p_1}{\rho g} + \frac{v_1{}^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2{}^2}{2g} + z_2 + h_L$$

- **F.dt = d(mv)** is Known as **impulse- momentum equation**

- The force exerted by the bend on the fluid in the directions of x and y will be

$$F_x = \rho Q(V_1 - V_2 \cos\theta) + p_1 A_1 - p_2 A_2 \cos\theta$$

$$F_y = \rho Q(-V_2 \ sin\theta) - p_2 A_2 \ sin\theta$$

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REYNOLD'S EXPERIMENT AND DARCY WEISBACH EQUATION

Presented By:
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Lecture Details:
Unit-III (Closed Conduit Flow) , Reynold's Experiment and Darcy Weisbach Equation
FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
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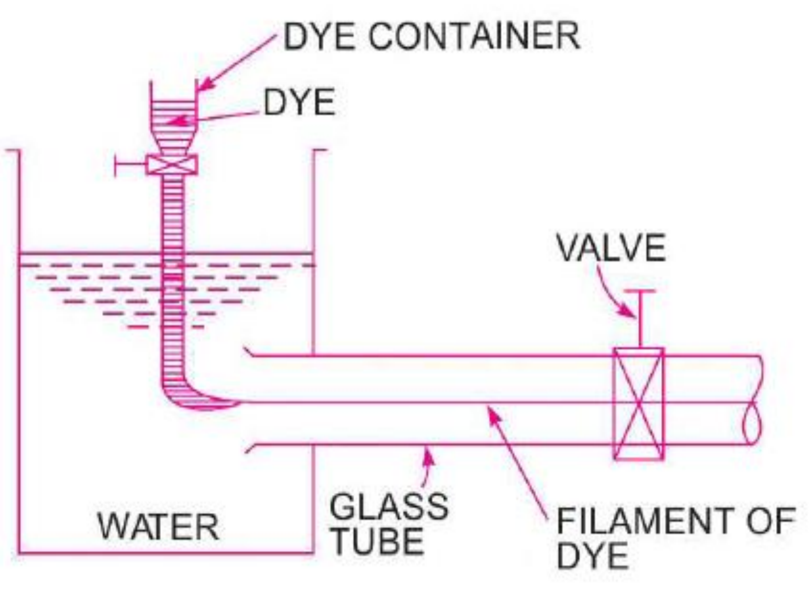
REYNOLD'S EXPERIMENT AND DARCY WEISBACH EQUATION

Contents

- Reynold's experiment
- Frictional loss in pipe flow
- Darcy-Weisbach equation
- Expression for Co-efficient of Friction in Terms of Shear Stress
- Shear Stress in Turbulent Flow
- Summary

Reynold's Experiment

- The Type of flow (laminar or Turbulent) is determined by the Reynold's number($R_e = \frac{\rho V d}{\mu}$)
- O. Reynold conducted an experiment in 1883
- The water from the tank was allowed to flow through the glass tube

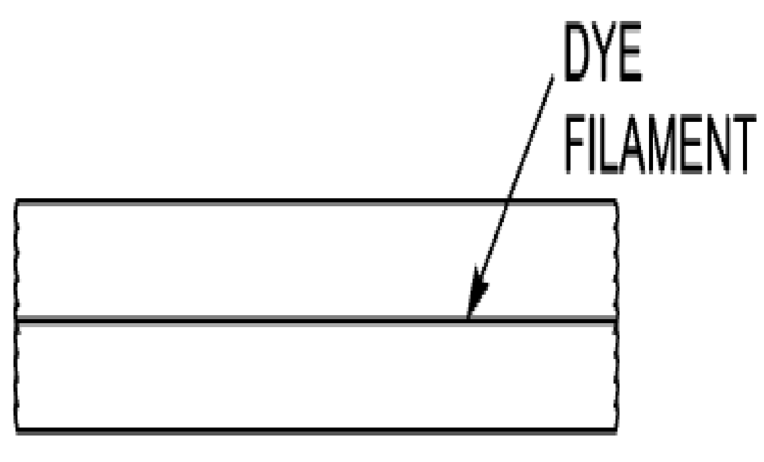


Reynold apparatus.

Reynold’s Experiment(Cont…)

- Velocity of flow was varied by the regulating valve
- A liquid dye having same specific weight as water was introduced into the glass tube
- It is observed that

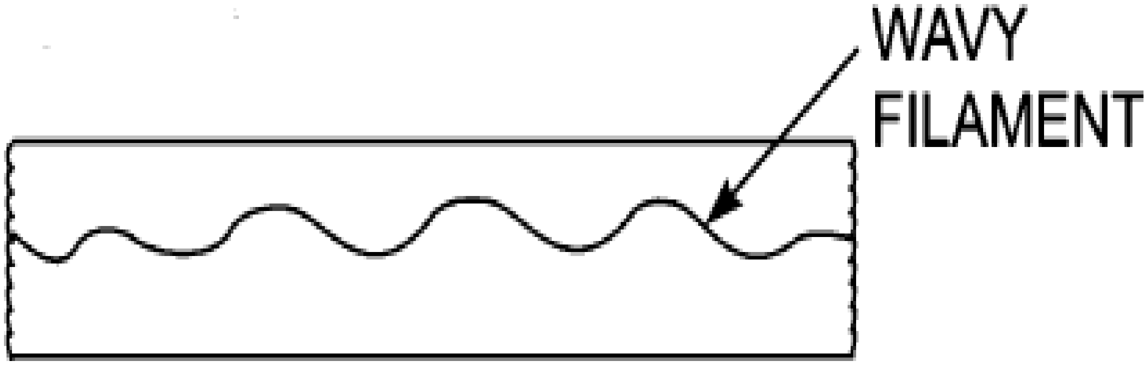
i) When the velocity of flow was low , then dye filament is straight line and parallel to the glass tube. This type of flow is called ”Laminar Flow”



(a) Laminar flow

Reynold's experiment(Cont...)

ii) With the increase of velocity of flow , the dye filament was no longer a straight-line and it becomes wavy in nature. This is called " Transition "

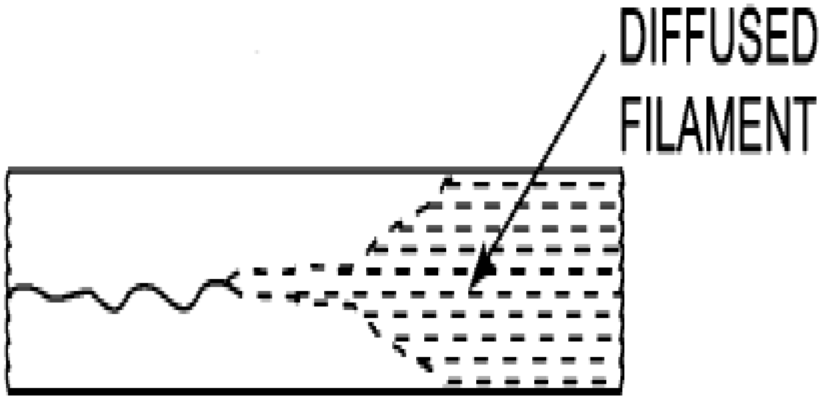


(b) Transition

Reynold’s experiment(Cont…)

iii) With further increase of velocity of flow, the fluid particles of the dye moving in random fashion. This type of flow is called “Turbulent Flow”

- For laminar flow the loss of pressure head is proportional to the velocity
- For turbulent flow loss of pressure head is proportional to the square of velocity



(c) Turbulent flow

Reynold’s experiment(Cont…)

Frictional loss in pipe flow

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- The viscous action which causes loss of energy in the pipe flow is called the frictional loss
- The frictional resistance for turbulent flow is:

i) Proportional to V^n, where n varies from 1.5 to 2.0

ii) Proportional to the density of fluid

iii) Proportional to the area of surface in cantact

iv) Independent of pressure

v) Dependent on the nature of the surface in contact

Darcy-Weisbach equation

Let 1-1 and 2-2 are two sections of pipe

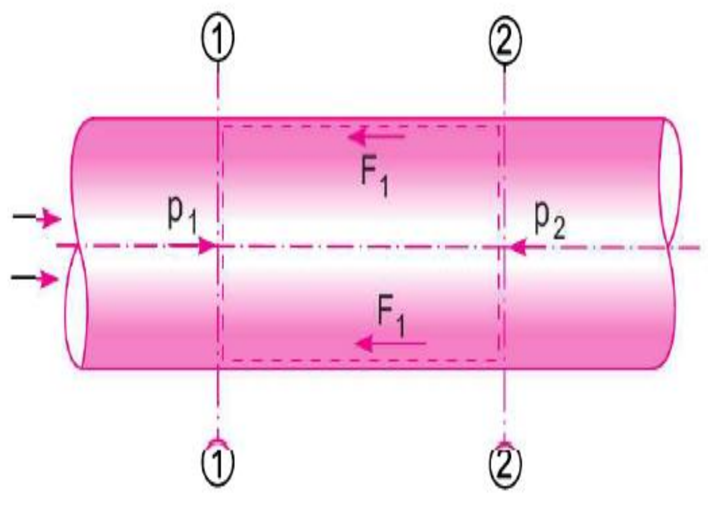
Let p_1= pressure intensity at section 1-1

V_1= velocity of flow at section 1-1

L=length of the pipe between section 1-1 and 2-2

d= diameter of pipe

f'= frictional resistance per unit wetted area per unit velocity



Uniform horizontal pipe.

Darcy- Weisbach equation(Cont…)

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h_f = loss of head due to friction

and p_2, V_2 = are values of pressure intensity and velocity at section 2-2

- Applying Bernoulli's equation between sections 1-1 and 2-2
- Total head at 1-1 = Total head at 2-2 + loss of head due to friction between 1-1 and 2-2

$$\frac{p_1}{\rho g} + \frac{{V_1}^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{{V_2}^2}{2g} + z_2 + h_f$$

Darcy -Weisbach equation(Cont…)

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If pipe is horizontal then , $z_1 = z_2$

If diameter of pipe is same at 1-1 and 2-2 , then $V_1 = V_2$

$$\frac{p_1}{\rho g} = \frac{p_2}{\rho g} + h_f$$

$$h_f = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} \text{------------}(1)$$

- But h_f is the head lost due to friction and hence intensity of pressure will be reduced in the direction of flow by frictional resistance

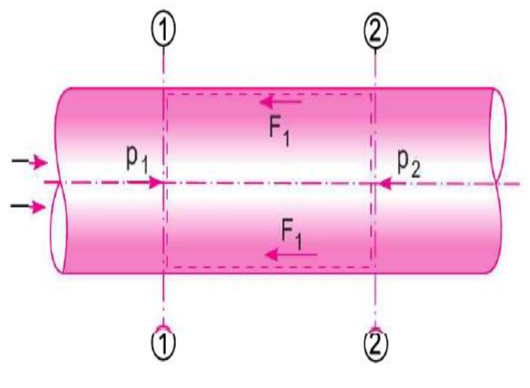
Darcy -Weisbach equation(Cont…)

- Now frictional resistance = frictional resistance per unit wetted area per unit velocity X wetted area X $velocity^2$
- Wetted area = πdL
- Velocity =V=$V_1 = V_2$

$$F_1 = f' X\ \pi dL\ X\ V^2$$

- Perimeter = P =πd

$$F_1 = f' X\ PXL\ X\ V^2$$



Uniform horizontal pipe.

Darcy-Weisbach equation(Cont…)

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- The forces acting on the fluid between section 1-1 and 2-2 are:

i) Pressure force at section 1-1 = p_1X A

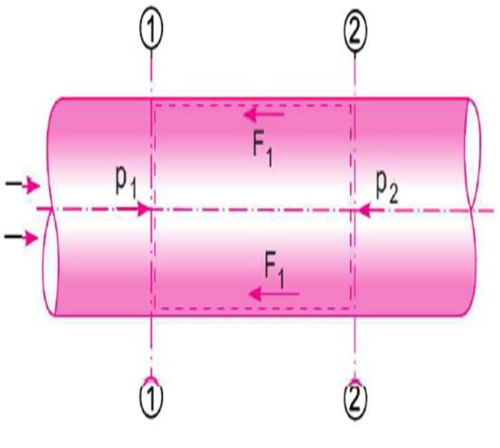
Where A = Area of pipe

ii) Pressure force at section 2-2 =p_2X A

iii) Frictional force F_1

Resolving all forces in the horizontal direction , then

$$p_1 \text{X A} - p_2 \text{X A} - F_1 = 0$$



Uniform horizontal pipe.

Darcy- Weisbach equation(Cont…)

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$$(p_1 - p_2) \mathrm{X}\ \mathrm{A} = F_1$$

- But $F_1 = f' X\ PXL\ X\ V^2$

$$(p_1 - p_2)\ \mathrm{A} = f' X\ PXL\ X\ V^2$$

$$(p_1 - p_2) = \frac{f' X\ PXL\ X\ V^2}{A}$$

- But from eq(1) $h_f = \frac{p_1}{\rho g} - \frac{p_2}{\rho g}$

$$p_1 - p_2 = h_f\ \rho g$$

Darcy-Weisbach equation(Cont…)

$$h_f \ \rho g = \frac{f' X \ PXL \ X \ V^2}{A}$$

$$h_f = \frac{f' X \ PXL \ X \ V^2}{\rho g A}$$

$$h_f = \frac{f'}{\rho g} \text{X} \ \frac{P}{A} \text{XL X} \ V^2$$

But P = wetted perimeter = πd, A = Area = $\frac{\pi}{4} d^2$

$$h_f = \frac{f'}{\rho g} \text{X} \ \frac{\pi d}{\frac{\pi}{4} d^2} \text{XL X} \ V^2$$

Darcy-Weisbach equation(Cont…)

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$$h_f = \frac{f'}{\rho g} X \frac{4}{d} XL\ X\ V^2$$

$$h_f = \frac{f'}{\rho g} X \frac{4Lv^2}{d}$$

- Putting $\frac{f'}{\rho} = \frac{f}{2}$, where f is known as co-efficient of friction

$$h_f = \frac{4f}{2g} . \frac{Lv^2}{d}$$

Darcy-Weisbach equation(Cont…)

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$$h_f = \frac{4f\,L\,V^2}{dX2g} \text{-----------}(2)$$

- Equation (2) is known as ”**Darcy-Weisbach equation**”. This equation is commonly used for finding loss of head due to friction in pipes
- Sometimes equation(2) is written as

$$h_f = \frac{fLV^2}{dX2g}$$

- Then “f “is known as friction factor

Expression for Co-efficient of Friction in Terms of Shear Stress

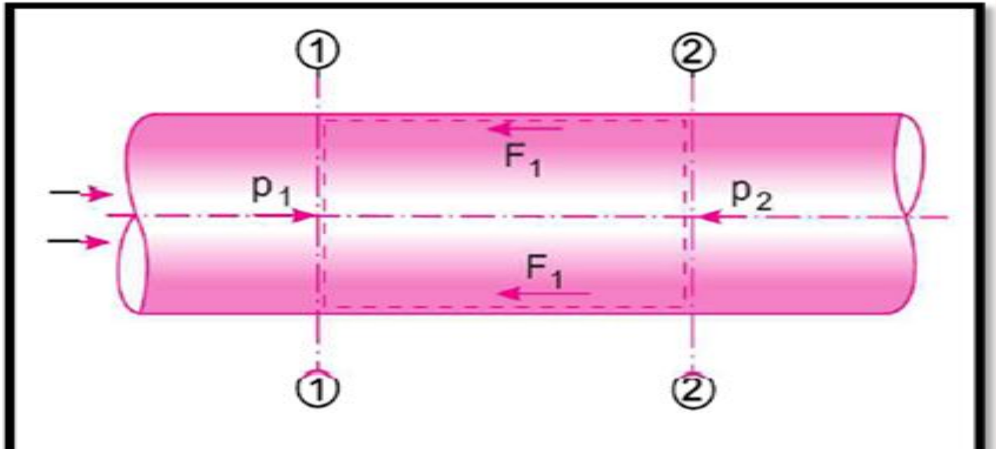
- Force acting on a fluid between section 1-1 and 2-2 is given by

$$(p_1 - p_2)\text{X A} - F_1 = 0$$

$(p_1 - p_2)\text{X A} = F_1$ = force due to shear stress τ_0

= shear stress X surface area

= τ_0X πd X L



Uniform horizontal pipe.

Expression for Co-efficient of Friction in Terms of Shear Stress(Cont…)

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$$(p_1 - p_2)\frac{\pi}{4}d^2 = \tau_0 \mathrm{X}\, \pi d\ X\ L$$

$$(p_1 - p_2)\frac{d}{4} = \tau_0\ \ L$$

$$(p_1 - p_2) = \frac{4\tau_0\ \ L}{d} \text{ -----(I)}$$

But $h_f = \frac{p_1}{\rho g} - \frac{p_2}{\rho g}$ and $h_f = \frac{4f\,L\,V^2}{dX2g}$

Expression for Co-efficient of Friction in Terms of Shear Stress(Cont...)

$$\frac{p_1 - p_2}{\rho g} = \frac{4f\,L\,V^2}{d X 2g}$$

$$p_1 - p_2 = \frac{4f\,L\,V^2}{d X 2g} \text{X}\,\rho g\text{------(II)}$$

From eq(I), $(p_1 - p_2) = \frac{4\tau_0\ L}{d}$

substituting $(p_1 - p_2)$ in eq(II)

$$\frac{4\tau_0\ L}{d} = = \frac{4f\,L\,V^2}{d X 2g} \text{X}\,\rho g$$

Expression for Co-efficient of Friction in Terms of Shear Stress(Cont...)

$$\frac{4\tau_0\ L}{d} = = \frac{4f\,L\,V^2}{dX2g}\,\mathrm{X}\,\rho g$$

$$\tau_0 = f\,\frac{\rho V^2}{2}$$

$$\boldsymbol{f = \frac{2\tau_0}{\rho V^2}}$$

$\boldsymbol{f}$ **– Coefficient of friction**

Shear Stress in Turbulent Flow

- The shear stress in viscous flow is given by Newton's law of viscosity as

$$\boldsymbol{\tau_v} = \boldsymbol{\mu}\frac{\boldsymbol{du}}{\boldsymbol{dy}}$$

Where $\boldsymbol{\tau_v}$ = shear stress due to viscosity

- Similarly to the expression for viscous shear, for turbulent flow

$$\boldsymbol{\tau_t} = \eta\frac{d\bar{\boldsymbol{u}}}{dy}$$

Shear Stress in Turbulent Flow(Cont…)

Where $\boldsymbol{\tau_t}$ = shear stress due to turbulence

η = eddy visosity

$\overline{\boldsymbol{u}}$ =average velocity at a distance y from boundary

$\frac{\eta}{\rho}$ is known as kinetic eddy viscosity(ϵ) i.e

$$\boldsymbol{\epsilon = \frac{\eta}{\rho}}$$

Shear Stress in Turbulent Flow(Cont...)

- If the shear stress due to viscous flow is also considered, then total shear stress becomes as:

$$\tau = \tau_v + \tau_t$$

$$\tau = \mu \frac{du}{dy} + \eta \frac{d\bar{u}}{dy}$$

Summary

- Laminar or Turbulent flow is determined by the Reynolds experiment
- Darcy-Weisbach equation is commonly used for finding loss of head due to friction in pipes
- Coefficient of friction is given by the equaiton $f = \frac{2\tau_0}{\rho V^2}$
- Turbulent shear stress is given by the expression $\tau_t = \eta \frac{d\bar{u}}{dy}$
- Total shear stress is given by the expression $\tau = \mu \frac{du}{dy} + \eta \frac{d\bar{u}}{dy}$

LOSS OF ENERGY IN PIPES

Presented By:
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Lecture Details:
Unit-III (Closed conduit flow) , Loss of Energy in pipes
FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

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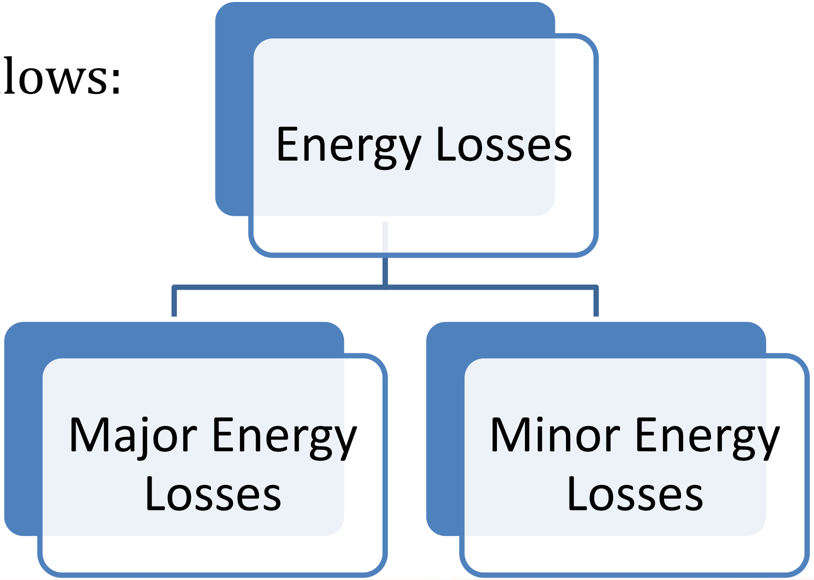
LOSS OF ENERGY IN PIPES

Contents

- Loss of Energy in Pipes
- Major Energy Losses
- Summary

Loss of Energy in Pipes

- When a fluid is flowing through a pipe, the fluid experiences some resistance, due to which some of energy of fluid is lost
- This loss of energy is classified as follows:



Major Energy Losses

- This is due to friction and it is calculated by the following formulae:

a) Darcy-Weisbach Formula

b) Chezy's Formula

Major Energy Losses(Cont...)

a) **Darcy-Weisbach Formula:**

The loss of head (or energy) in pipes due to friction is calculated from Darcy-Weisbach equation as:

$$\boldsymbol{h_f = \frac{4fLV^2}{d X 2g}}$$

Where $\boldsymbol{h_f}$= loss of head due to friction

$\boldsymbol{f}$= co-efficient of friction which is a function of Reynolds number

Major Energy Losses(Cont…)

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$$\boldsymbol{f} = \frac{16}{R_e} \text{ for } R_e < 2000 \text{(viscous flow)}$$

$$\boldsymbol{f} = \frac{0.079}{{R_e}^{1/4}} \text{ for } R_e \text{ varying from 4000 to } 10^6$$

L= Length of pipe

V= mean velocity of flow

d= diameter of pipe

Major Energy Losses(Cont...)

b) Chezy's formula for loss of head due to friction in pipes:

Another expression for loss of head due to friction is given as:

$$h_f = \frac{f'}{\rho g} X \frac{P}{A} XL \ X \ V^2 \text{-------}(1)$$

Where h_f = loss of head due to friction

P = wetted perimeter of pipe

A= area of cross- section of pipe

Major Energy Losses(Cont…)

L= length of the pipe

V= mean velocity of flow

- Now the ratio of $\frac{A}{P}\left(=\frac{Area\ of\ flow}{Perimeter(wetted)}\right)$ is called hydraulic mean depth or hydraulic radius and is denoted by **m**

Hydraulic mean depth, m = $\frac{A}{P}=\frac{\frac{\pi}{4}\boldsymbol{d^2}}{\pi d}=\frac{d}{4}$

Major Energy Losses(Cont...)

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Substituting $\frac{A}{P}$ = m or $\frac{P}{A} = \frac{1}{m}$ in equation(1)

$$h_f = \frac{f'}{\rho g} \text{X L X } V^2 \text{X} \frac{1}{m}$$

$$V^2 = h_f \text{ X } \frac{\rho g}{f'} \text{ X m X } \frac{1}{L}$$

$$V^2 = \frac{\rho g}{f'} \text{ X m X } \frac{h_f}{L}$$

Major Energy Losses(Cont…)

$$V = \sqrt{\frac{\rho g}{f'} \text{ X m X } \frac{h_f}{L}}$$

$$V = \sqrt{\frac{\rho g}{f'}} \sqrt{m \frac{h_f}{L}}$$

$$\text{Let } \sqrt{\frac{\rho g}{f'}} = C, \; \frac{h_f}{L} = i$$

- Where **C** is a constant known as Chezy's constant and **i** is loss of head per unit length of pipe

Major Energy Losses(Cont…)

$$V = C\sqrt{m\,i} \text{-------}(2)$$

- Equation (2) is known as Chezy’s formula
- Thus the loss of head due to friction in pipe from Chezy’s formula can be obtained if the velocity of flow through pipe and also the value of C is known
- The value of **m** for pipe is always equal to d/4

Problem:1

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Find the head lost due to friction in a pipe of diameter 300 mm and length 50 m, through which water is flowing at a velocity of 3 m/s using (i) Darcy formula, (ii) Chezy's formula for which C = 60.

Take ν for water = 0.01 stoke.

Solution. Given :

Dia. of pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$

Length of pipe, $L = 50 \text{ m}$

Velocity of flow, $V = 3 \text{ m/s}$

Chezy's constant, $C = 60$

Kinematic viscosity, $\nu = 0.01 \text{ stoke} = 0.01 \text{ cm}^2/\text{s}$

$= 0.01 \times 10^{-4} \text{ m}^2/\text{s}.$

Problem:1(Cont…)

(*i*) **Darcy Formula** is given

$$h_f = \frac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g}$$

where 'f' = co-efficient of friction is a function of Reynolds number, R_e

But R_e is given by $$R_e = \frac{V \times d}{\nu} = \frac{3.0 \times 0.30}{.01 \times 10^{-4}} = 9 \times 10^5$$

$\therefore$ Value of $$f = \frac{0.079}{R_e^{1/4}} = \frac{0.079}{\left(9 \times 10^5\right)^{1/4}} = .00256$$

Problem:1(Cont…)

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$\therefore$ Head lost, $$h_f = \frac{4 \times .00256 \times 50 \times 3^2}{0.3 \times 2.0 \times 9.81} = \textbf{.7828 m. Ans.}$$

(*ii*) **Chezy's Formula.** Using equation

$$V = C\sqrt{mi}$$

where $C = 60$, $m = \dfrac{d}{4} = \dfrac{0.30}{4} = 0.075$ m

Problem:1(Cont…)

$\therefore$ $$3 = 60\sqrt{.075 \times i} \text{ or } i = \left(\frac{3}{60}\right)^2 \times \frac{1}{.075} = 0.0333$$

But $$i = \frac{h_f}{L} = \frac{h_f}{50}$$

Equating the two values of i, we have $\frac{h_f}{50} = .0333$

$\therefore$ $$h_f = 50 \times .0333 = \mathbf{1.665\ m.\ Ans.}$$

Problem:2

Find the diameter of a pipe of length 2000 m when the rate of flow of water through the pipe is 200 litres/s and the head lost due to friction is 4 m. Take the value of C = 50 in Chezy's formulae.

Solution. Given :

Length of pipe, $L = 2000$ m

Discharge, $Q = 200$ litre/s $= 0.2$ m^3/s

Head lost due to friction, $h_f = 4$ m

Value of Chezy's constant, $C = 50$

Let the diameter of pipe $= d$

Problem:2(Cont…)

Velocity of flow, $$V = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{\frac{\pi}{4} d^2} = \frac{0.2}{\frac{\pi}{4} d^2} = \frac{0.2 \times 4}{\pi d^2}$$

Hydraulic mean depth, $$m = \frac{d}{4}$$

Loss of head per unit length, $$i = \frac{h_f}{L} = \frac{4}{2000} = .002$$

Problem:2(Cont…)

Chezy's formula is given by equation as $V = C\sqrt{mi}$

Substituting the values of V, m, i and C, we get

$$\frac{0.2\times 4}{\pi d^2} = 50\sqrt{\frac{d}{4}\times .002} \text{ or } \sqrt{\frac{d}{4}\times .002} = \frac{0.2\times 4}{\pi d^2\times 50} = \frac{.00509}{d^2}$$

Squaring both sides, $\dfrac{d}{4}\times .002 = \dfrac{.00509^2}{d^4} = \dfrac{.0000259}{d^4}$ or $d^5 = \dfrac{4\times .0000259}{.002} = 0.0518$

$\therefore$ $d = \sqrt[5]{0.0518} = (.0518)^{1/5} = 0.553$ m = **553 mm. Ans.**

Summary

- Loss of energy in pipe flow is divided into major and minor energy losses
- Major energy losses is due to friction only and it is given by the Darcy-Weisbach Formula and Chezy's Formula
- Darcy- Weisbach formula for loss of head due to friction in pipes is given by $h_f = \frac{4fLV^2}{dX2g}$
- Chezy's formula for loss of head due to friction in pipes is given by **V= C** $\sqrt{\mathbf{m}\, i}$

MINOR ENERGY LOSSES: PART-1

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Lecture Details:

Unit-III (Closed Conduit Flow) , Minor Energy Losses

FM & HM /Mechanical, I -Semester.

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MINOR ENERGY LOSSES: PART-1

Contents

- Minor Energy Losses
- Loss of Head due to Sudden Enlargement
- Summary

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Minor Energy Losses

- The loss of energy due to change of velocity of the flowing fluid in magnitude or direction is called minor energy losses
- The minor loss of energy (or head) includes the following cases:

Minor Energy Losses(Cont…)

- Loss of head due to sudden enlargement
- Loss of head due to sudden contraction
- Loss of head at the entrance of a pipe
- Loss of head at the exit of a pipe
- Loss of head due to an obstruction in a pipe
- Loss of head due to bend in the pipe
- Loss of head in various pipe fittings

Minor Energy Losses(Cont…)

- In case of long pipe the minor energy losses are small compared to major energy losses and they can be neglected without serious error
- But in case of a short pipe, these losses are comparable with the major losses

Loss of Head due to Sudden Enlargement(h_e)

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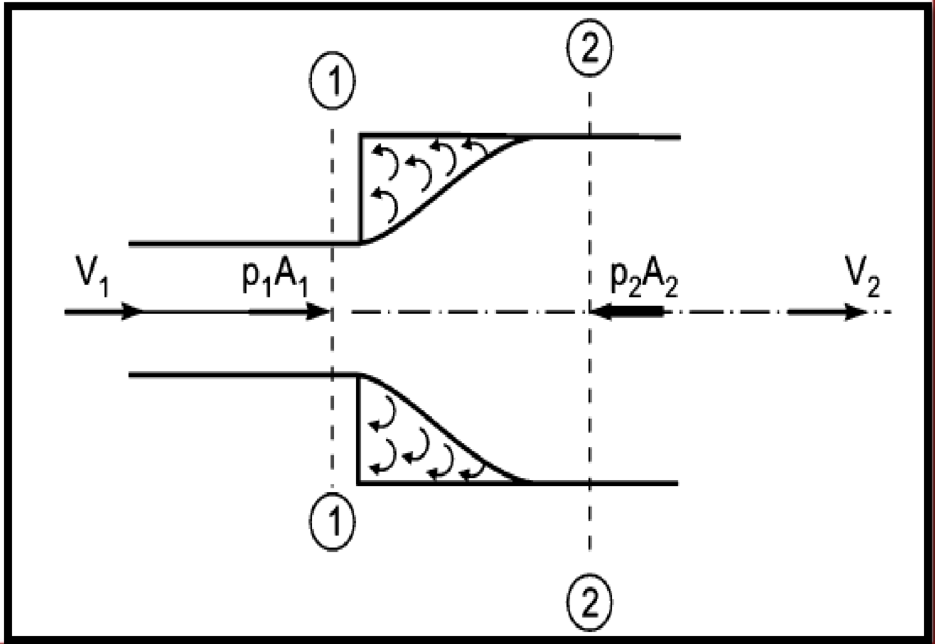
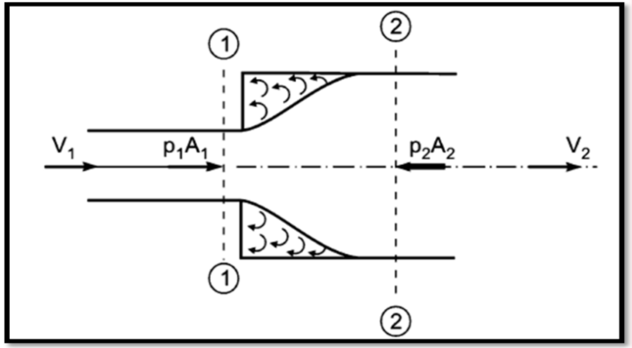
- Consider a liquid flowing through a pipe which has sudden enlargement as shown in the fig
- Consider two sections 1-1 and 2-2 before and after the enlargement



Loss of Head due to Sudden Enlargement(Cont…)



- Let p_1 = pressure intensity at section 1-1

  V_1 = velocity of flow at section 1-1

  A_1 = area of pipe at section 1-1

  p_2, V_1 *and* A_2 = corresponding values at section 2-2

- Due to sudden change of diameter of the pipe from D_1 to D_2, the liquid flowing from the smaller pipe is not able to follow the abrupt change of the boundary

Loss of Head due to Sudden Enlargement(Cont…)

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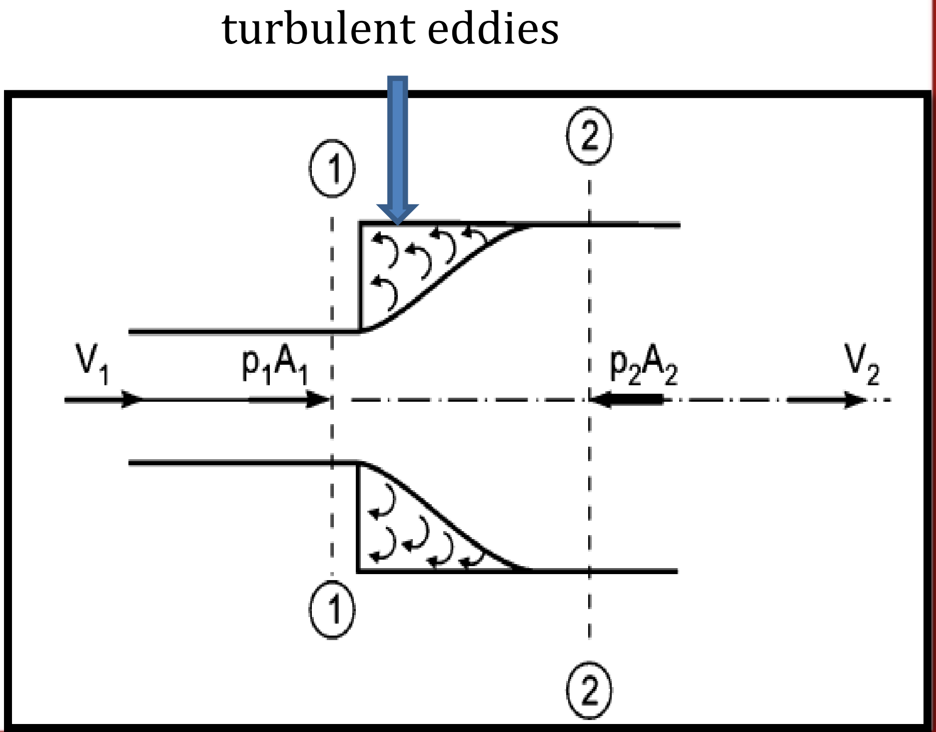
- The flow separates from the boundary and turbulent eddies are formed
- The loss of head (or energy) takes place due to the formation of these eddies

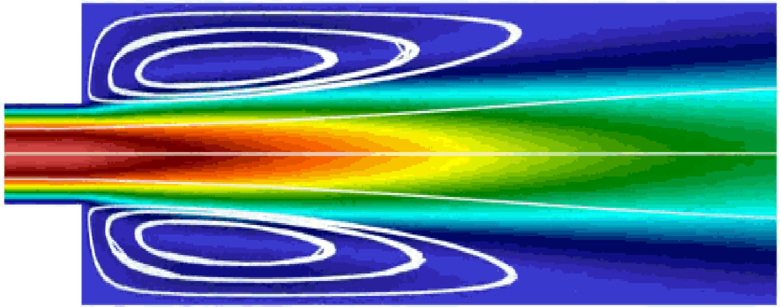


Loss of Head due to Sudden Enlargement(Cont…)

- Let p'= pressure intensity of the liquid eddies on the area(A_2- A_1)

  h_e = loss of head due to sudden enlargement

- Applying Bernoulli's equation at section 1-1 and 2-2



Loss of Head due to Sudden Enlargement(Cont...)

$$\frac{p_1}{\rho g} + \frac{{V_1}^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{{V_2}^2}{2g} + z_2 + \text{loss of head due to sudden enlargement}$$

But $z_1 = z_2$ as pipe is horizontal

$$\frac{p_1}{\rho g} + \frac{{V_1}^2}{2g} = \frac{p_2}{\rho g} + \frac{{V_2}^2}{2g} + h_e$$

$$h_e = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + \left(\frac{{V_1}^2}{2g} - \frac{{V_2}^2}{2g}\right) \text{----------}(1)$$

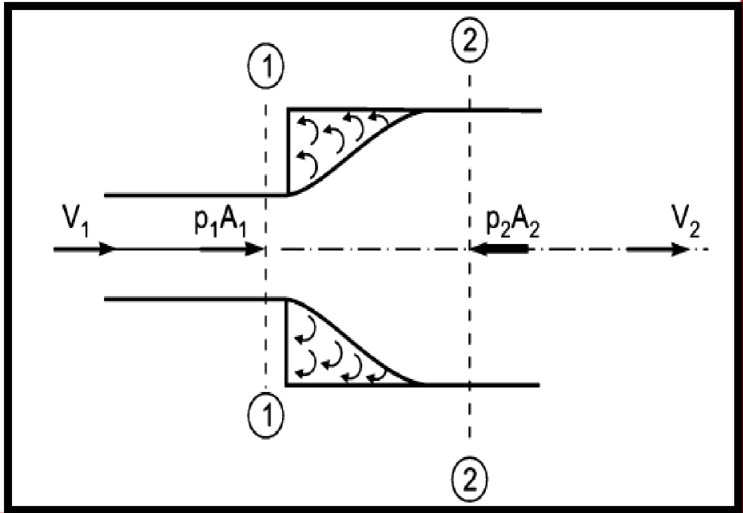
Loss of Head due to Sudden Enlargement(Cont…)

- Considering the control volume of liquid between sections 1-1 and 2-2
- The force acting on the liquid in the control volume in the direction of flow is given by:

$$F_x = p_1A_1 + p'(A_2 - A_1) - p_2A_2$$

But experimentally it is found that $p' = p_1$

$$F_x = p_1A_1 + p_1(A_2 - A_1) - p_2A_2$$



Loss of Head due to Sudden Enlargement(Cont…)

$$F_x = p_1 A_2 - p_2 A_2$$

$$F_x = (p_1 - p_2) A_2$$

Momentum of liquid/sec at section 1-1 = mass flow rate X velocity

$$= \rho\, A_1\, V_1 \text{X}\, V_1$$

$$= \rho\, A_1 V_1{}^2$$

Similarly, Momentum of liquid/sec at section 2-2 = $\rho\, A_2 V_2{}^2$

Loss of Head due to Sudden Enlargement(Cont…)

Change of Momentum of liquid/sec $= \rho\, A_2 {V_2}^2 - \rho\, A_1 {V_1}^2$

But from continuity equation, $A_1 V_1 = A_2 V_2$

$$A_1 = \frac{A_2 V_2}{V_1}$$

Change of Momentum of liquid/sec $= \rho\, A_2 {V_2}^2 - \rho\, \frac{A_2 V_2}{V_1} {V_1}^2$

$$= \rho\, A_2\left({V_2}^2 - V_1 V_2\right)$$

Loss of Head due to Sudden Enlargement(Cont...)

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Now net force acting on the control volume in the direction of flow must be equal to the rate of change of momentum

$$(p_1 - p_2)A_2 = \rho\, A_2\left({V_2}^2 - V_1V_2\right)$$

$$\frac{(p_1 - p_2)}{\rho} = {V_2}^2 - V_1V_2$$

Dividing by g on both sides, $\frac{(p_1 - p_2)}{\rho g} = \frac{{V_2}^2 - V_1V_2}{g}$

Loss of Head due to Sudden Enlargement(Cont...)

$$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \frac{{V_2}^2 - V_1 V_2}{g} \text{----------}(2)$$

Substituting eq(2) in eq(1)

$$\left\{ h_e = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g} \right) + \left(\frac{{V_1}^2}{2g} - \frac{{V_2}^2}{2g} \right) \text{------------}(1) \right\}$$

$$h_e = \frac{{V_2}^2 - V_1 V_2}{g} + \left(\frac{{V_1}^2}{2g} - \frac{{V_2}^2}{2g} \right)$$

Loss of Head due to Sudden Enlargement(Cont…)

$$h_e = \frac{{V_2}^2 - V_1 V_2}{g} + \left(\frac{{V_1}^2}{2g} - \frac{{V_2}^2}{2g}\right)$$

$$= \frac{{V_2}^2 + {V_1}^2 - 2V_1 V_2}{2g}$$

Loss of head due to sudden enlargement $\boldsymbol{h_e = \frac{(V_1 - V_2)^2}{2g}}$

Problem:1

Find the loss of head when a pipe of diameter 200 mm is suddenly enlarged to a diameter of 400 mm. The rate of flow of water through the pipe is 250 litres/s.

Solution. Given :

Dia. of smaller pipe, $D_1 = 200 \text{ mm} = 0.20 \text{ m}$

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.03141 \text{ m}^2$

Dia. of large pipe, $D_2 = 400 \text{ mm} = 0.4 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} \times (0.4)^2 = 0.12564 \text{ m}^2$

Discharge, $Q = 250 \text{ litres/s} = 0.25 \text{ m}^3\text{/s}$

Problem:1(Cont…)

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Velocity, $V_1 = \dfrac{Q}{A_1} = \dfrac{0.25}{.03141} = 7.96 \text{ m/s}$

Velocity, $V_2 = \dfrac{Q}{A_2} = \dfrac{0.25}{.12564} = 1.99 \text{ m/s}$

Loss of head due to enlargement is given by equation

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2g} = \mathbf{1.816 \text{ m of water. Ans.}}$$

Problem:2

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At a sudden enlargement of a water main from 240 mm to 480 mm diameter, the hydraulic gradient rises by 10 mm. Estimate the rate of flow.

Solution. Given :

Dia. of smaller pipe, $D_1 = 240 \text{ mm} = 0.24 \text{ m}$

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.24)^2$

Dia. of large pipe, $D_2 = 480 \text{ mm} = 0.48 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} (0.48)^2$

Problem:2(Cont…)

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Rise of hydraulic gradient*, *i.e.*, $\left(z_2 + \frac{p_2}{\rho g}\right) - \left(\frac{p_1}{\rho g} + z_1\right) = 10 \text{ mm} = \frac{10}{1000} = \frac{1}{100} \text{ m}$

Let the rate of flow = Q

Applying Bernoulli's equation to both sections, *i.e.*, smaller pipe section, and large pipe section.

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{Head loss due to enlargement}$$

But head loss due to enlargement,

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

Problem:2(Cont...)

From continuity equation, we have $A_1V_1 = A_2V_2$

$$\therefore \quad V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}D_2^2 \times V_2}{\frac{\pi}{4}D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{.48}{.24}\right)^2 \times V_2 = 2^2 \times V_2 = 4V_2$$

Substituting this value

$$h_e = \frac{(4V_2 - V_2)^2}{2g} = \frac{(3V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

Problem:2(Cont…)

Now substituting the value of h_e and V_1 in equation

$$\frac{p_1}{\rho g} + \frac{(4V_2)^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{9V_2^2}{2g}$$

or $$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \left(\frac{p_2}{\rho g} + z_2\right) - \left(\frac{p_1}{\rho g} + z_1\right)$$

But hydraulic gradient rise $$= \left(\frac{p_2}{\rho g} + z_2\right) - \left(\frac{p_1}{\rho g} + z_1\right) = \frac{1}{100}$$

Problem:2(Cont…)

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$$\therefore \quad \frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \frac{1}{100} \text{ or } \frac{6V_2^2}{2g} = \frac{1}{100}$$

$$\therefore \quad V_2 = \sqrt{\frac{2 \times 9.81}{6 \times 100}} = 0.1808 \simeq 0.181 \text{ m/s}$$

$\therefore$ Discharge,

$$Q = A_2 \times V_2$$

$$= \frac{\pi}{4} D_2^{\,2} \times V_2 = \frac{\pi}{4} (.48)^2 \times .181 = 0.03275 \text{ m}^3\text{/s}$$

$$= \mathbf{32.75 \text{ litres/s. Ans.}}$$

Problem:3

The rate of flow of water through a horizontal pipe is 0.25 m^3/s. The diameter of the pipe which is 200 mm is suddenly enlarged to 400 mm. The pressure intensity in the smaller pipe is 11.772 N/cm^2. Determine :

(i) loss of head due to sudden enlargement, (ii) pressure intensity in the large pipe,

(iii) power lost due to enlargement.

Solution. Given :

Discharge, $Q = 0.25\ m^3/s$

Dia. of smaller pipe, $D_1 = 200\ mm = 0.20\ m$

$\therefore$ Area, $$A_1 = \frac{\pi}{4}(.2)^2 = 0.03141\ m^2$$

Problem:3(Cont…)

Dia. of large pipe, $D_2 = 400 \text{ mm} = 0.40 \text{ m}$

$\therefore$ Area, $\quad A_2 = \dfrac{\pi}{4}(0.4)^2 = 0.12566 \text{ m}^2$

Pressure in smaller pipe, $\quad p_1 = 11.772 \text{ N/cm}^2 = 11.772 \times 10^4 \text{ N/m}^2$

Now velocity, $\quad V_1 = \dfrac{Q}{A_1} = \dfrac{0.25}{.03141} = 7.96 \text{ m/s}$

Velocity, $\quad V_2 = \dfrac{Q}{A_2} = \dfrac{0.25}{.12566} = 1.99 \text{ m/s}$

Problem:3(Cont…)

(*i*) Loss of head due to sudden enlargement,

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2 \times 9.81} = \textbf{1.816 m. Ans.}$$

(*ii*) Let the pressure intensity in large pipe = p_2.

Then applying Bernoulli's equation before and after the sudden enlargement,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_e$$

But $z_1 = z_2$ (Given horizontal pipe)

Problem:3(Cont…)

$$\therefore \quad \frac{p_1}{\rho g}+\frac{V_1^2}{2g}=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+h_e \text{ or } \frac{p_2}{\rho g}=\frac{p_1}{\rho g}+\frac{V_1^2}{2g}-\frac{V_2^2}{2g}-h_e$$

$$=\frac{11.772\times10^4}{1000\times9.81}+\frac{7.96^2}{2\times9.81}-\frac{1.99^2}{2\times9.81}-1.816$$

$$= 12.0 + 3.229 - 0.2018 - 1.8160$$

$$= 15.229 - 2.0178 = 13.21 \text{ m of water}$$

$$\therefore \quad p_2 = 13.21 \times \rho g = 13.21 \times 1000 \times 9.81 \text{ N/m}^2$$

$$= 13.21 \times 1000 \times 9.81 \times 10^{-4} \text{ N/cm}^2 = \mathbf{12.96\ N/cm^2.\ Ans.}$$

(*iii*) Power lost due to sudden enlargement,

$$P=\frac{\rho g\,.\,Q\,.\,h_e}{1000}=\frac{1000\times9.81\times0.25\times1.816}{1000}=\mathbf{4.453\ kW.\ Ans.}$$

Summary

- The loss of energy due to change of velocity of the flowing fluid in magnitude or direction is called minor energy losses
- For long pipe the minor energy losses neglected
- For short pipe, minor losses are comparable with the major losses
- Due to sudden change of pipe diameter from smaller to larger diameter, the smaller pipe is not able to follow the abrupt change of the boundary , thus turbulent eddies are formed
- The loss of head (or energy) takes place due to the formation of eddies

MINOR ENERGY LOSSES: PART-2

Presented By:

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Mechanical Engineering

GIET(A)

Lecture Details:

Unit-III (Closed Conduit Flow) , Minor Energy Losses

FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
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- RECIPROCATING PUMPS

MINOR ENERGY LOSSES: PART-2

Contents

- Loss of Head due to Sudden Contraction
- Summary

Loss of Head due to Sudden Contraction

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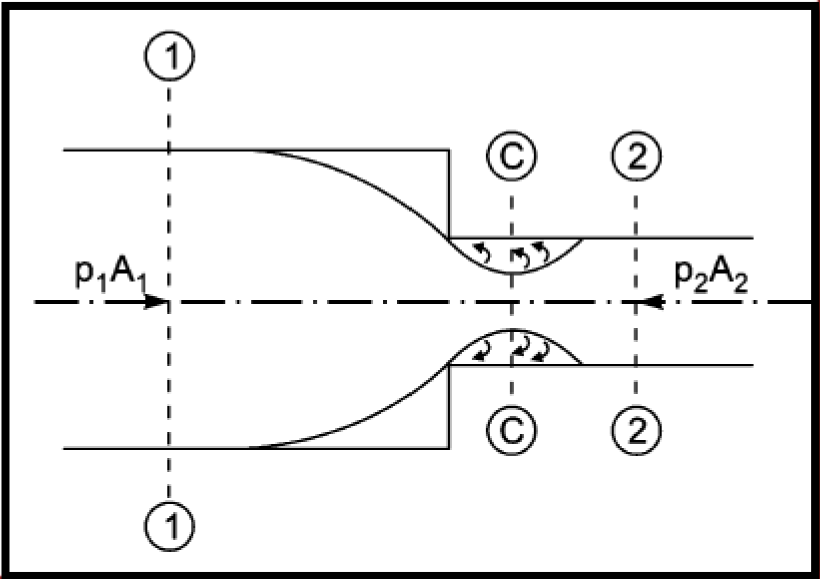
- Consider a liquid flowing in a pipe which has a sudden contraction in area as shown in the fig
- As liquid flows from large pipe to smaller pipe, the area of flow goes on decreasing and becomes minimum at section C-C



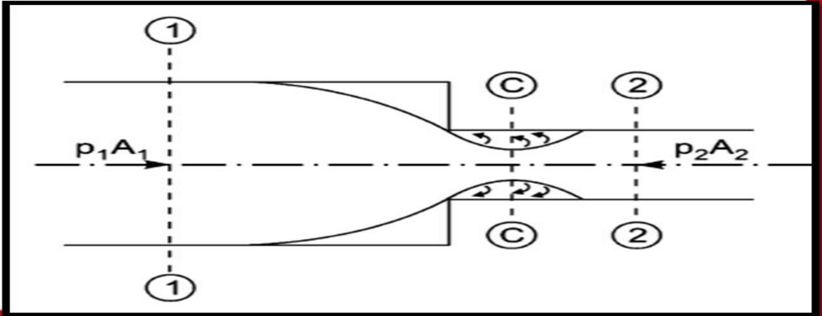
Loss of Head due to Sudden Contraction(Cont...)

- The section C-C is called Vena-contracta
- After section C-C , a sudden enlargement of the area takes place
- The loss of head due to sudden contraction is actually due to sudden enlargement form Vena-contracta to smaller pipe

Let A_c = Area of flow at section C-C

V_c = Velocity of flow at section C-C

A_2 = Area of flow at section 2-2



Loss of Head due to Sudden Contraction(Cont...)

V_2 = Velocity of flow at section 2-2

h_c = Loss of head due to sudden contraction

Now h_c= actual loss of head due to sudden enlargement from section C-C to section 2-2 and given as:

$$\left\{\textbf{But we know Loss of head due to sudden enlargement } \boldsymbol{h_e} = \frac{(\boldsymbol{V_1} - \boldsymbol{V_2})^2}{\boldsymbol{2g}}\right\}$$

Loss of Head due to Sudden Contraction(Cont…)

$$h_c = \frac{(V_c - V_2)^2}{2g}$$

$$= \frac{1}{2g}\left[(V_2)\left(\frac{V_c}{V_2} - 1\right)\right]^2$$

$$= \frac{{V_2}^2}{2g}\left[\frac{V_c}{V_2} - 1\right]^2 \text{-------}(1)$$

From continuity equation, $A_c V_c = A_2 V_2 \implies \frac{V_c}{V_2} = \frac{A_2}{A_c}$

Loss of Head due to Sudden Contraction(Cont…)

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$$\frac{V_c}{V_2} = \frac{1}{A_c/A_2}$$

$$\frac{V_c}{V_2} = \frac{1}{C_c}$$

Let $A_c/A_2 = C_c$

Substituting $\frac{V_c}{V_2}$ in equation (1)

$$\left\{ h_c = \frac{{V_2}^2}{2g}\left[\frac{V_c}{V_2} - 1\right]^2 ---(1) \right\}$$

Loss of Head due to Sudden Contraction(Cont…)

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$$h_c = \frac{{V_2}^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

Let k= $\left[\frac{1}{C_c} - 1\right]^2$

Loss of head due to sudden contraction $h_c = \frac{{V_2}^2}{2g}k$

If the value of C_c= 0.62 , k= $\left[\frac{1}{0.62} - 1\right]^2$=0.375

Loss of Head due to Sudden Contraction(Cont…)

$$h_c = \frac{V_2{}^2}{2g} k$$

$$h_c = 0.375 \frac{V_2{}^2}{2g}$$

If the C_c is not given then head loss due to contraction is taken as:

$$h_c = 0.5 \frac{V_2{}^2}{2g}$$

Problem:1

A horizontal pipe of diameter 500 mm is suddenly contracted to a diameter of 250 mm. The pressure intensities in the large and smaller pipe is given as 13.734 N/cm^2 and 11.772 N/cm^2 respectively. Find the loss of head due to contraction if C_c = 0.62. Also determine the rate of flow of water.

Solution. Given :

Dia. of large pipe, $D_1 = 500 \text{ mm} = 0.5 \text{ m}$

Area, $A_1 = \dfrac{\pi}{4}(0.5)^2 = 0.1963 \text{ m}^2$

Problem:1(Cont…)

Dia. of smaller pipe, $D_2 = 250 \text{ mm} = 0.25 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4}(.25)^2 = 0.04908 \text{ m}^2$

Pressure in large pipe, $p_1 = 13.734 \text{ N/cm}^2 = 13.734 \times 10^4 \text{ N/m}^2$

Pressure in smaller pipe, $p_2 = 11.772 \text{ N/cm}^2 = 11.772 \times 10^4 \text{ N/m}^2$

$C_c = 0.62$

$$\text{Head lost due to contraction} = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1.0\right]^2 = \frac{V_2^2}{2g}\left[\frac{1}{0.62} - 1.0\right]^2 = 0.375\,\frac{V_2^2}{2g}$$

From continuity equation, we have $A_1V_1 = A_2V_2$

Problem:1(Cont…)

or $$V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}D_2^2 \times V_2}{\frac{\pi}{4}D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{0.25}{0.50}\right)^2 V_2 = \frac{V_2}{4}$$

Applying Bernoulli's equation before and after contraction,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_c$$

But $$z_1 = z_2$$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_c$$

Problem:1(Cont…)

But $$h_c = 0.375\,\frac{V_2^2}{2g} \text{ and } V_1 = \frac{V_2}{4}$$

Substituting these values in the above equation, we get

$$\frac{13.734\times 10^4}{9.81\times 1000}+\frac{(V_2/4)^2}{2g}=\frac{11.772\times 10^4}{1000\times 9.81}+\frac{V_2^2}{2g}+0.375\,\frac{V_2^2}{2g}$$

or $$14.0+\frac{V_2^2}{16\times 2g}=12.0+1.375\,\frac{V_2^2}{2g}$$

Problem:1(Cont…)

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or $$14 - 12 = 1.375\,\frac{V_2^2}{2g} - \frac{1}{16}\,\frac{V_2^2}{2g} = 1.3125\,\frac{V_2^2}{2g}$$

or $$2.0 = 1.3125 \times \frac{V_2^2}{2g} \text{ or } V_2 = \sqrt{\frac{2.0 \times 2 \times 9.81}{1.3125}} = 5.467 \text{ m/s.}$$

(*i*) Loss of head due to contraction, $h_c = 0.375\,\dfrac{V_2^2}{2g} = \dfrac{0.375 \times (5.467)^2}{2 \times 9.81}$ = **0.571 m. Ans.**

(*ii*) Rate of flow of water, $Q = A_2V_2 = 0.04908 \times 5.467 = 0.2683 \text{ m}^3\text{/s}$ = **268.3 lit/s. Ans.**

Summary

- The loss of head due to sudden contraction is actually due to sudden enlargement form Vena-contracta to smaller pipe
- Loss of head due to sudden contraction is given by the expression as

$$\boldsymbol{h_c = \frac{{V_2}^2}{2g} k}$$

MINOR ENERGY LOSSES: PART-3

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Lecture Details:
Unit-III (Closed Conduit Flow) , Minor Energy Losses
FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
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MINOR ENERGY LOSSES: PART-3

Contents

- Loss of Head at the Entrance of Pipe
- Loss of Head at the Exit of Pipe
- Loss of Head due to an Obstruction in a Pipe
- Summary

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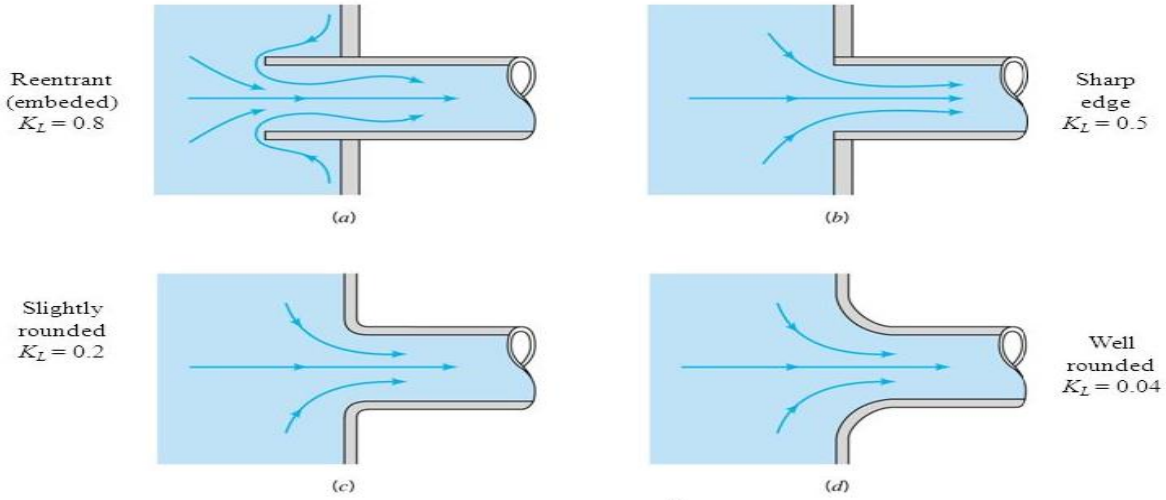
Loss of Head at the Entrance of Pipe

- This is loss of energy which occurs when a liquid enters a pipe which is connected to a large tank or reservoir
- This loss is similar to the loss of head due to sudden contraction
- This loss depends on the form of entrance
- For sharp entrance, this loss is slightly more than a rounded or bell mounted entrance

Loss of Head at the Entrance of Pipe(Cont…)

(flow leaving a tank)



$$h_L = K_L \frac{V^2}{2g}$$

Loss of Head at the Entrance of Pipe(Cont...)

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- In practice the value of loss of head at the entrance with sharp cornered

  is taken as $0.5\frac{V_2{}^2}{2g}$

- Where V = velocity of liquid in pipe

Thus, loss of head at the entrance of pipe $\boldsymbol{h_i = 0.5\frac{V_2{}^2}{2g}}$

Loss of Head at the Exit of Pipe

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- This head loss due to the velocity of liquid at the outlet of pipe which is dissipated either in the form of a free jet(if outlet of the pipe is free) or it is lost in the tank or reservoir

- This loss is equal to $\frac{V^2}{2g}$, where V= velocity of liquid at the outlet of pipe

Thus, loss of head at the exit of pipe $h_o = \frac{{V_2}^2}{2g}$

Loss of Head at the Exit of Pipe(Cont…)

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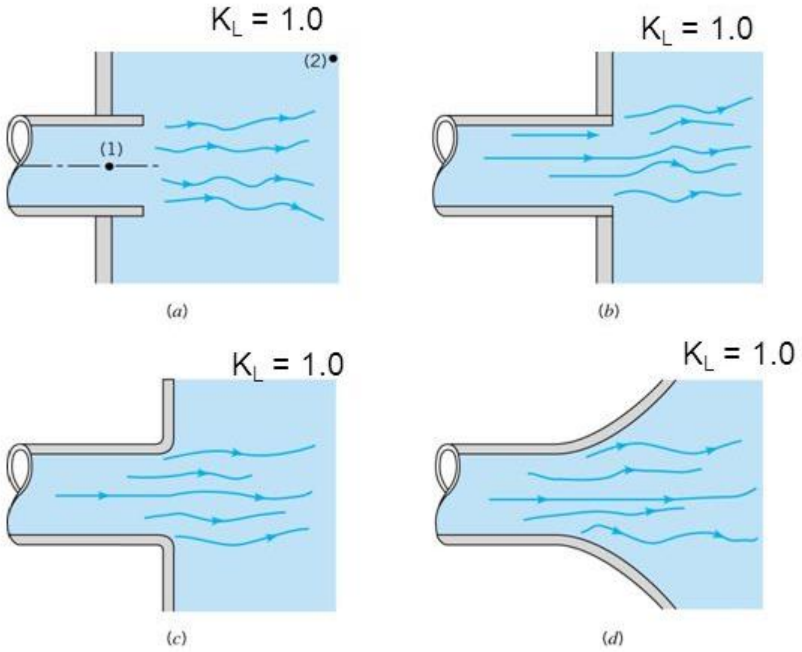
(flow entering a tank)

$$h_L = \frac{V^2}{2g}$$



Loss of Head at the Exit of Pipe(Cont…)

Loss of Head due to an Obstruction in a Pipe

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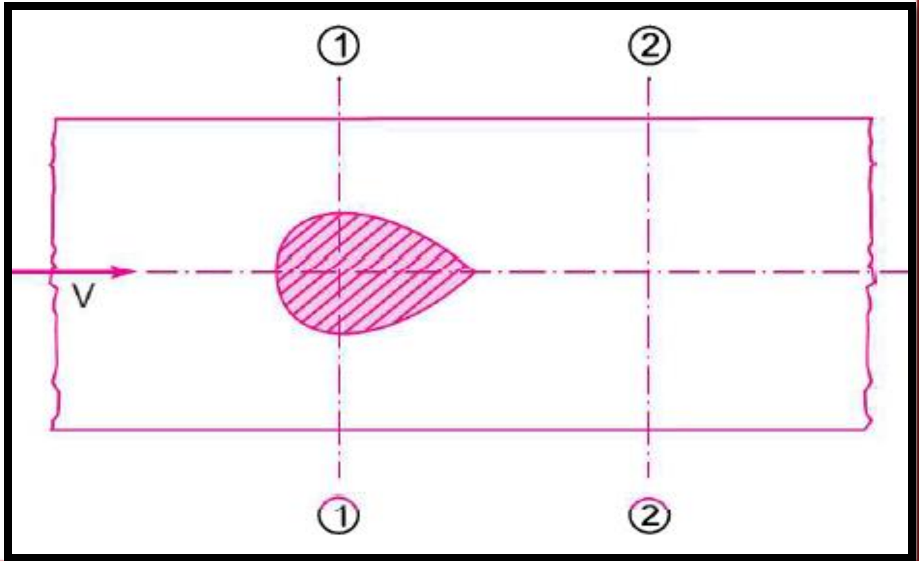
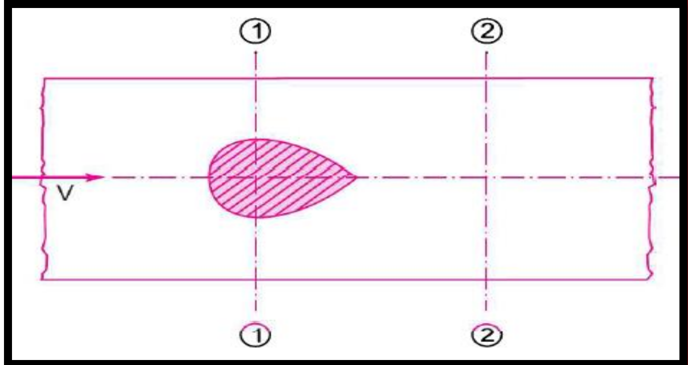
- Here loss of head (or energy) takes place due to reduction of area of the cross section of the pipe where the obstruction present
- There is sudden enlargement of the area of the flow beyond the obstruction
- Due to this loss of head takes place



Loss of Head due to an Obstruction in a Pipe(Cont...)



Let a = Maximum area of obstruction

A= Area of pipe,V= Velocity of liquid in pipe

(A-a)= Area of flow of liquid at section 1-1

- As the liquid flows and passes through section 1-1, a vena-contracta is formed beyond section 1-1
- After which the stream of liquid widens again and velocity of flow at section 2-2 becomes uniform and equal to the velocity , V in the pipe

Loss of Head due to an Obstruction in a Pipe(Cont…)

- This is similar to the flow of liquid through sudden enlargement
- Let V_c =Velocity of liquid at vena- contracta
- Thus loss of head due to an obstruction =loss of head due to enlargement from vena-contracta to section 2-2

$$\left\{\textbf{Loss of head due to sudden enlargement } \boldsymbol{h_e} = \frac{(V_1-V_2)^2}{2g}\right\}$$

$$= \frac{(V_c-V)^2}{2g} \text{-----------} (1)$$

Loss of Head due to an Obstruction in a Pipe(Cont…)

- From continuity equation, $a_c X\ V_c = AXV$ ------------(2)
- Let C_c= coefficient of contractin

$$C_c = \frac{area\ at\ vena\ contracta}{A - a} = \frac{a_c}{A - a}$$

$$a_c = C_c \mathrm{X}(A - a)$$

Substituting a_c in equatin(2)

$$C_c \mathrm{X}(A - a)V_c = AXV$$

Loss of Head due to an Obstruction in a Pipe(Cont…)

$$V_c = \frac{AV}{C_c(A-a)}$$

Substituting V_c in equation (1)

$$\left\{ \frac{(\boldsymbol{V_c - V})^{\mathbf{2}}}{\mathbf{2g}} \text{-------------} (1) \right\}$$

$$= \frac{\left(\frac{\mathrm{AV}}{C_c\left(A-a\right)} - \boldsymbol{V} \right)^{\mathbf{2}}}{\mathbf{2g}}$$

Loss of Head due to an Obstruction in a Pipe(Cont…)

Loss of head due to an obstruction = $\frac{V^2}{2g}\left(\frac{A}{C_c(A-a)} - 1\right)^2$

Problem:1

Water is flowing through a horizontal pipe of diameter 200 mm at a velocity of 3 m/s. A circular solid plate of diameter 150 mm is placed in the pipe to obstruct the flow. Find the loss of head due to obstruction in the pipe if $C_c = 0.62$.

Solution. Given :

Dia. of pipe, $D = 200 \text{ mm} = 0.20 \text{ m}$

Velocity, $V = 3.0 \text{ m/s}$

Area of pipe, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.2)^2 = 0.03141 \text{ m}^2$

Dia. of obstruction, $d = 150 \text{ mm} = 0.15 \text{ m}$

$\therefore$ Area of obstruction, $a = \frac{\pi}{4} (.15)^2 = 0.01767 \text{ m}^2$

Problem:1(Cont…)

$$C_c = 0.62$$

The head lost due to obstruction is given by equation as

$$= \frac{V^2}{2g}\left(\frac{A}{C_c\,(A-a)} - 1.0\right)^2$$

$$= \frac{3\times 3}{2\times 9.81}\left[\frac{.03141}{0.62\,[.03141 - .01767]} - 1.0\right]^2$$

$$= \frac{9}{2\times 9.81}\,[3.687 - 1.0]^2 = \mathbf{3.311\ m.\ Ans.}$$

Summary

- The Loss of Head at the Entrance of Pipe is similar to the loss of head due to sudden contraction
- Loss of head at the entrance of pipe is $\boldsymbol{h_i = 0.5\frac{{V_2}^2}{2g}}$
- Loss of head at the exit of pipe is $\boldsymbol{h_o = \frac{{V_2}^2}{2g}}$
- Loss of head due to an obstruction = $\boldsymbol{\frac{V^2}{2g}\left(\frac{A}{C_c(A-a)} - 1\right)^2}$

MINOR ENERGY LOSSES: PART-4

Presented By:
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Lecture Details:
Unit-III (Closed Conduit Flow) , Minor Energy Losses
FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
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MINOR ENERGY LOSSES: PART-4

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Contents

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- Loss of Head due to Bend in Pipe
- Loss of Head In Various Pipe Fittings
- Summary

Loss of Head due to Bend in Pipe

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- When there is any bend in a pipe, the velocity of flow changes, due to which the separation of flow from the boundary and also formation of eddies takes place

Loss of head in pipe due to bend is: $\boldsymbol{h_b = \frac{kV^2}{2g}}$

Where V= velocity of flow, k= co-efficient of bend

The value of k depends on (i) Angle of bend (ii) Radius of curvature of bend (iii) Diameter of pipe

Loss of Head In Various Pipe Fittings

- The loss of head in various pipe fittings such as valves, couplings etc., is equal to $\frac{kV^2}{2g}$

Where V= velocity of flow

k= co-efficient of pipe fitting

Problem:1

Determine the rate of flow of water through a pipe of diameter 20 cm and length 50 m when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The pipe is horizontal and the height of water in the tank is 4 m above the centre of the pipe. Consider all minor losses and take f = .009 in the formula $h_f = \dfrac{4 . f . L . V^2}{d \times 2g}$.

Solution. Dia. of pipe, $d = 20 \text{ cm} = 0.20 \text{ m}$

Length of pipe, $L = 50 \text{ m}$

Height of water, $H = 4 \text{ m}$

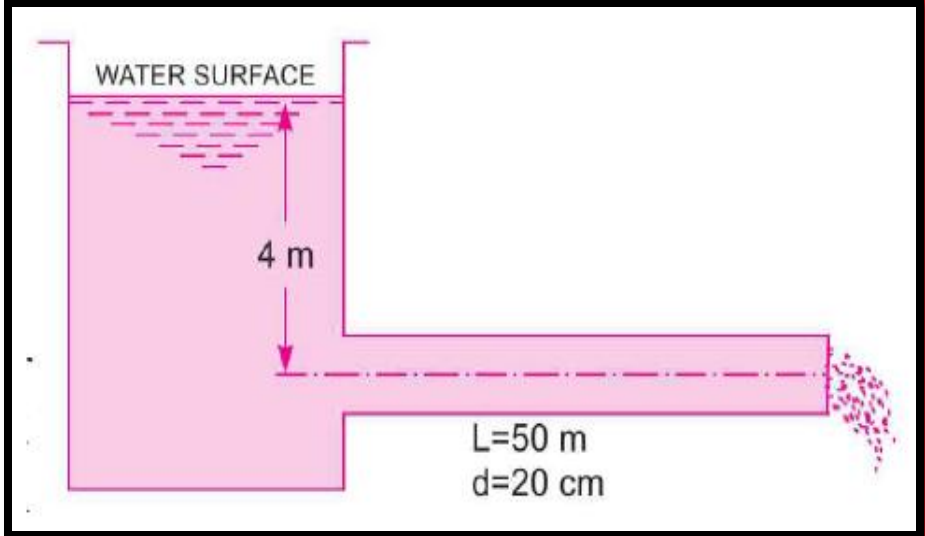
Co-efficient of friction, $f = .009$

Let the velocity of water in pipe $= V$ m/s.

Problem:1(Cont...)

Applying Bernoulli's equation at the top of the water surface in the tank and at the outlet of pipe, we have [Taking point 1 on the top and point 2 at the outlet of pipe].

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{all losses}$$



Problem:1(Cont...)

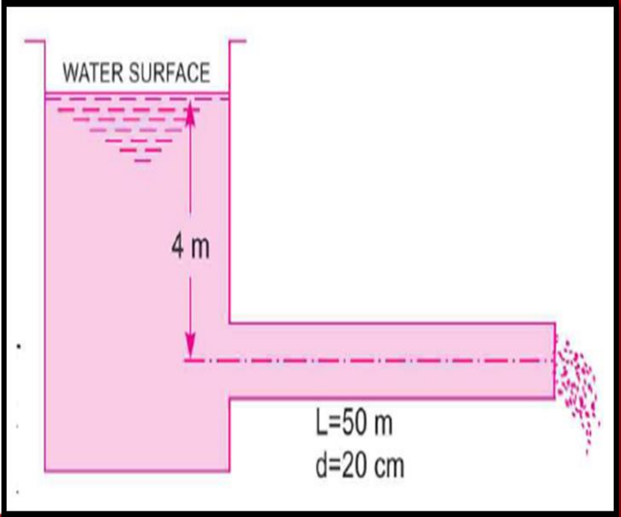
Considering datum line passing through the centre of pipe

$$0 + 0 + 4.0 = 0 + \frac{V_2^2}{2g} + 0 + (h_i + h_f)$$

$$4.0 = \frac{V_2^2}{2g} + h_i + h_f$$

But the velocity in pipe = V, $\quad \therefore V = V_2$

$$\therefore \quad 4.0 = \frac{V^2}{2g} + h_i + h_f$$



Problem:1(Cont...)

From equation $h_i = 0.5\dfrac{V^2}{2g}$; $h_f = \dfrac{4.f.L.V^2}{d \times 2g}$

Substituting these values, we have

$$4.0 = \frac{V^2}{2g} + \frac{0.5\,V^2}{2g} + \frac{4.f.L.V^2}{d \times 2g}$$

$$= \frac{V^2}{2g}\left[1.0 + 0.5 + \frac{4 \times .009 \times 50}{0.2}\right] = \frac{V^2}{2g}\,[1.0 + 0.5 + 9.0]$$

$$= 10.5 \times \frac{V^2}{2g}$$

Problem:1(Cont...)

$\therefore$ $$V = \sqrt{\frac{4 \times 2 \times 9.81}{10.5}} = 2.734 \text{ m/sec}$$

$\therefore$ Rate of flow, $$Q = A \times V = \frac{\pi}{4} \times (0.2)^2 \times 2.734 = 0.08589 \text{ m}^3\text{/s}$$

$$= \mathbf{85.89 \text{ litres/s. Ans.}}$$

Problem:2

A horizontal pipe line 40 m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 25 m of its length from the tank, the pipe is 150 mm diameter and its diameter is suddenly enlarged to 300 mm. The height of water level in the tank is 8 m above the centre of the pipe. Considering all losses of head which occur, determine the rate of flow. Take f = .01 for both sections of the pipe.

Solution. Given :

Total length of pipe, $L = 40$ m

Length of 1st pipe, $L_1 = 25$ m

Dia. of 1st pipe, $d_1 = 150$ mm $= 0.15$ m

Problem:2(Cont...)

Length of 2nd pipe, $L_2 = 40 - 25 = 15$ m

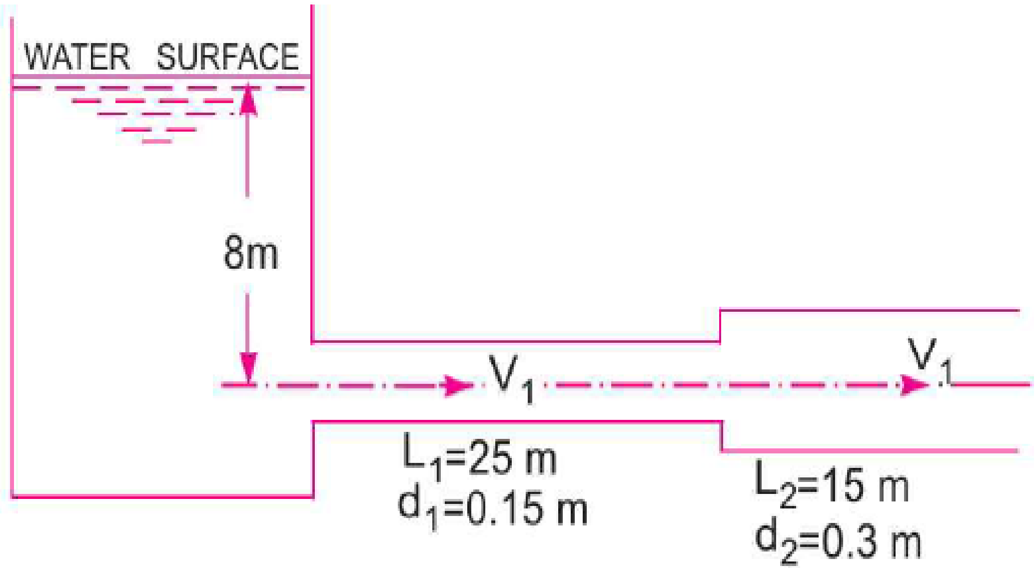
Dia. of 2nd pipe, $d_2 = 300$ mm $= 0.3$ m

Height of water, $H = 8$ m

Co-efficient of friction, $f = 0.01$

Applying Bernoulli's theorem to the free surface of water in the tank and outlet of pipe as shown in Fig. and taking reference line passing through the centre of pipe.

Problem:2(Cont...)



$$0 + 0 + 8 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + 0 + \text{all losses}$$

Problem:2(Cont...)

or $$8.0 = 0 + \frac{V_2^2}{2g} + h_i + h_{f_1} + h_e + h_{f_2} \quad \ldots(i)$$

where h_i = loss of head at entrance = $0.5 \dfrac{V_1^2}{2g}$

h_{f_1} = head lost due to friction in pipe 1 = $\dfrac{4 \times f \times L_1 \times V_1^2}{d_1 \times 2g}$

h_e = loss head due to sudden enlargement = $\dfrac{(V_1 - V_2)^2}{2g}$

h_{f_2} = Head lost due to friction in pipe 2 = $\dfrac{4 \times f \times L_2 \times V_2^2}{d_2 \times 2g}$

Problem:2(Cont...)

But from continuity equation, we have

$$A_1V_1 = A_2V_2$$

$$\therefore \qquad V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}d_2^2 \times V_2}{\frac{\pi}{4}d_1^2} = \left(\frac{d_2}{d_1}\right)^2 \times V_2 = \left(\frac{0.3}{.15}\right)^2 \times V_2 = 4V_2$$

Substituting the value of V_1 in different head losses, we have

$$h_i = \frac{0.5\,V_1^2}{2g} = \frac{0.5 \times (4V_2)^2}{2g} = \frac{8V_2^2}{2g}$$

Problem:2(Cont…)

$$h_{f_1} = \frac{4\times 0.01\times 25\times (4V_2)^2}{0.15\times 2\times g}$$

$$= \frac{4\times .01\times 25\times 16}{0.15}\times \frac{V_2^2}{2g} = 106.67\ \frac{V_2^2}{2g}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(4V_2 - V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

$$h_{f_2} = \frac{4\times .01\times 15\times V_2^2}{0.3\times 2g} = \frac{4\times .01\times 15}{0.3}\times \frac{V_2^2}{2g} = 2.0\times \frac{V_2^2}{2g}$$

Problem:2(Cont...)

Substituting the values of these losses in equation (*i*), we get

$$8.0 = \frac{V_2^2}{2g} + \frac{8V_2^2}{2g} + 106.67\,\frac{V_2^2}{2g} + \frac{9V_2^2}{2g} + 2 \times \frac{V_2^2}{2g}$$

$$= \frac{V_2^2}{2g}\,[1 + 8 + 106.67 + 9 + 2] = 126.67\,\frac{V_2^2}{2g}$$

$$\therefore \quad V_2 = \sqrt{\frac{8.0 \times 2 \times g}{126.67}} = \sqrt{\frac{8.0 \times 2 \times 9.81}{126.67}} = \sqrt{1.2391} = 1.113 \text{ m/s}$$

$\therefore$ Rate of flow, $Q = A_2 \times V_2 = \frac{\pi}{4}(0.3)^2 \times 1.113 = 0.07867 \text{ m}^3\text{/s}$ = **78.67 litres/s. Ans.**

Summary

- The Loss of head in pipe due to bend is: $\boldsymbol{h_b = \frac{kV^2}{2g}}$

- The loss of head in various pipe fittings is: $\boldsymbol{\frac{kV^2}{2g}}$

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HYDRAULIC GRADIENT AND TOTAL ENERGY LINE

Presented By:
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Lecture Details:
Unit-III (Closed Conduit Flow) , Minor Energy Losses
FM & HM /Mechanical, I -Semester.

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HYDRAULIC GRADIENT AND TOTAL ENERGY LINE

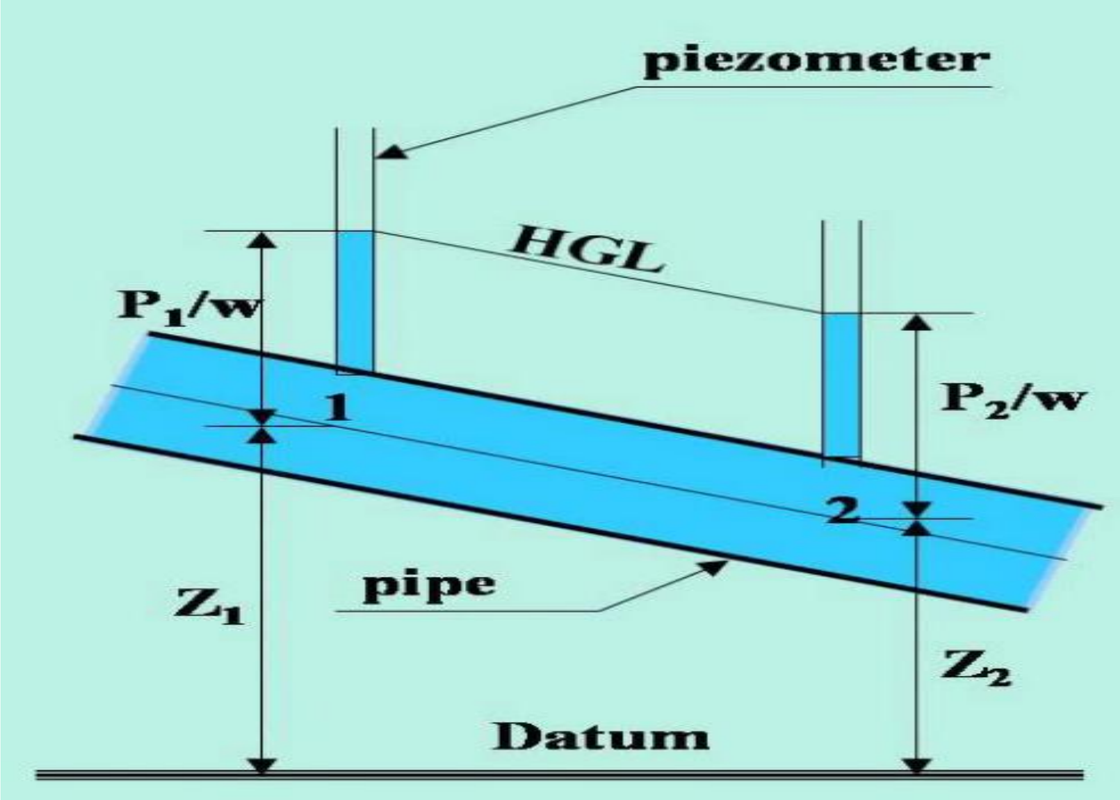
Contents

- Hydraulic Gradient Line(H.G.L)
- Total Energy Line(T.E.L)
- Summary

Hydraulic Gradient Line(H.G.L)

- It is defined as the line which gives the sum of pressure head $(\frac{p}{w})$ and datum head(z) of a flowing fluid in a pipe with respect to some reference line which is obtained by joining the top of all vertical ordinates, showing the pressure head $(\frac{p}{w})$ of a flowing fluid in a pipe from the centre of the pipe

Hydraulic Gradient Line(H.G.L)(Cont…)



Hydraulic Gradient Line(H.G.L)(Cont…)

Hydraulic gradient line(HGL)

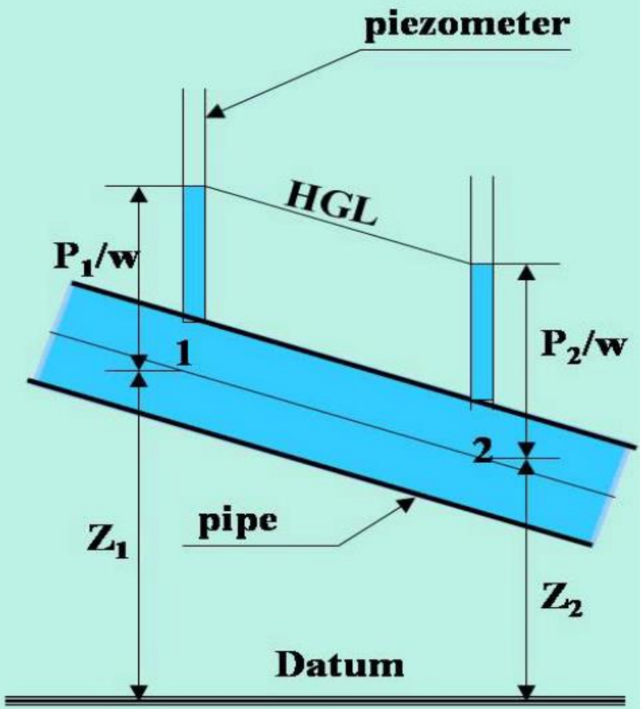
Piezometric head at section 1

$$= P_1/w + Z_1$$

Piezomtric head at section 2

$$= P_2/w + Z_2$$

PZL joins $P_1/w + Z_1$ with $P_2/w + Z_2$ as shown in fig.



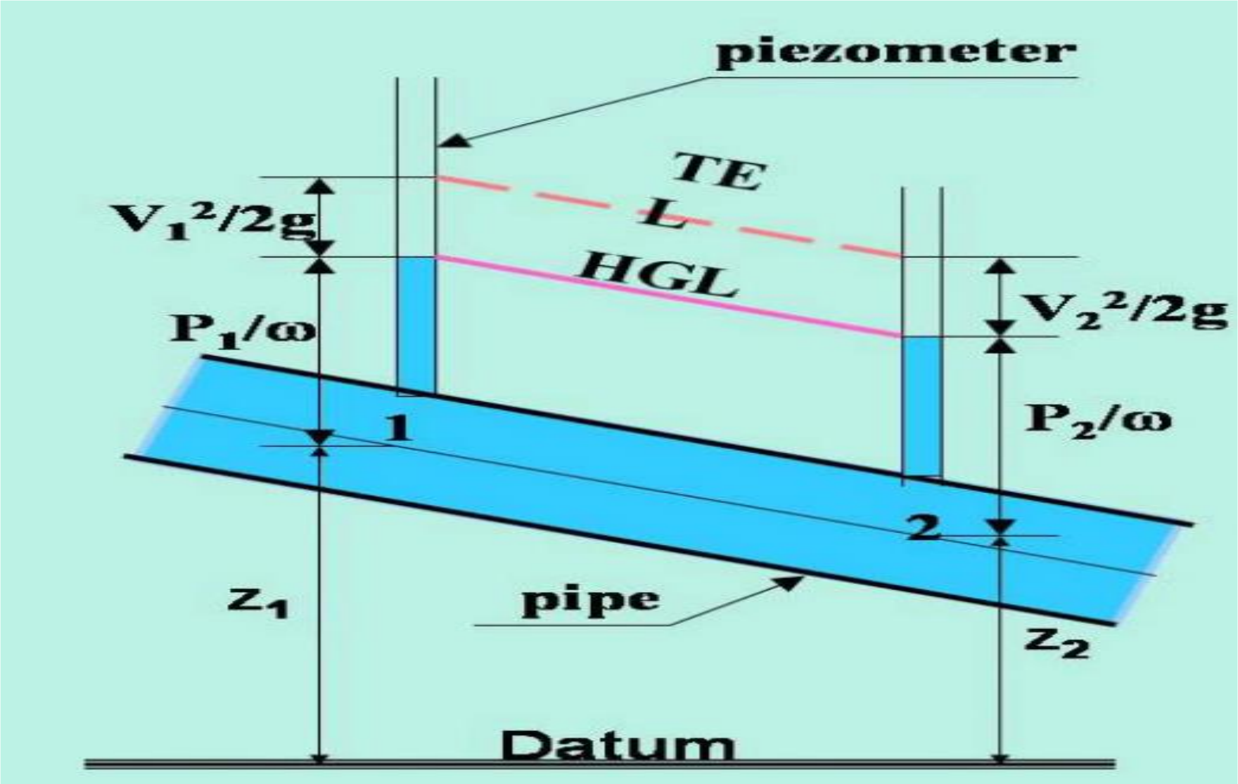
Total Energy Line(T.E.L)

- It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line

or

- It is defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe

Total Energy Line(T.E.L)(Cont…)



Total Energy Line(T.E.L)(Cont…)

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Total energy line(TEL)

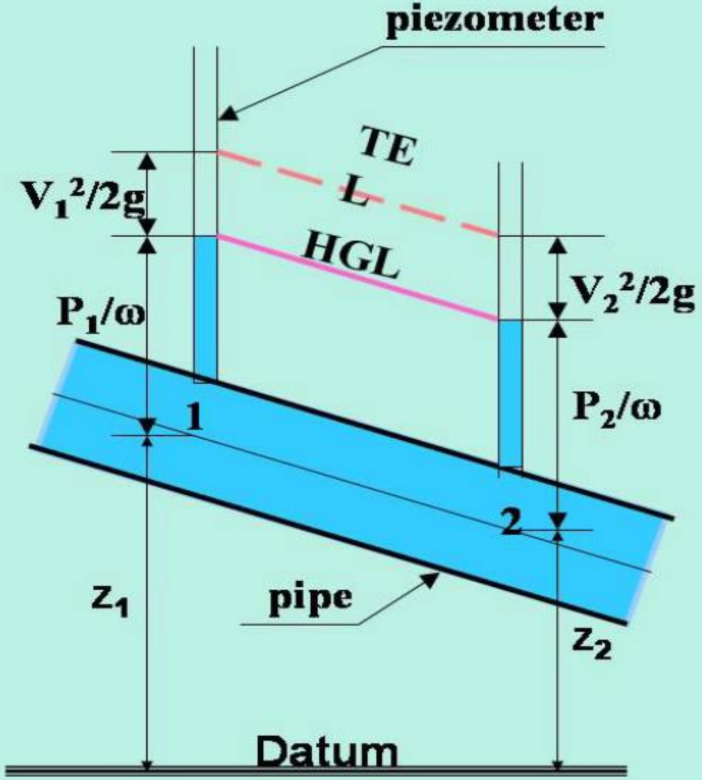
Total energy at section 1

$$= P_1/\omega + V_1^2/2g + Z_1$$

Total energy at section 2

$$= P_2/\omega + V_2^2/2g + Z_2$$

TEL joins $P_1/\omega + V_1^2/2g + Z_1$ **with** $P_2/+ V_2^2/2g + Z_2$ **as shown in fig.**



Problem:1

Determine the rate of flow of water through a pipe of diameter 20 cm and length 50 m when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The pipe is horizontal and the height of water in the tank is 4 m above the centre of the pipe. Consider all minor losses and take $f = .009$ in the formula $h_f = \dfrac{4 \cdot f \cdot L \cdot V^2}{d \times 2g}$.

For the problem , draw the Hydraulic Gradient Line (H.G.L.) and Total Energy Line (T.E.L.).

Problem:1(Cont…)

Solution. Dia. of pipe, $d = 20 \text{ cm} = 0.20 \text{ m}$

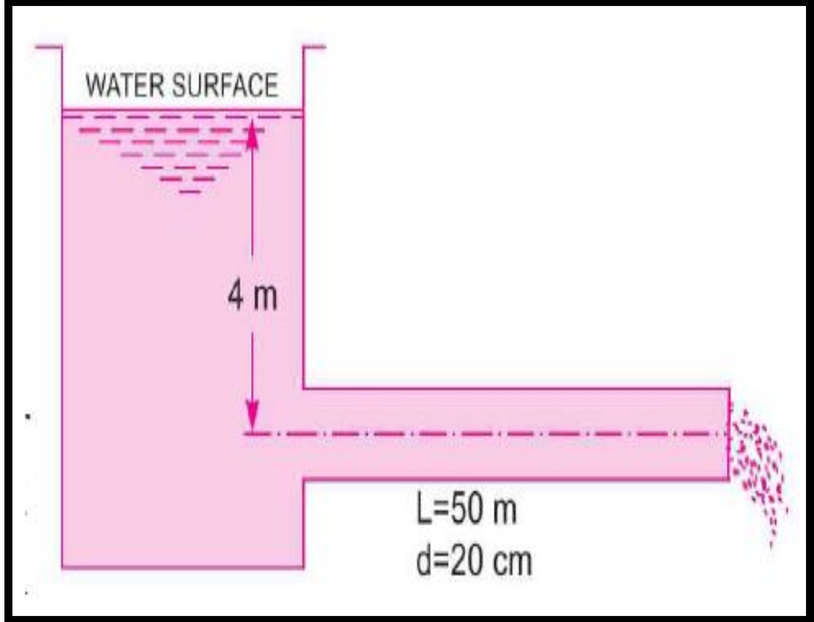
Length of pipe, $L = 50 \text{ m}$

Height of water, $H = 4 \text{ m}$

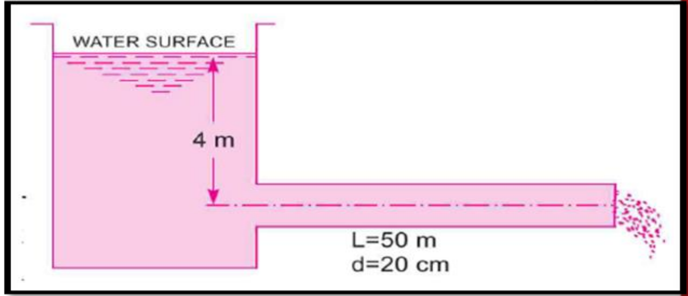
Co-efficient of friction, $f = .009$

Let the velocity of water in pipe $= V$ m/s.

Applying Bernoulli's equation at the top of the water surface in the tank and at the outlet of pipe, we have [Taking point 1 on the top and point 2 at the outlet of pipe].



Problem:1(Cont…)



$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{all losses}$$

Considering datum line passing through the centre of pipe

$$0 + 0 + 4.0 = 0 + \frac{V_2^2}{2g} + 0 + (h_i + h_f)$$

$$4.0 = \frac{V_2^2}{2g} + h_i + h_f$$

Problem:1(Cont...)

But the velocity in pipe = V, $\quad \therefore V = V_2$

$$\therefore \qquad 4.0 = \frac{V^2}{2g} + h_i + h_f$$

From equation $h_i = 0.5\,\frac{V^2}{2g}$; $\quad h_f = \frac{4\,.\,f\,.\,L\,.\,V^2}{d \times 2g}$

Substituting these values, we have

$$4.0 = \frac{V^2}{2g} + \frac{0.5\,V^2}{2g} + \frac{4\,.\,f\,.\,L\,.\,V^2}{d \times 2g}$$

Problem:1(Cont...)

$$= \frac{V^2}{2g}\left[1.0 + 0.5 + \frac{4 \times .009 \times 50}{0.2}\right] = \frac{V^2}{2g}\;[1.0 + 0.5 + 9.0]$$

$$= 10.5 \times \frac{V^2}{2g}$$

$\therefore$ $$V = \sqrt{\frac{4 \times 2 \times 9.81}{10.5}} = 2.734 \text{ m/sec}$$

$\therefore$ Rate of flow, $$Q = A \times V = \frac{\pi}{4} \times (0.2)^2 \times 2.734 = 0.08589 \text{ m}^3\text{/s}$$

$$= \mathbf{85.89 \text{ litres/s. Ans.}}$$

Problem:1(Cont...)

h_i = Head lost at entrance of pipe

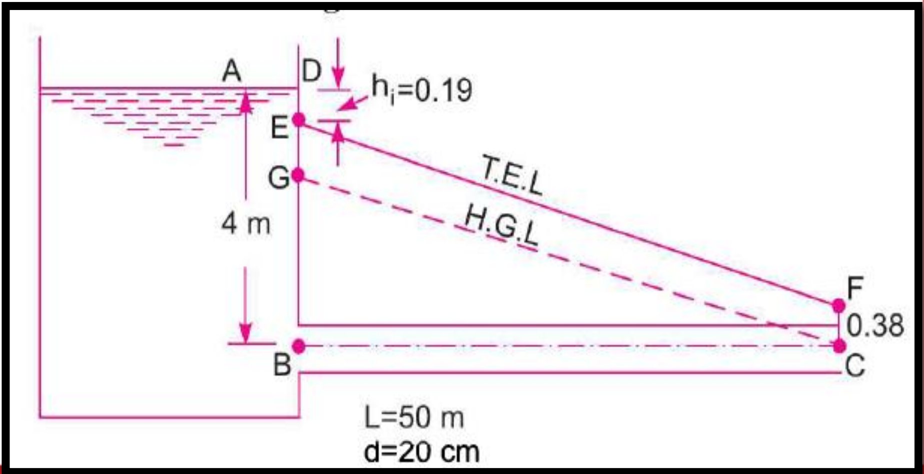
$$= 0.5\,\frac{V^2}{2g} = \frac{0.5 \times 2.734^2}{2 \times 9.81} = 0.19 \text{ m}$$

and h_f = Head loss due to friction

$$= \frac{4 \times f \times L \times V^2}{d \times 2g} = \frac{4 \times 0.009 \times 50 \times (2.734)^2}{0.2 \times 2 \times 9.81} = 3.428 \text{ m.}$$

Problem:1(Cont...)

(*a*) **Total Energy Line (T.E.L.).** Consider three points, *A*, *B* and *C* on the free surface of water in the tank, at the inlet of the pipe and at the outlet of the pipe respectively as shown in Fig. Let us find total energy at these points, taking the centre of pipe as reference line.

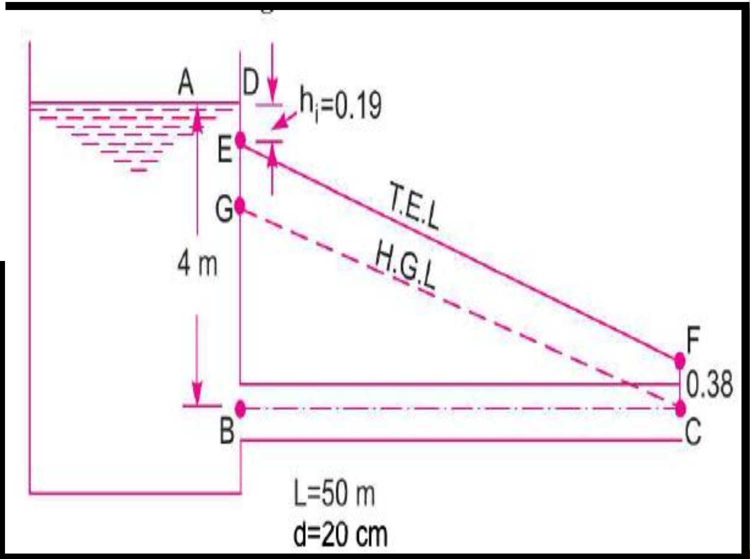


Problem:1(Cont…)

1. $\text{Total energy at } A = \frac{p}{\rho g} + \frac{V^2}{2g} + z = 0 + 0 + 4.0 = 4 \text{ m}$

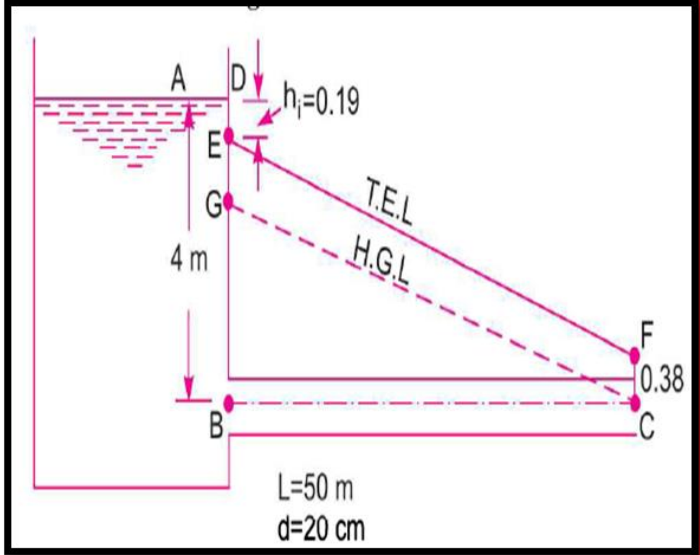
2. $\text{Total energy at } B = \text{Total energy at } A - h_i = 4.0 - 0.19 = 3.81 \text{ m}$

3. $\text{Total energy at } C = \frac{p_c}{\rho g} + \frac{V_c^2}{2g} + z_c = 0 + \frac{V^2}{2g} + 0 = \frac{2.734^2}{2 \times 9.81} = 0.38 \text{ m.}$



Problem:1(Cont...)

Hence total energy line will coincide with free surface of water in the tank. At the inlet of the pipe, it will decrease by h_i (= 0.19 m) from free surface and at outlet of pipe total energy is 0.38 m. Hence in Fig.



(*i*) Point D represents total energy at A

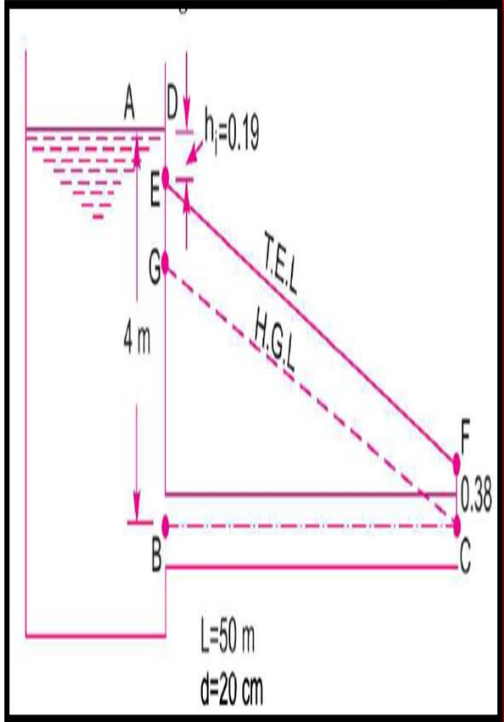
(*ii*) Point E, where $DE = h_i$, represents total energy at inlet of the pipe

(*iii*) Point F, where $CF = 0.38$ represents total energy at outlet of pipe.

Join D to E and E to F. Then DEF represents the total energy line.

Problem:1(Cont...)

(*b*) **Hydraulic Gradient Line (H.G.L.).** H.G.L. gives the sum of $(p/w + z)$ with reference to the datum-line. Hence hydraulic gradient line is obtained by subtracting $\frac{V^2}{2g}$ from total energy line. At outlet of the pipe, total energy = $\frac{V^2}{2g}$. By subtracting $\frac{V^2}{2g}$ from total energy at this point, we shall get point *C*, which lies on the centre line of pipe. From *C*, draw a line *CG* parallel to *EF*. Then *CG* represents the hydraulic gradient line.



Problem:2

A horizontal pipe line 40 m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 25 m of its length from the tank, the pipe is 150 mm diameter and its diameter is suddenly enlarged to 300 mm. The height of water level in the tank is 8 m above the centre of the pipe. Considering all losses of head which occur, determine the rate of flow. Take f = .01 for both sections of the pipe.

For the problem draw the hydraulic gradient and total energy line.

Solution. Given :

Total length of pipe, $L = 40$ m

Length of 1st pipe, $L_1 = 25$ m

Dia. of 1st pipe, $d_1 = 150 \text{ mm} = 0.15$ m

Problem:2(Cont...)

Length of 2nd pipe, $L_2 = 40 - 25 = 15$ m

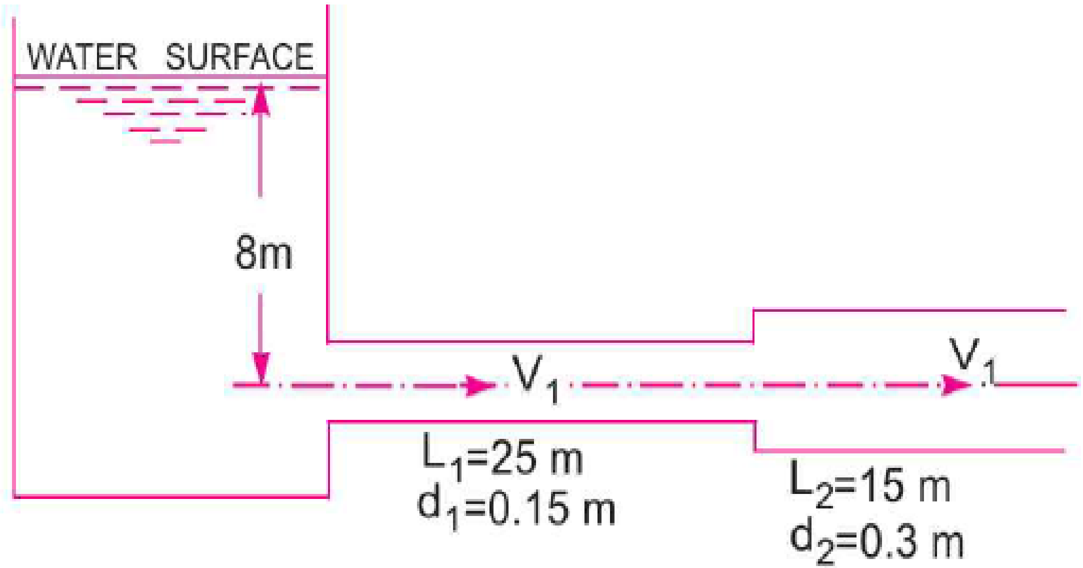
Dia. of 2nd pipe, $d_2 = 300$ mm $= 0.3$ m

Height of water, $H = 8$ m

Co-efficient of friction, $f = 0.01$

Applying Bernoulli's theorem to the free surface of water in the tank and outlet of pipe as shown in Fig. and taking reference line passing through the centre of pipe.

Problem:2(Cont...)



$$0 + 0 + 8 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + 0 + \text{all losses}$$

Problem:2(Cont...)

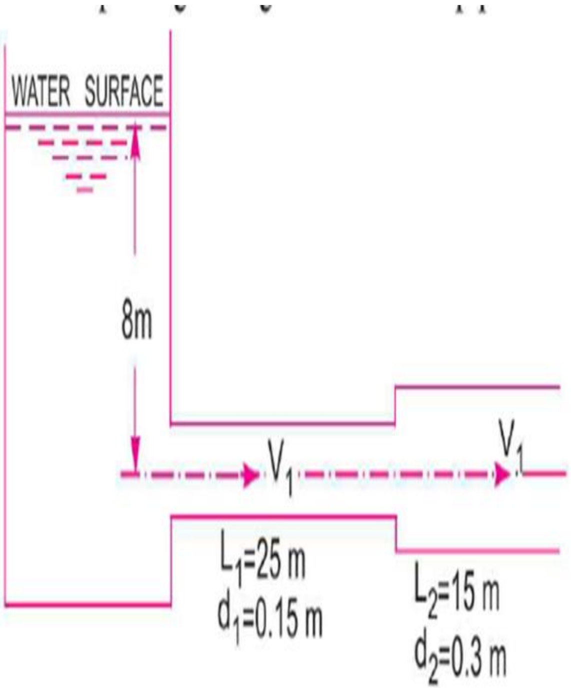
or $$8.0 = 0 + \frac{V_2^2}{2g} + h_i + h_{f_1} + h_e + h_{f_2}$$

where h_i = loss of head at entrance $= 0.5 \dfrac{V_1^2}{2g}$

h_{f_1} = head lost due to friction in pipe 1 $= \dfrac{4 \times f \times L_1 \times V_1^2}{d_1 \times 2g}$

h_e = loss head due to sudden enlargement $= \dfrac{(V_1 - V_2)^2}{2g}$

h_{f_2} = Head lost due to friction in pipe 2 $= \dfrac{4 \times f \times L_2 \times V_2^2}{d_2 \times 2g}$



Problem:2(Cont…)

But from continuity equation, we have

$$A_1V_1 = A_2V_2$$

$$\therefore \qquad V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}d_2^2 \times V_2}{\frac{\pi}{4}d_1^2} = \left(\frac{d_2}{d_1}\right)^2 \times V_2 = \left(\frac{0.3}{.15}\right)^2 \times V_2 = 4V_2$$

Substituting the value of V_1 in different head losses, we have

$$h_i = \frac{0.5\,V_1^2}{2g} = \frac{0.5 \times (4V_2)^2}{2g} = \frac{8V_2^2}{2g}$$

Problem:2(Cont…)

$$h_{f_1} = \frac{4 \times 0.01 \times 25 \times (4V_2)^2}{0.15 \times 2 \times g}$$

$$= \frac{4 \times .01 \times 25 \times 16}{0.15} \times \frac{V_2^2}{2g} = 106.67\ \frac{V_2^2}{2g}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(4V_2 - V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

$$h_{f_2} = \frac{4 \times .01 \times 15 \times V_2^2}{0.3 \times 2g} = \frac{4 \times .01 \times 15}{0.3} \times \frac{V_2^2}{2g} = 2.0 \times \frac{V_2^2}{2g}$$

Problem:2(Cont…)

Substituting the values of these losses in equation (*i*), we get

$$8.0 = \frac{V_2^2}{2g} + \frac{8V_2^2}{2g} + 106.67\frac{V_2^2}{2g} + \frac{9V_2^2}{2g} + 2 \times \frac{V_2^2}{2g}$$

$$= \frac{V_2^2}{2g}[1 + 8 + 106.67 + 9 + 2] = 126.67\frac{V_2^2}{2g}$$

$$\therefore \quad V_2 = \sqrt{\frac{8.0 \times 2 \times g}{126.67}} = \sqrt{\frac{8.0 \times 2 \times 9.81}{126.67}} = \sqrt{1.2391} = 1.113 \text{ m/s}$$

$\therefore$ Rate of flow, $Q = A_2 \times V_2 = \frac{\pi}{4}(0.3)^2 \times 1.113 = 0.07867 \text{ m}^3/\text{s} =$ **78.67 litres/s. Ans.**

Problem:2(Cont...)

Total Energy Line

(*i*) Point *A* lies on free surface of water.

(*ii*) Take $AB = h_i = 0.5$ m.

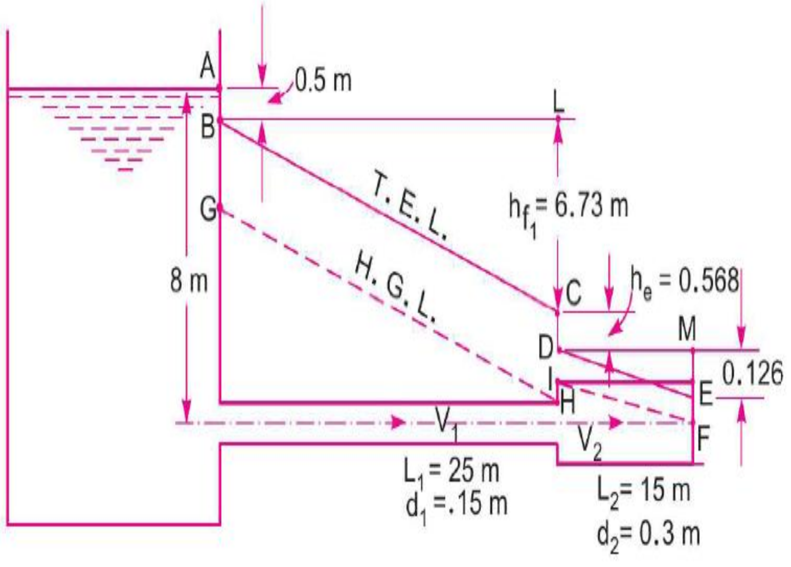
(*iii*) From *B*, draw a horizontal line. Take *BL* equal to the length of pipe, *i.e.*, L_1. From *L* draw a vertical line downward.

(*iv*) Cut the line $LC = h_{f_1} = 6.73$ m.

(*v*) Join the point *B* to *C*. From *C*, take a line *CD* vertically downward equal to $h_e = 0.568$ m.

(*vi*) From *D*, draw *DM* horizontal and from point *F* which is lying on the centre of the pipe, draw a vertical line in the upward direction, meeting at *M*. From *M*, take a distance $ME = h_{f_2} = 0.126$. Join *DE*.

Then line *ABCDE* represents the total energy line.



Problem:2(Cont…)

Hydraulic Gradient Line (H.G.L.)

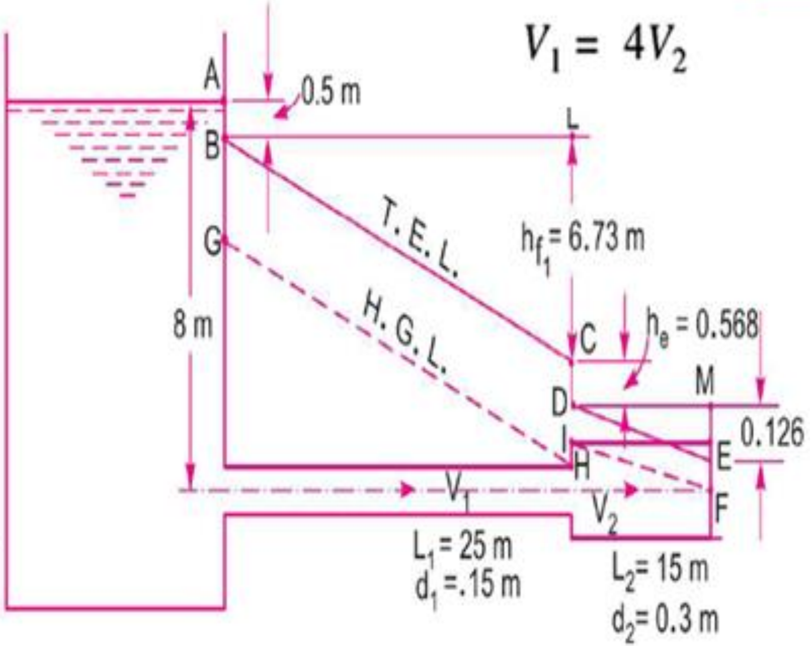
(i) From B, take $BG = V_1^2/2g = \dfrac{4.452^2}{2 \times 9.81} = 1.0$ m.

(ii) Draw the line GH parallel to the line BC.

(iii) From F, draw a line FI parallel to the line ED.

(iv) Join the point H and I.

Then the line $GHIF$ represents the hydraulic gradient line (H.G.L.).



Summary

- Hydraulic Gradient Line is sum of pressure head $(\frac{p}{w})$ and datum head(z)
- Total Energy Line is sum of pressure head, datum head and kinetic head

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FLOW THROUGH PIPES IN SERIES AND PARALLEL

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Lecture Details:
Unit-III (Closed Conduit Flow) , Flow through pipes in series and parallel
FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
- FLUID DYNAMICS
- CLOSED CONDUIT FLOW
- BOUNDARY LAYER THEORY AND APPLICATIONS
- BASICS OF TURBO MACHINERY
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FLOW THROUGH PIPES IN SERIES AND PARALLEL

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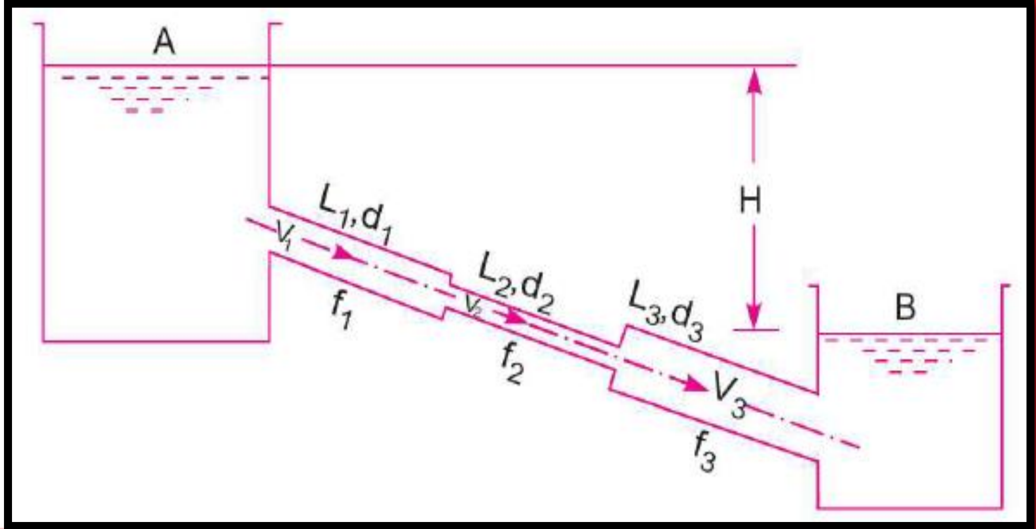
Contents

- Flow through Pipes in series or Flow through Compound Pipes
- **Flow through Parallel Pipes**
- Summary

Flow through Pipes in series or Flow through Compound Pipes

- Pipes in series or compound pipes are defined as the pipe of different lengths and different diameters connected end to end(in series) to form a pipe line as shown in fig
- Let L_1, L_2, L_3 = length of pipes 1,2 and 3 respectively

d_1, d_2, d_3 = diamter of pipes 1,2 and 3 respectively



Flow through Pipes in series or Flow through Compound Pipes(Cont…)

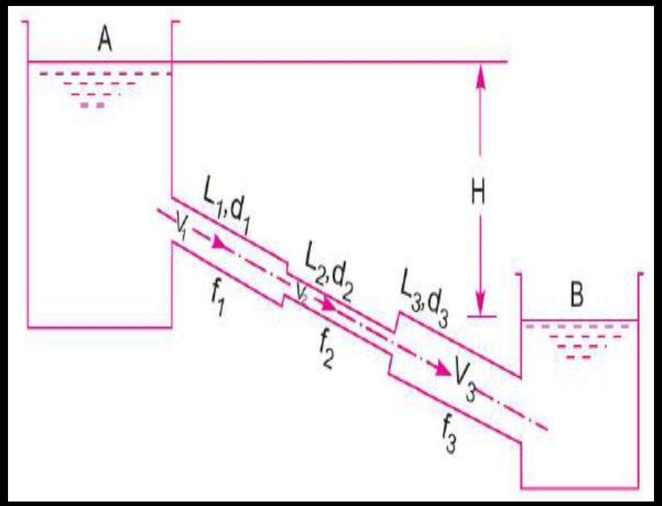
V_1, V_2, V_3 =velocity of flow through pipes 1,2 and 3 respectively

f_1, f_2, f_3 = coefficient of frictions for pipes 1,2 and 3 respectively

H= difference of water level in the two tanks

The discharge passing through each pipe is same

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$



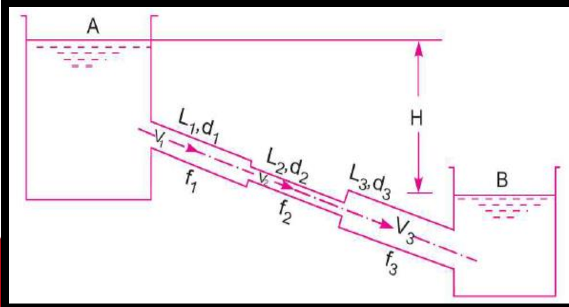
Flow through Pipes in series or Flow through Compound Pipes(Cont…)

- The difference in liquid surface levels is equal to sum of the total head loss in the pipes

$$H = 0.5\frac{V_1^2}{2g} + \frac{4f_1L_1V_1^2}{d_1 X 2g} + 0.5\frac{V_2^2}{2g} + \frac{4f_2L_2V_2^2}{d_2 X 2g} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{d_3 X 2g} + \frac{V_3^2}{2g}$$

- If the minor losses are neglected, then the equation becomes as

$$H = \frac{4f_1L_1V_1^2}{d_1 X 2g} + \frac{4f_2L_2V_2^2}{d_2 X 2g} + \frac{4f_3L_3V_3^2}{d_3 X 2g}$$



Flow through Pipes in series or Flow through Compound Pipes(Cont...)

- If the coefficient of friction is same for all pipes, i.e $\boldsymbol{f_1}$=$\boldsymbol{f_2 = f_3 = f}$

$$H=\frac{4fL_1{V_1}^2}{d_1X2g}+\frac{4fL_2{V_2}^2}{d_2X2g}+\frac{4fL_3{V_3}^2}{d_3X2g}$$

$$\mathbf{H}=\frac{4f}{2g}\left[\frac{L_1{V_1}^2}{d_1}+\frac{L_2{V_2}^2}{d_2}+\frac{L_3{V_3}^2}{d_3}\right]$$

Problem:1

The difference in water surface levels in two tanks, which are connected by three pipes in series of lengths 300 m, 170 m and 210 m and of diameters 300 mm, 200 mm and 400 mm respectively, is 12 m. Determine the rate of flow of water if co-efficient of friction are .005, .0052 and .0048 respectively, considering : (i) minor losses also (ii) neglecting minor losses.

Solution. Given :

Difference of water level, $H = 12$ m

Length of pipe 1, $L_1 = 300$ m and dia., $d_1 = 300$ mm $= 0.3$ m

Length of pipe 2, $L_2 = 170$ m and dia., $d_2 = 200$ mm $= 0.2$ m

Length of pipe 3, $L_3 = 210$ m and dia., $d_3 = 400$ mm $= 0.4$ m

Also, $f_1 = .005$, $f_2 = .0052$ and $f_3 = .0048$

Problem:1(Cont...)

(*i*) **Considering Minor Losses.** Let V_1, V_2 and V_3 are the velocities in the 1st, 2nd and 3rd pipe respectively.

From continuity, we have $A_1V_1 = A_2V_2 = A_3V_3$

$$V_2 = \frac{A_1V_1}{A_2} = \frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_2^2}V_1 = \frac{d_1^2}{d_2^2}V_1 = \left(\frac{0.3}{.2}\right)^2 \times V_1 = 2.25\ V_1$$

$$V_3 = \frac{A_1V_1}{A_3} = \frac{d_1^2}{d_3^2}V_1 = \left(\frac{0.3}{0.4}\right)^2 V_1 = 0.5625\ V_1$$

Problem:1(Cont...)

Now using equation

$$H = \frac{0.5\,V_1^2}{2g} + \frac{4 f_1 L_1 V_1^2}{d_1 \times 2g} + \frac{0.5\,V_2^2}{2g} + \frac{4 f_2 L_2 V_2^2}{d_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4 f_3 L_3 V_3^2}{d_3 \times 2g} + \frac{V_3^2}{2g}$$

Substituting V_2 and V_3, $$12.0 = \frac{0.5\,V_1^2}{2g} + \frac{4 \times .005 \times 300 \times V_1^2}{0.3 \times 2g} + \frac{0.5 \times \left(2.25 V_1^2\right)^2}{2g}$$

$$+ 4 \times 0.0052 \times 170 \times \frac{(2.25\,V_1)^2}{0.2 \times 2g} + \frac{(2.25\,V_1 - .562\,V_1)^2}{2g} + \frac{4 \times .0048 \times 210 \times (.5625\,V_1)^2}{0.4 \times 2g} + \frac{(.5625\,V_1)^2}{2g}$$

Problem:1(Cont...)

or $$12.0 = \frac{V_1^2}{2g}[0.5 + 20.0 + 2.53 + 89.505 + 2.847 + 3.189 + 0.316]$$

$$= \frac{V_1^2}{2g}[118.887]$$

$\therefore$ $$V_1 = \sqrt{\frac{12 \times 2 \times 9.81}{118.887}} = 1.407 \text{ m/s}$$

$\therefore$ Rate of flow, Q = Area $\times$ Velocity = $A_1 \times V_1$

$$= \frac{\pi}{4}(d_1)^2 \times V_1 = \frac{\pi}{4}(.3)^2 \times 1.407 = 0.09945 \text{ m}^3\text{/s}$$

$=$ **99.45 litres/s. Ans.**

Problem:1(Cont...)

(*ii*) **Neglecting Minor Losses.** Using equation

$$H = \frac{4 f_1 L_1 V_1^2}{d_1 \times 2g} + \frac{4 f_2 L_2 V_2^2}{d_2 \times 2g} + \frac{4 f_3 L_3 V_3^2}{d_3 \times 2g}$$

or $$12.0 = \frac{V_1^2}{2g}\left[\frac{4 \times .005 \times 300}{0.3} + \frac{4 \times .0052 \times 170 \times (2.25)^2}{0.2} + \frac{4 \times .0048 \times 210 \times (.5625)^2}{0.4}\right]$$

$$= \frac{V_1^2}{2g}[20.0 + 89.505 + 3.189] = \frac{V_1^2}{2g} \times 112.694$$

Problem:1(Cont…)

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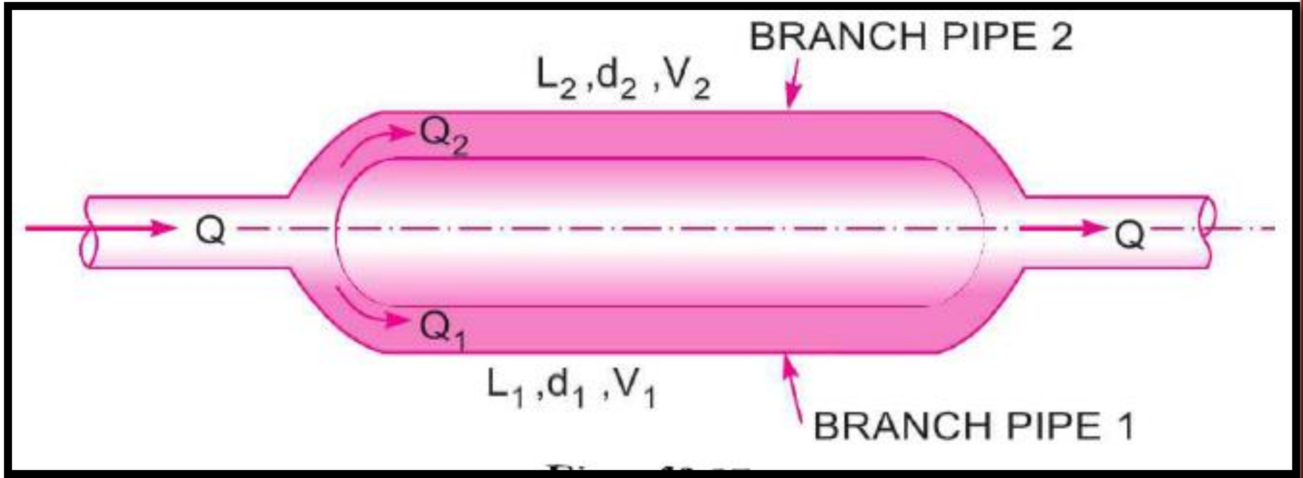
$$V_1 = \sqrt{\frac{2 \times 9.81 \times 12.0}{112.694}} = 1.445 \text{ m/s}$$

Discharge, $Q = V_1 \times A_1 = 1.445 \times \frac{\pi}{4} (.3)^2 = 0.1021 \text{ m}^3\text{/s} = $ **102.1 litres/s. Ans.**

Flow through Parallel Pipes

- Consider a main pipe which divides into two or more branches as shown in fig and again join together at downstream to form a single pipe, then the branch pipes are said to be connected in parallel
- The discharge through the main pipe is increased by connecting pipes in parallel



Flow through Parallel Pipes(Cont…)

- The rate of flow in main pipe is equal to the sum of rate of flow through branch pipes

$$Q = Q_1 + Q_2$$

- The loss of head for each branch pipe is same

Loss of head for branch pipe 1= Loss of head for branch pipe 2

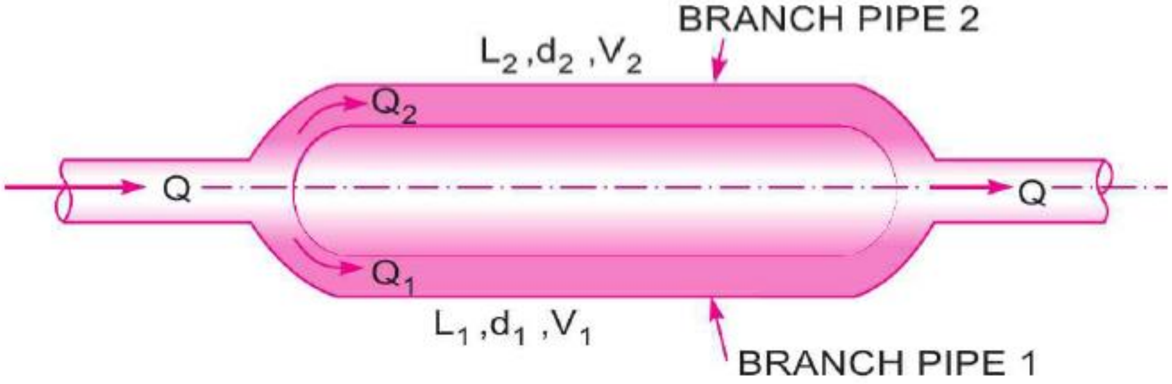
$$\frac{4f_1 L_1 {V_1}^2}{d_1 X 2g} = \frac{4f_2 L_2 {V_2}^2}{d_2 X 2g}$$

- If $f_1 = f_2$

$$\frac{L_1 {V_1}^2}{d_1 X 2g} = \frac{L_2 {V_2}^2}{d_2 X\, 2g}$$

Problem:2

A main pipe divides into two parallel pipes which again forms one pipe as shown in Fig. . The length and diameter for the first parallel pipe are 2000 m and 1.0 m respectively, while the length and diameter of 2nd parallel pipe are 2000 m and 0.8 m. Find the rate of flow in each parallel pipe, if total flow in the main is 3.0 m^3/s. The co-efficient of friction for each parallel pipe is same and equal to .005.



Problem:2(Cont...)

Solution. Given :

Length of pipe 1, $L_1 = 2000$ m

Dia. of pipe 1, $d_1 = 1.0$ m

Length of pipe 2, $L_2 = 2000$ m

Dia. of pipe 2, $d_2 = 0.8$ m

Total flow, $Q = 3.0\ m^3/s$

$f_1 = f_2 = f = .005$

Let Q_1 = discharge in pipe 1

Q_2 = discharge in pipe 2

From equation $Q = Q_1 + Q_2 = 3.0$

Problem:2(Cont...)

Using equation (

$$\frac{4 f_1 L_1 V_1^2}{d_1 \times 2g} = \frac{4 f_2 L_2 V_2^2}{d_2 \times 2g}$$

$$\frac{4 \times .005 \times 2000 \times V_1}{1.0 \times 2 \times 9.81} = \frac{4 \times .005 \times 2000 \times V_2^2}{0.8 \times 2 \times 9.81}$$

or $$\frac{V_1^2}{1.0} = \frac{V_2^2}{0.8} \text{ or } V_1^2 = \frac{V_2^2}{0.8}$$

$\therefore$ $$V_1 = \frac{V_2}{\sqrt{0.8}} = \frac{V_2}{.894}$$

Problem:2(Cont...)

Now $$Q_1 = \frac{\pi}{4} d_1^2 \times V_1 = \frac{\pi}{4} (1)^2 \times \frac{V_2}{.894} \qquad \left[\because V_1 = \frac{V_2}{.894}\right]$$

and $$Q_2 = \frac{\pi}{4} d_2^2 \times V_2 = \frac{\pi}{4} (.8)^2 \times V_2 = \frac{\pi}{4} \times .64 \times V_2$$

Substituting the value of Q_1 and Q_2 in equation $\bar{Q} = Q_1 + Q_2 = 3.0$

$$\frac{\pi}{4} \times \frac{V_2}{0.894} + \frac{\pi}{4} \times .64\ V_2 = 3.0 \text{ or } 0.8785\ V_2 + 0.5026\ V_2 = 3.0$$

or $$V_2[.8785 + .5026] = 3.0 \text{ or } V = \frac{3.0}{1.3811} = 2.17 \text{ m/s.}$$

Problem:2(Cont…)

Substituting this value in equation $V_1 = \frac{V_2}{\sqrt{0.8}} = \frac{V_2}{.894}$

$$V_1 = \frac{V_2}{.894} = \frac{2.17}{0.894} = 2.427 \text{ m/s}$$

Hence $Q_1 = \frac{\pi}{4} d_1{}^2 \times V_1 = \frac{\pi}{4} \times 1^2 \times 2.427 = \mathbf{1.906\ m^3/s.\ Ans.}$

$\therefore$ $Q_2 = Q - Q_1 = 3.0 - 1.906 = \mathbf{1.094\ m^3/s.\ Ans.}$

Summary

- The discharge passing through each pipe is same in case of pipes in series is Q= $A_1V_1 = A_2V_2 = A_3V_3$ and coefficient of friction is same for all pipes
- The difference in liquid surface levels in case of pipes in series is

$$H = 0.5\frac{V_1{}^2}{2g} + \frac{4f_1L_1V_1{}^2}{d_1X2g} + 0.5\frac{V_2{}^2}{2g} + \frac{4f_2L_2V_2{}^2}{d_2X2g} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3{}^2}{d_3X2g} + \frac{V_3{}^2}{2g}$$

- The rate of flow in main pipe in case of parallel pipes is $Q = Q_1 + Q_2$ and loss of head for each branch pipe is same

VENTURIMETER

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Lecture Details:
Unit-III (Closed Conduit Flow) , Venturimeter
FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
- FLUID DYNAMICS
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VENTURIMETER

Contents

- Practical Applications of Bernoulli’s Equation
- Venturimeter
- Summary

Practical Applications of Bernoulli's Equation

- Bernoulli's equation is applied in all problems of incompressible fluid flow where energy considerations are involved

- We shall consider its application to the following measuring devices like:

Venturimeter

Orifice meter

Pitot-tube

Venturimeter

- It is used for measuring the rate of flow of a fluid flowing through a pipe
- It consist of (i) A short converging part (ii) Throat and (iii) Diverging part

<u>Expression for rate of flow through venturimeter</u>

- Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing(say water)

Venturimeter(Cont…)

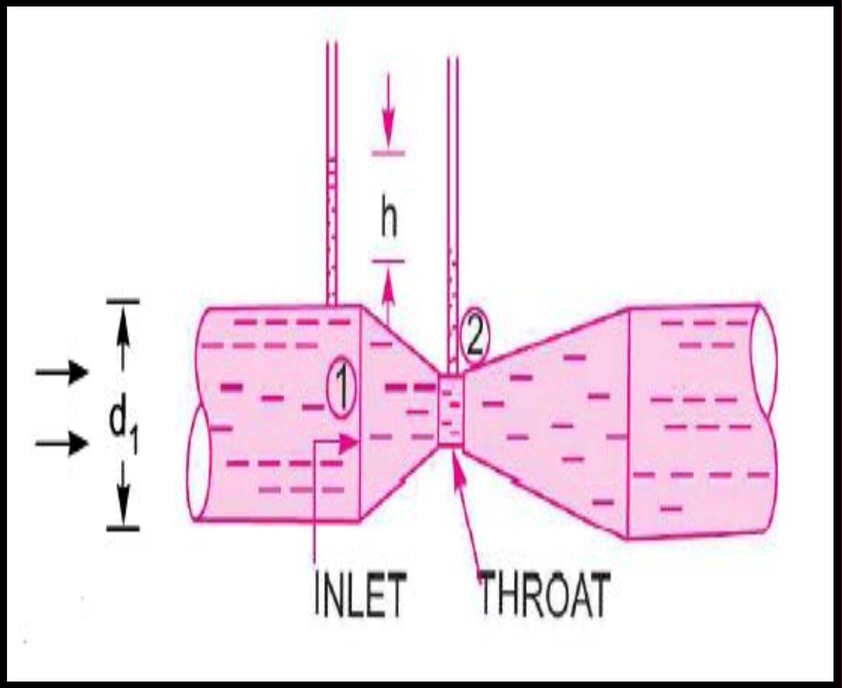
- Let d_1 = diameter at inlet or at section (1)

$p_1 = pressure\ at\ section(1)$

$v_1 = velocity\ of\ fluid\ at\ section(1)$

$a = area\ at\ section(1) = \frac{\pi}{4} d^2$

d_2, p_2, v_2, a_2 are corresponding values at section (2)



Venturimeter(Cont...)

- Applying Bernoulli's equation at section(1) and (2)

$$\frac{p_1}{\rho g}+\frac{{v_1}^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{{v_2}^2}{2g}+z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\frac{p_1}{\rho g}+\frac{{v_1}^2}{2g}=\frac{p_2}{\rho g}+\frac{{v_2}^2}{2g}$$

$$\frac{p_1-p_2}{\rho g}=\frac{{v_2}^2}{2g}-\frac{{v_1}^2}{2g}$$

Venturimeter(Cont…)

$$\frac{p_1 - p_2}{\rho g} = \frac{{v_2}^2}{2g} - \frac{{v_1}^2}{2g}$$

- But $\frac{p_1 - p_2}{\rho g} = h$= difference of pressure heads at section 1 and 2

$$h = \frac{{v_2}^2}{2g} - \frac{{v_1}^2}{2g} \text{-----------}(1)$$

- Now applying continuity equation at sections 1 and 2

$$a_1\, v_1 = a_2\, v_2, \qquad v_1 = \frac{a_2\, v_2}{a_1}$$

Venturimeter(Cont…)

$$v_1 = \frac{a_2\, v_2}{a_1}$$

- Substituting v_1 in equation(h= $\frac{{v_2}^2}{2g} - \frac{{v_1}^2}{2g}$-----------(1))

$$h = \frac{{v_2}^2}{2g} - \frac{\left(\frac{a_2\, v_2}{a_1}\right)^2}{2g}$$

$$h = \frac{{v_2}^2}{2g}\left[1 - \frac{{a_2}^2}{{a_1}^2}\right] = \frac{{v_2}^2}{2g}\left[\frac{{a_1}^2 - {a_2}^2}{{a_1}^2}\right]$$

Venturimeter(Cont…)

$$h = \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$v_2 = \sqrt{2gh\frac{a_1^2}{a_1^2 - a_2^2}}$$

$$v_2 = \frac{a_1}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$$

Venturimeter(Cont...)

Discharge Q=$a_2 v_2$

$$Q=a_2 \frac{a_1}{\sqrt{{a_1}^2-{a_2}^2}} \sqrt{2gh}$$

$$Q=\frac{a_1 a_2}{\sqrt{{a_1}^2-{a_2}^2}} \sqrt{2gh}\text{------------}(2)$$

- Equation (2) gives discharge under ideal conditions and is called, theoretical discharge

Venturimeter(Cont...)

- Actual discharge will be less than theoretical discharge

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

- Where C_d = coefficient of venturimeter and its value less than 1

Venturimeter(Cont…)

Value of 'h' given by differential U-tube manometer

Case I: Differential manometer contains a liquid which is heavier than the liquid flowing through the pipe

Let S_h = Sp. gravity of the heavier liquid

S_o = Sp. gravity of the liquid flowing through pipe

x = Difference of the heavier liquid column in U-tube

Venturimeter(Cont...)

$$h = x\left[\frac{S_h}{S_o} - 1\right]$$

Case II: If the differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given by

$$h = x\left[1 - \frac{S_l}{S_o}\right]$$

Where S_l= Sp. gravity of lighter liquid in U-tube

S_o = Sp. gravity of the liquid flowing through pipe

x= Difference of lighter liquid column in U-tube

Venturimeter(Cont…)

Case III. Inclined venturimeter with differential manometer: Let the differential manometer contains heavier liquid ,then h is given by

$$h=\left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)=x\left[\frac{S_h}{S_o}-1\right]$$

Case IV: For inclined venturimeter in which differential manometer contains a liquid which is lighter than the liquid flowing through the pipe

$$h=\left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)=x\left[1-\frac{S_l}{S_o}\right]$$

Problem:1

A horizontal venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and the throat is 20 cm of mercury. Determine the rate of flow. Take $C_d = 0.98$.

Solution. Given :

Dia. at inlet, $d_1 = 30 \text{ cm}$

$\therefore$ Area at inlet, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} (30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15 \text{ cm}$

$\therefore$ $a_2 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ cm}^2$

$C_d = 0.98$

Problem:1(Cont...)

Reading of differential manometer = x = 20 cm of mercury.

$\therefore$ Difference of pressure head is given by

or
$$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gravity of mercury = 13.6, S_o = Sp. gravity of water = 1

$$= 20\left[\frac{13.6}{1} - 1\right] = 20 \times 12.6 \text{ cm} = 252.0 \text{ cm of water.}$$

The discharge through venturimeter is given by eqn.

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

Problem:1(Cont...)

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}} = \frac{86067593.36}{684.4}$$

$$= 125756 \text{ cm}^3/\text{s} = \frac{125756}{1000} \text{ lit/s} = \mathbf{125.756 \text{ lit/s. Ans.}}$$

Problem:2

A horizontal venturimeter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of water. The pressure at inlet is 17.658 N/cm^2 and the vacuum pressure at the throat is 30 cm of mercury. Find the discharge of water through venturimeter. Take C_d = 0.98.

Solution. Given :

Dia. at inlet, $d_1 = 20$ cm

$$\therefore \quad a_1 = \frac{\pi}{4} \times (20)^2 = 314.16 \text{ cm}^2$$

Dia. at throat, $d_2 = 10$ cm

$$\therefore \quad a_2 = \frac{\pi}{4} \times 10^2 = 78.74 \text{ cm}^2$$

Problem:2(Cont...)

$$p_1 = 17.658 \text{ N/cm}^2 = 17.658 \times 10^4 \text{ N/m}^2$$

ρ for water $= 1000 \dfrac{\text{kg}}{\text{m}^3}$ and $\therefore \dfrac{p_1}{\rho g} = \dfrac{17.658 \times 10^4}{9.81 \times 1000} = 18$ m of water

$$\frac{p_2}{\rho g} = -30 \text{ cm of mercury}$$

$$= -0.30 \text{ m of mercury} = -0.30 \times 13.6 = -4.08 \text{ m of water}$$

$\therefore$ Differential head $= h = \dfrac{p_1}{\rho g} - \dfrac{p_2}{\rho g} = 18 - (-4.08)$

$$= 18 + 4.08 = 22.08 \text{ m of water} = 2208 \text{ cm of water}$$

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Problem:2(Cont...)

The discharge Q is given by equation

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.74)^2}} \times \sqrt{2 \times 981 \times 2208}$$

$$= \frac{50328837.21}{304} \times 165555 \text{ cm}^3/\text{s} = \mathbf{165.555\ lit/s.\ Ans.}$$

Summary

- Bernoulli's equation is applied in all problems of incompressible fluid flow where energy considerations are involved
- Venturimeter is used for measuring the rate of flow of a fluid flowing through a pipe
- Theoretical discharge for venturimeter is : $Q=\frac{a_1 a_2}{\sqrt{a_1{}^2-a_2{}^2}}\sqrt{2gh}$
- Actual discharge will be less than theoretical discharge: $Q=C_d\,\frac{a_1 a_2}{\sqrt{a_1{}^2-a_2{}^2}}\sqrt{2gh}$

ORIFICE METER

Presented By:
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Lecture Details:
Unit-III (Closed Conduit Flow) , Orifice Meter
FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

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- FLUID STATICS
- FLUID KINEMATICS
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ORIFICE METER

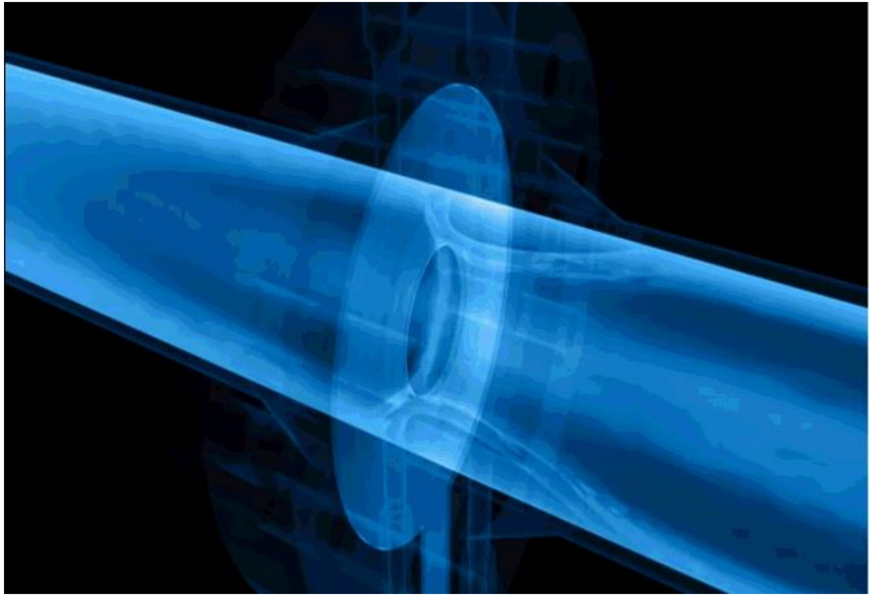
Contents

- Orifice Meter or Orifice Plate
- Summary

Orifice Meter or Orifice Plate

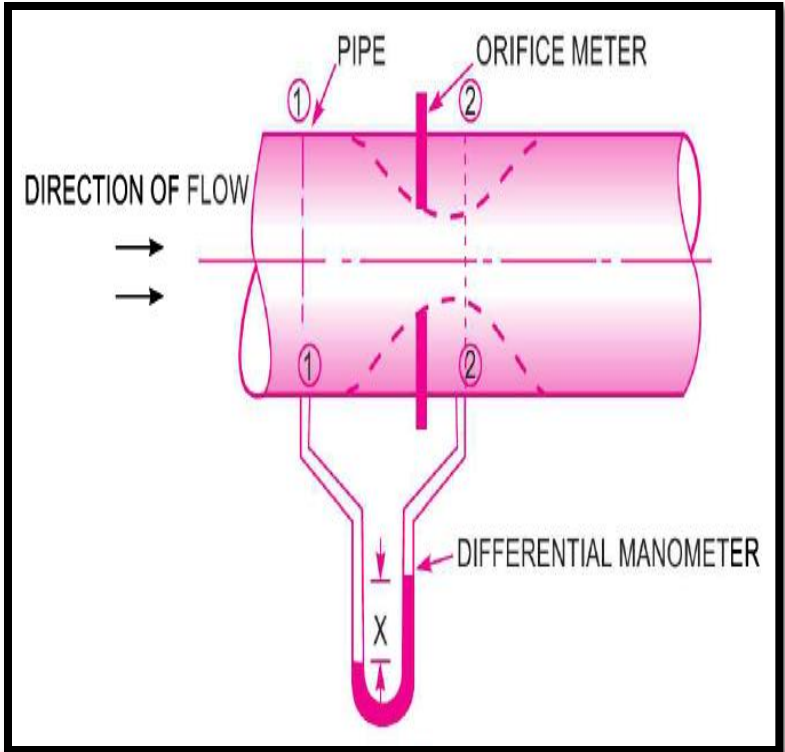
- It is used for measuring the rate of flow of a fluid through a pipe
- It is a cheaper device as compared to venturimeter
- It also works on the same principle as that of venturimeter
- It consists of a flat circular plate which has a circular sharp edged hole called orifice, which is concentric with the pipe
- The orifice diameter is generally 0.5 times the diameter of the pipe, it may vary from 0.4 to 0.8 times the pipe diameter

Orifice Meter or Orifice Plate(Cont...)



Orifice Meter or Orifice Plate(Cont...)

- A differential manometer is connected at section (1), which is at a distance of about 1.5 to 2.0 times the pipe diameter upstream from the orifice plate
- At section (2) which is at distance of about half the diameter of the orifice on the downstream side from orifice plate



Orifice Meter or Orifice Plate(Cont…)

- Let $p_1 = pressure\ at\ section\ (1)$

$v_1 = velocity\ at\ section\ (1)$

$a_1 = area\ of\ pipe\ at\ section\ (1)$

p_2, v_2, a_2 are corresponding values at section (2)

Applying Bernoulli's equation at section (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{{v_2}^2}{2g} + z_2$$

Orifice Meter or Orifice Plate(Cont...)

$$\left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)=\frac{{v_2}^2}{2g}-\frac{{v_1}^2}{2g}$$

But $\left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)=h$ = Differential head

$$h=\frac{{v_2}^2}{2g}-\frac{{v_1}^2}{2g}$$

$$2\text{gh}={v_2}^2-{v_1}^2$$

$$v_2=\sqrt{2gh+{v_1}^2}\text{---------}(1)$$

Orifice Meter or Orifice Plate(Cont...)

- Now section (2) is at the vena- contracta and a_2 represents area at vena – contracta and a_0 is the area of orifice, then

$$C_c = \frac{a_2}{a_0}$$

C_c= Coefficient of contraction

$$a_2 = a_0 \, C_c$$

By continuity equation

$$a_1 v_1 = a_2 v_2$$

Orifice Meter or Orifice Plate(Cont…)

$$v_1 = \frac{a_2}{a_1} v_2 = \frac{a_0\, C_c}{a_1} v_2$$

Substituting v_1 in equation $(v_2 = \sqrt{2gh + {v_1}^2}$---------(1))

$$v_2 = \sqrt{2gh + \left(\frac{a_0\, C_c}{a_1} v_2\right)^2}$$

$$ {v_2}^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 {C_c}^2 {v_2}^2$$

Orifice Meter or Orifice Plate(Cont…)

$$v_2{}^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 C_c{}^2 v_2{}^2$$

$$v_2{}^2 \left[1 - \left(\frac{a_0}{a_1}\right)^2 C_c{}^2\right] = 2gh$$

$$v_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c{}^2}}$$

Orifice Meter or Orifice Plate(Cont...)

The discharge $Q = v_2 a_2 = v_2 a_0 C_c$

$$Q = \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c{}^2}} \text{-------------}(2)$$

The above expression is simplified by using

Orifice Meter or Orifice Plate(Cont…)

$$C_d = C_c \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 {C_c}^2}}$$

$$C_c = C_d \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 {C_c}^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}$$

Orifice Meter or Orifice Plate(Cont…)

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Substituting this value of C_c in equation ($Q = \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 {C_c}^2}}$-------------(2))

$$Q = a_0 C_d \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 {C_c}^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}} X \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 {C_c}^2}}$$

Orifice Meter or Orifice Plate(Cont…)

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$$Q = \frac{a_0 C_d \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}$$

$$Q = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1{}^2 - a_0{}^2}}$$

- Where C_d= Co – efficient of discharge for orifice meter
- The co-efficient of discharge for orifice meter is much smaller than that for a venturimeter

Problem:1

An orifice meter with orifice diameter 10 cm is inserted in a pipe of 20 cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter gives readings of 19.62 N/cm^2 and 9.81 N/cm^2 respectively. Co-efficient of discharge for the orifice meter is given as 0.6. Find the discharge of water through pipe.

Solution. Given :

Dia. of orifice, $d_0 = 10$ cm

$\therefore$ Area, $a_0 = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$

Dia. of pipe, $d_1 = 20$ cm

$\therefore$ Area, $a_1 = \frac{\pi}{4}(20)^2 = 314.16 \text{ cm}^2$

Problem:1(Cont...)

$$p_1 = 19.62 \text{ N/cm}^2 = 19.62 \times 10^4 \text{ N/m}^2$$

$$\therefore \qquad \frac{p_1}{\rho g} = \frac{19.62 \times 10^4}{1000 \times 9.81} = 20 \text{ m of water}$$

Similarly $$\frac{p_2}{\rho g} = \frac{9.81 \times 10^4}{1000 \times 9.81} = 10 \text{ m of water}$$

$$h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 20.0 - 10.0 = 10 \text{ m of water} = 1000 \text{ cm of water}$$

$$C_d = 0.6$$

Problem:1(Cont…)

The discharge, Q is given by equation :

$$Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$= 0.6 \times \frac{78.54 \times 314.16}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 1000}$$

$$= \frac{20736838.09}{304} = 68213.28 \text{ cm}^3\text{/s} = \mathbf{68.21 \text{ litres/s. Ans.}}$$

Problem:2

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An orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure difference measured by a mercury oil differential manometer on the two sides of the orifice meter gives a reading of 50 cm of mercury. Find the rate of flow of oil of sp. gr. 0.9 when the co-efficient of discharge of the orifice meter = 0.64.

Solution. Given :

Dia. of orifice, $d_0 = 15$ cm

$\therefore$ Area, $a_0 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

Dia. of pipe, $d_1 = 30$ cm

$\therefore$ Area, $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Problem:2(Cont…)

Sp. gr. of oil, $S_o = 0.9$

Reading of diff. manometer, $x = 50$ cm of mercury

$\therefore$ Differential head, $h = x\left[\dfrac{S_g}{S_o} - 1\right] = 50\left[\dfrac{13.6}{0.9} - 1\right]$ cm of oil

$= 50 \times 14.11 = 705.5$ cm of oil

$C_d = 0.64$

$\therefore$ The rate of the flow, Q is given by equation ·

$$Q = C_d \cdot \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

Problem:2(Cont...)

$$= 0.64 \times \frac{176.7 \times 706.85}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 705.5}$$

$$= \frac{94046317.78}{684.4} = 137414.25 \text{ cm}^3/\text{s} = \mathbf{137.414 \text{ litres/s. Ans.}}$$

Summary

- Orifice meter also works on the same principle as that of venturimeter

- The discharge $Q = \dfrac{a_0 C_c \sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c{}^2}}$

- The discharge can also be expressed as $Q = \dfrac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1{}^2 - a_0{}^2}}$

- The co-efficient of discharge for orifice meter is much smaller than that for a venturimeter

PITOT- TUBE

Presented By:
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Lecture Details:
Unit-III (Closed Conduit Flow) , Pitot -tube
FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

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PITOT- TUBE

Contents

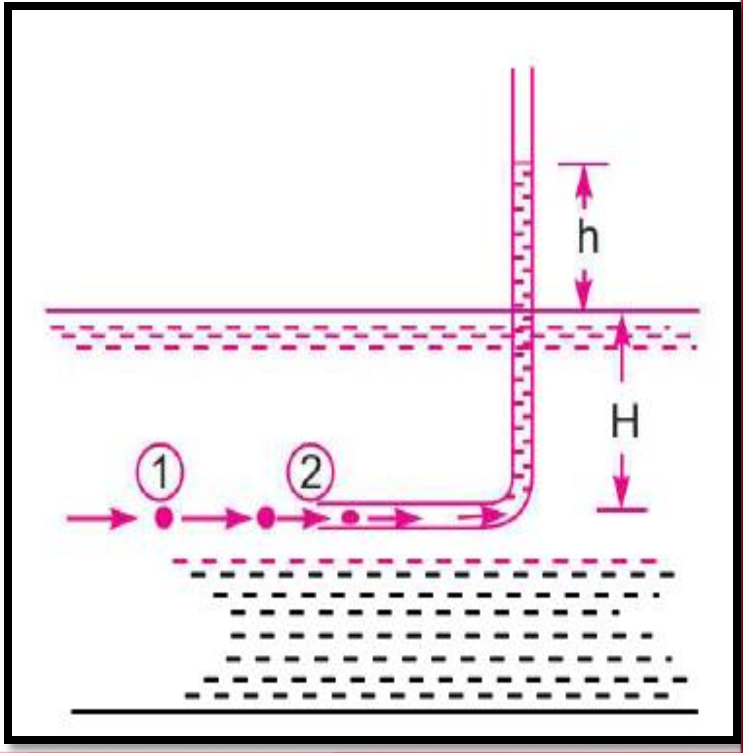
- Pitot- tube
- Summary

Pitot- tube

- It is used for measuring the velocity of flow at any point in a pipe or a channel
- It is based on the principle that , if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy
- Pitot –tube consists of a glass tube, bent at right angles

Pitot- tube (Cont…)

- The lower end, which is bent through 90^0 is directed in the upstream direction
- The liquid rises up in the tube due to the conversion of kinetic energy into pressure energy
- The velocity is determined by measuring the rise of liquid in the tube
- Point (2) is just as the inlet of the pitot tube
- Point (1) is far away from the tube



Pitot- tube (Cont…)

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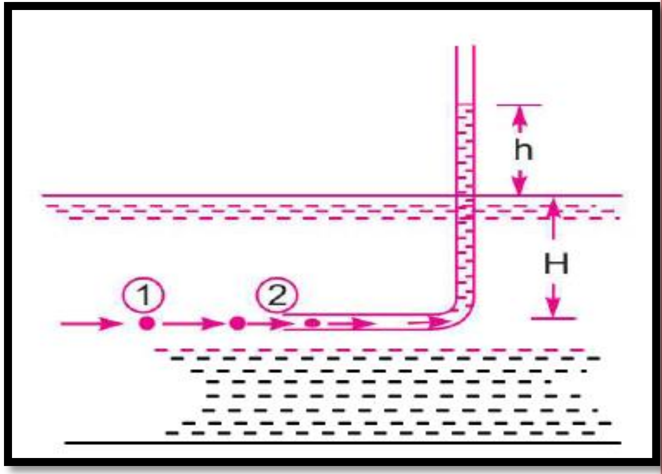
- Let p_1 =intensity of pressure at point (1)

v_1 =velocity of flow at point (1)

p_2 = pressure at point (2)

v_2 =velocity of flow at point (2), which is zero

h= rise of liquid in the tube above the free surface



Applying Bernoulli's equation at section (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{{v_2}^2}{2g} + z_2$$

Pitot- tube (Cont...)

But $z_1 = z_2$ and $v_2 = 0$

$\frac{p_1}{\rho g}$ =pressure head at (1)= H

$\frac{p_2}{\rho g}$ =pressure head at (2)= h + H

Substituting these values in $\frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{{v_2}^2}{2g} + z_2$, we get

$$H + \frac{{v_1}^2}{2g} = (h+H)$$

Pitot- tube (Cont…)

$$h = \frac{{v_1}^2}{2g}$$

$$v_1 = \sqrt{2gh}$$

- This is theoretical velocity. Actual velocity is given by

$$(v_1)_{act} = C_v\sqrt{2gh}$$

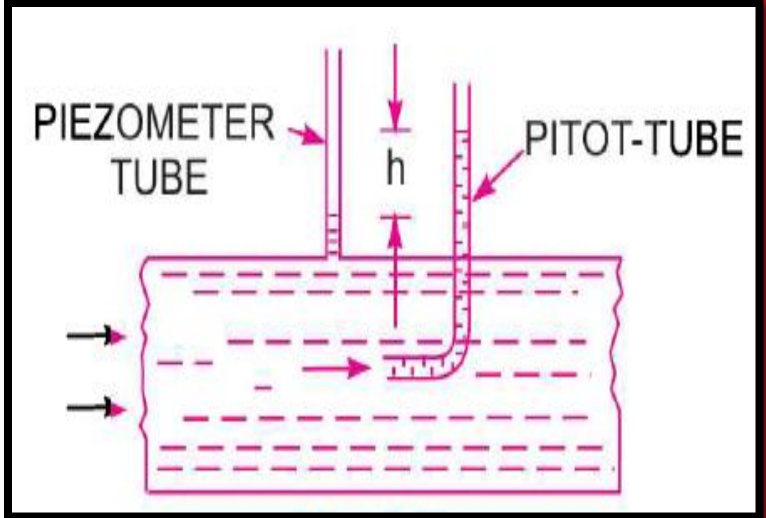
- C_v =Co-efficient of pitot –tube
- Velocity at any point $v = C_v\sqrt{2gh}$

Pitot- tube (Cont...)

Velocity of flow in a pipe by pitot-tube:

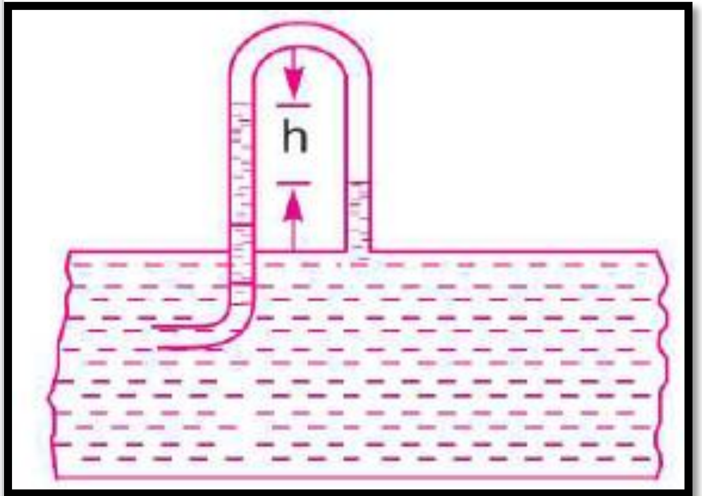
- For finding the velocity at any point in a pipe by pitot-tube, the following arrangements are adopted:

1) Pitot- tube along with a vertical piezometer tube

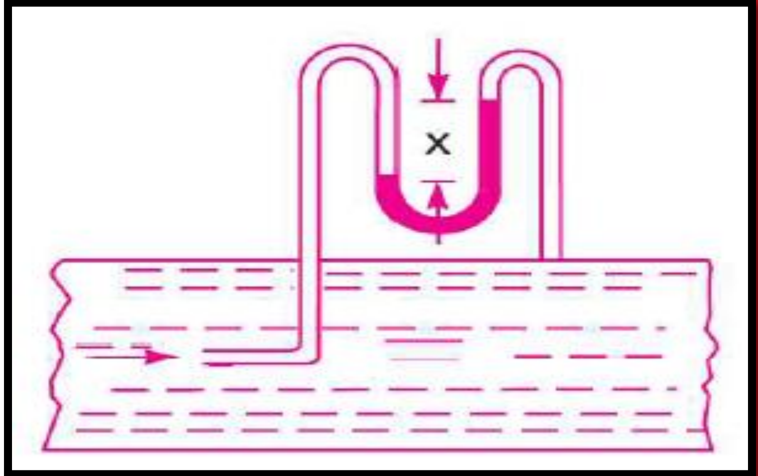


Pitot- tube (Cont...)

2) Pitot- tube connected with piezometer tube



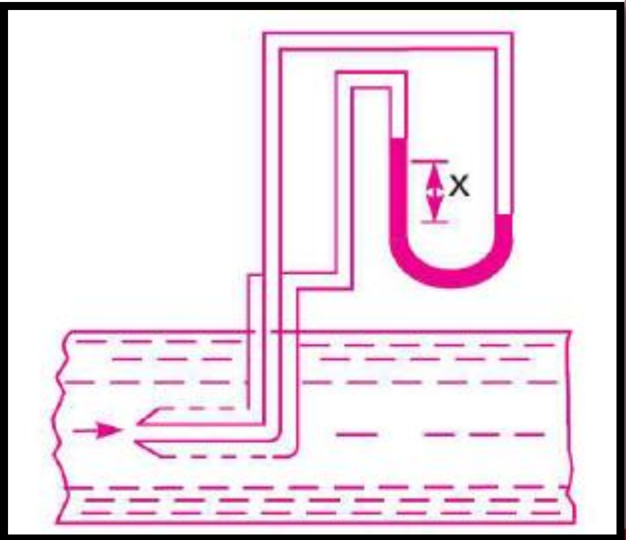
3) Pitot- tube and vertical piezometer tube connected with a differential U-tube manometer



Pitot- tube (Cont…)

4) Pitot- static tube, which consist of two circular concentric tubes one inside and other with some annular space in between

- The outlet of these two tubes are connected to the differential manometer where the difference of pressure head ‘h’ is measured by knowing the difference of the levels of the manometer liquid say x. Then $h = x\left[\frac{S_g}{S_0} - 1\right]$



Problem:1

A pitot-static tube placed in the centre of a 300 mm pipe line has one orifice pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe if the pressure difference between the two orifices is 60 mm of water. Take the co-efficient of pitot tube as $C_v = 0.98$.

Solution. Given :

Dia. of pipe, $d = 300$ mm $= 0.30$ m

Diff. of pressure head, $h = 60$ mm of water $= .06$ m of water

$C_v = 0.98$

Mean velocity, $\overline{V} = 0.80 \times$ Central velocity

Problem:1(Cont…)

Central velocity is given by equation

$$= C_v \sqrt{2gh} = 0.98 \times \sqrt{2 \times 9.81 \times .06} = 1.063 \text{ m/s}$$

$$\therefore \qquad \overline{V} = 0.80 \times 1.063 = 0.8504 \text{ m/s}$$

Discharge, $Q = \text{Area of pipe} \times \overline{V}$

$$= \frac{\pi}{4} d^2 \times \overline{V} = \frac{\pi}{4} (.30)^2 \times 0.8504 = \mathbf{0.06\ m^3/s.\ Ans.}$$

Problem:2

Find the velocity of the flow of an oil through a pipe, when the difference of mercury level in a differential U-tube manometer connected to the two tappings of the pitot-tube is 100 mm. Take co-efficient of pitot-tube 0.98 and sp. gr. of oil = 0.8.

Solution. Given :

Diff. of mercury level, $x = 100 \text{ mm} = 0.1 \text{ m}$

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_g = 13.6$

$C_v = 0.98$

Problem:2

Diff. of pressure head, $h = x\left[\frac{S_g}{S_o} - 1\right] = .1\left[\frac{13.6}{0.8} - 1\right] = 1.6$ m of oil

$\therefore$ Velocity of flow $= C_v\sqrt{2gh} = 0.98\sqrt{2 \times 9.81 \times 1.6}$ = **5.49 m/s. Ans.**

Summary

- Pitot –tube is used for measuring the velocity of flow at any point in a pipe or a channel
- Theoretical velocity: $v_1 = \sqrt{2gh}$
- Actual velocity: $(v_1)_{act} = C_v\sqrt{2gh}$

FLOW THROUGH NOZZLES

Presented By:
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Lecture Details:
Unit-III (Closed Conduit Flow) , Flow through nozzles
FM & HM /Mechanical, I -Semester.

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FLOW THROUGH NOZZLES

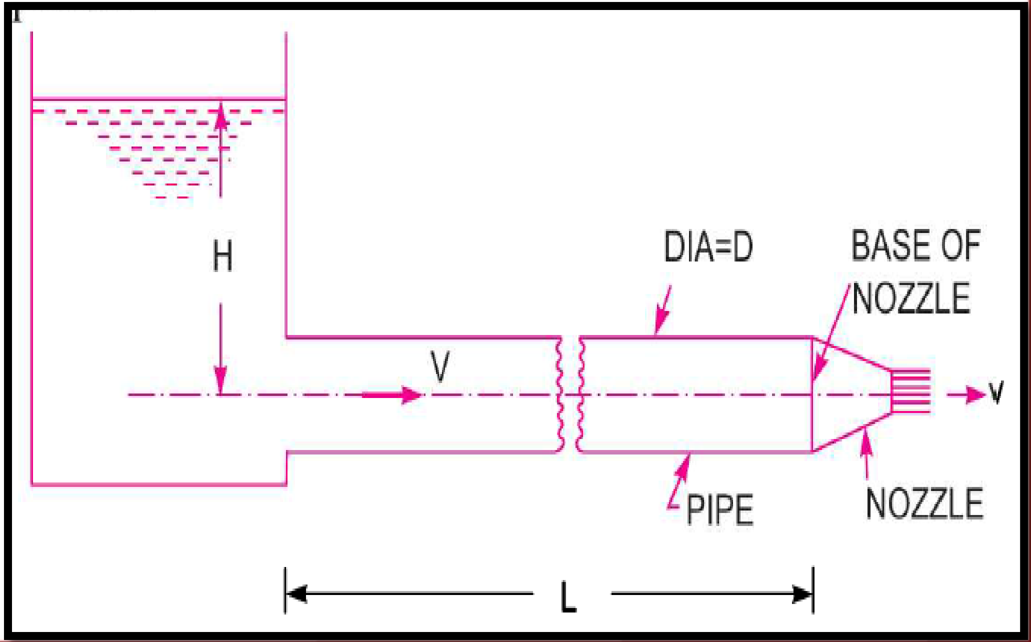
Contents

- Flow through Nozzles
- Summary

Flow through Nozzles

➢ The total energy at the end of the pipe consists of pressure energy and kinetic energy

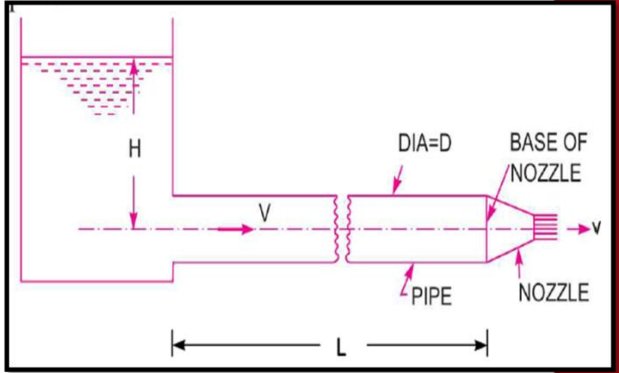
➢ By fitting the nozzle at the end of the pipe, the pressure energy is converted into kinetic energy



Flow through Nozzles(Cont…)

- Thus nozzles are used, where higher velocities of flow are required

- Let D= diameter of the pipe, L= length of the pipe



A= area of the pipe=$\frac{\pi}{4}D^2$, V= velocity of flow in pipe

H= total head at the inlet of the pipe ,

d= diameter of nozzle at outlet, v= velocity of flow at outlet of nozzle,

a= area of the nozzle at outlet=$\frac{\pi}{4}d^2, f$ =co-efficient of friction for pipe

Flow through Nozzles(Cont...)

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- Loss of head due to friction in pipe, $h_f = \frac{4fLV^2}{2gXD}$

- Head available at the end of the pipe or at the base of nozzle

  = Head at inlet of pipe −head lost due to friction

  $= H - h_f = \left(H - \frac{4fLV^2}{2gXD}\right)$

- Neglecting minor losses and also assuming losses in the nozzle negligible

Flow through Nozzles(Cont…)

- Total head at inlet of pipe= total head(energy) at the outlet of nozzle + Losses

- But total head at outlet of nozzle = kinetic head $=\frac{v^2}{2g}$

$$H= \frac{v^2}{2g} + h_f = \frac{v^2}{2g} + \frac{4fLV^2}{2gD} \text{--------}(1)$$

- From continuity equation in the pipe and outlet of nozzle

$$AV=av, \quad V=\frac{av}{A}$$

Flow through Nozzles(Cont…)

- Substituting V in equation $H = \frac{v^2}{2g} + \frac{4fLV^2}{2gD}$---------(1)

$$H = \frac{v^2}{2g} + \frac{4fL}{2gD} X \left(\frac{av}{A}\right)^2$$

$$H = \frac{v^2}{2g} + \frac{4fLa^2v^2}{2gDA^2}$$

$$H = \frac{v^2}{2g}\left(1 + \frac{4fLa^2}{DA^2}\right)$$

Flow through Nozzles(Cont...)

$$v = \sqrt{\frac{2gH}{1 + \frac{4fLa^2}{DA^2}}}$$

Discharge through nozzle= a v

$$Q = a\sqrt{\frac{2gH}{1+\frac{4fLa^2}{DA^2}}}$$

Problem:1

The head of water at the inlet of a pipe 2000 m long and 500 mm diameter is 60 m. A nozzle of diameter 100 mm at its outlet is fitted to the pipe. Find the velocity of water at the outlet of the nozzle if $f = .01$ for the pipe.

Solution. Given :

Head of water at inlet of pipe, $H = 60$ m

Length of pipe,	$L = 2000$ m
Dia. of pipe,	$D = 500$ mm $= 0.50$ m
Dia. of nozzle at outlet,	$d = 100$ mm $= 0.1$ m
Co-efficient of friction,	$f = .01$

Problem:1(Cont…)

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The velocity at outlet of nozzle is given by equation as

$$v = \sqrt{\frac{2gH}{\left(1 + \frac{4fL}{D} \times \frac{a^2}{A^2}\right)}} = \sqrt{\frac{2 \times 9.81 \times 60}{1 + \frac{4 \times .01 \times 2000}{0.5}\left(\frac{\frac{\pi}{4}d^2}{\frac{\pi}{4}D^2}\right)^2}}$$

$$= \sqrt{\frac{2 \times 9.81 \times 60}{1 + \frac{4 \times .01 \times 2000}{0.5} \times \left(\frac{0.1 \times .1}{0.5 \times .5}\right)^2}} = \mathbf{30.61\ m/s.\ Ans.}$$

Summary

- By fitting the nozzle at the end of the pipe, the total energy is converted into kinetic energy

- Discharge through nozzle: = a $\sqrt{\dfrac{2gH}{1+\dfrac{4fLa^2}{DA^2}}}$

BOUNDARY LAYER FLOW:PART-1

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Lecture Details:
Unit-III (Boundary layer theory and applications) ,
Boundary Layer Flow
FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
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- RECIPROCATING PUMPS

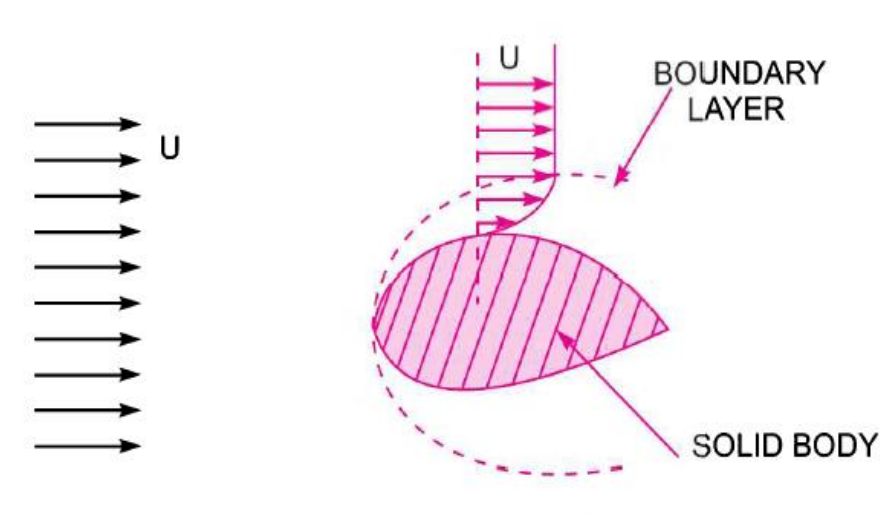
BOUNDARY LAYER FLOW:PART-1

Contents

- Introduction to Boundary Layer Flow
- Laminar Boundary Layer
- Turbulent Boundary Layer
- Laminar Sub-layer
- Summary

Introduction to Boundary Layer Flow

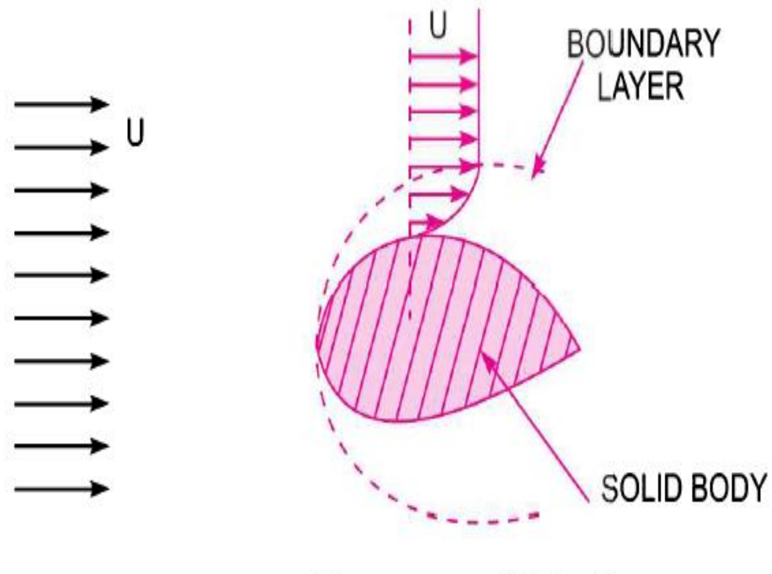
- When a real fluid flows past a solid body or a solid wall
- The fluid particles adhere to the boundary and condition of no slip occurs
- This means that the velocity of fluid close to the boundary will be same as that of the boundary



Flow over solid body.

Introduction to Boundary Layer Flow(Cont...)

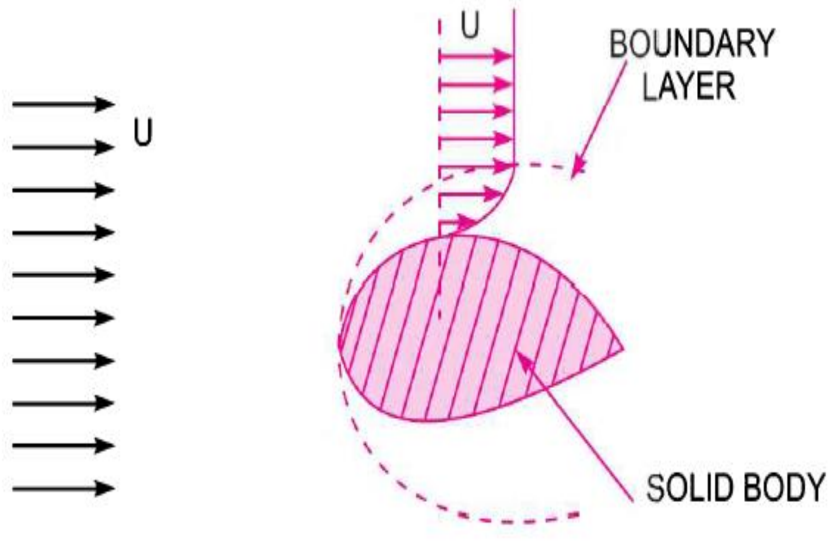
- If the boundary is stationary, the velocity of fluid at the boundary will be zero
- Farther away from the boundary, the velocity will be higher and as a result of this variation of velocity i.e the velocity gradient $\frac{du}{dy}$ will exist



Flow over solid body.

Introduction to Boundary Layer Flow(Cont...)

- The velocity of fluid increases from zero velocity on the stationary boundary to free-stream velocity (U) of the fluid in the direction normal to the boundary



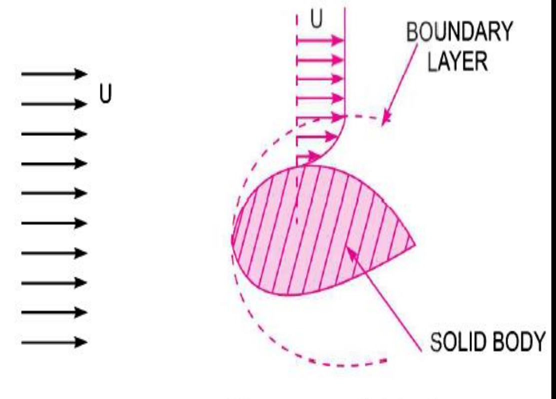
Flow over solid body.

Introduction to Boundary Layer Flow(Cont…)

- This variation of velocity from zero to free-stream velocity in the direction normal to the boundary takes place in a narrow region in the vicinity of solid boundary

- This narrow region of the fluid is called boundary layer

- The theory dealing with boundary layer flows is called boundary layer theory



Flow over solid body.

Introduction to Boundary Layer Flow(Cont...)

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- According to boundary layer theory , the flow of fluid may be divided into two regions

(1) A very thin layer of the fluid, called the boundary layer, in the immediate neighbourhood of the solid boundary, in this region $\frac{du}{dy}$ exists and hence the fluid exerts a shear stress on the wall in the direction of motion

The value of shear stress is given by $\tau = \mu \frac{du}{dy}$

Introduction to Boundary Layer Flow(Cont…)

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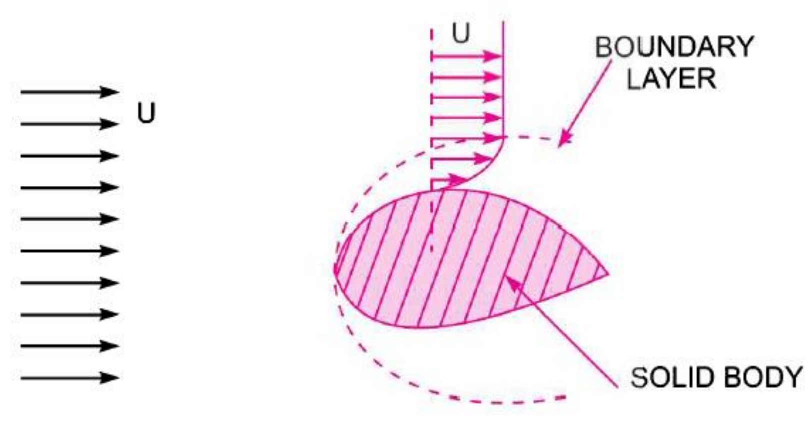
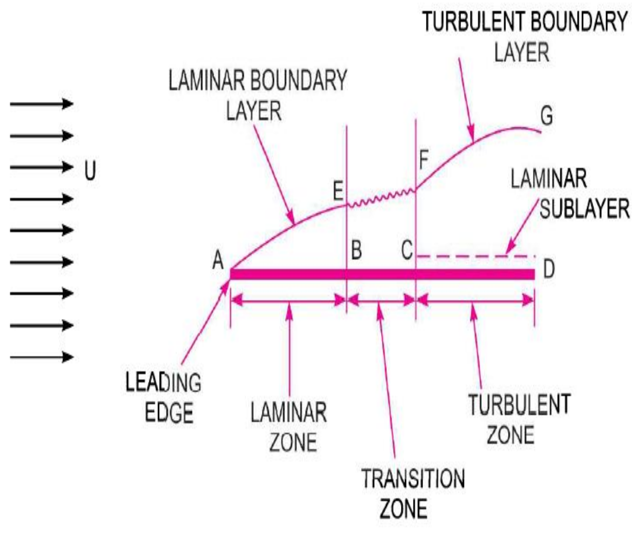
(2) The velocity outside the boundary layer is constant and equal to free-stream velocity. As there is no variation of velocity in this region , the velocity gradient $\frac{du}{dy}$ becomes zero



Flow over solid body.

Laminar Boundary Layer

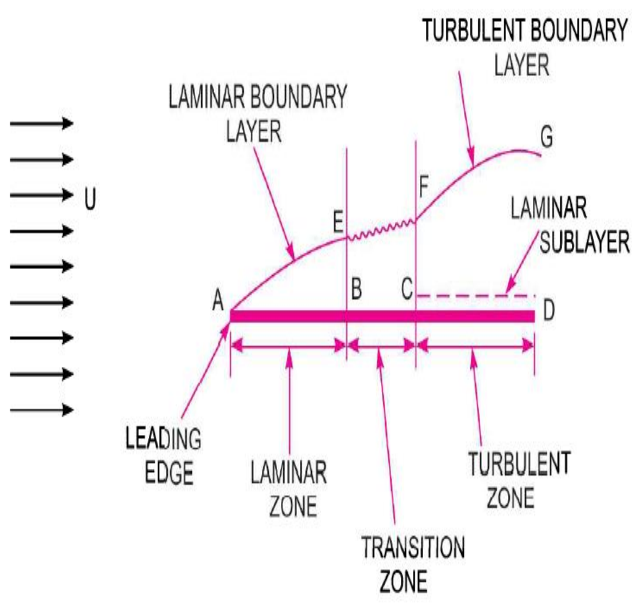
- Consider a fluid flow over a smooth thin plate which is flat and placed parallel to the direction of free stream of fluid
- A velocity gradient is set up in the fluid near the surface of the plate
- This velocity gradient develops shear resistance, which retards the fluid motion



Flow over a plate.

Laminar Boundary Layer(Cont...)

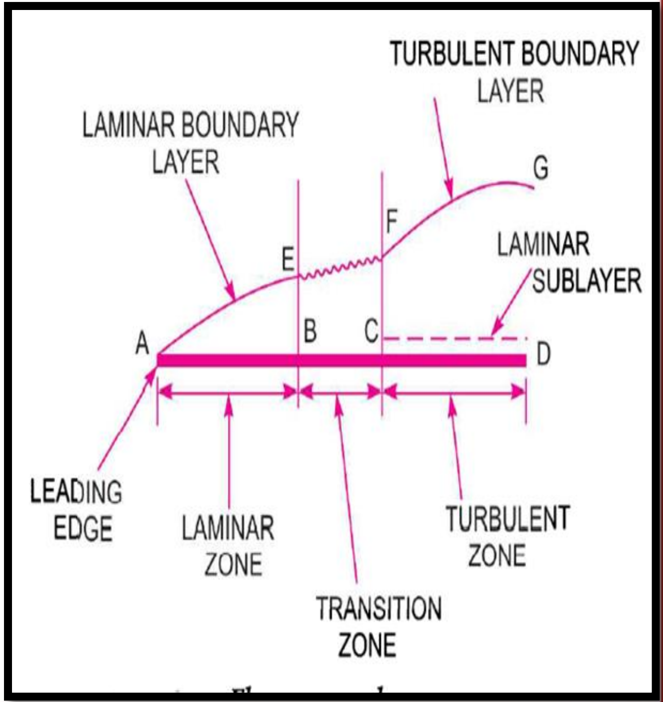
- Thus the fluid with a uniform free stream velocity(U) is retarded in the vicinity of the solid surface of the plate and boundary layer region begins at the sharp leading edge
- In the downstream the leading edge, the boundary layer region increases because the retarded fluid is further retarded



Flow over a plate.

Laminar Boundary Layer(Cont...)

- This is also referred as growth of boundary layer
- Near the leading edge of the surface of the plate, where the thickness is small, the flow in the boundary layer is laminar , so this layer of fluid is said to be laminar boundary layer(AE)



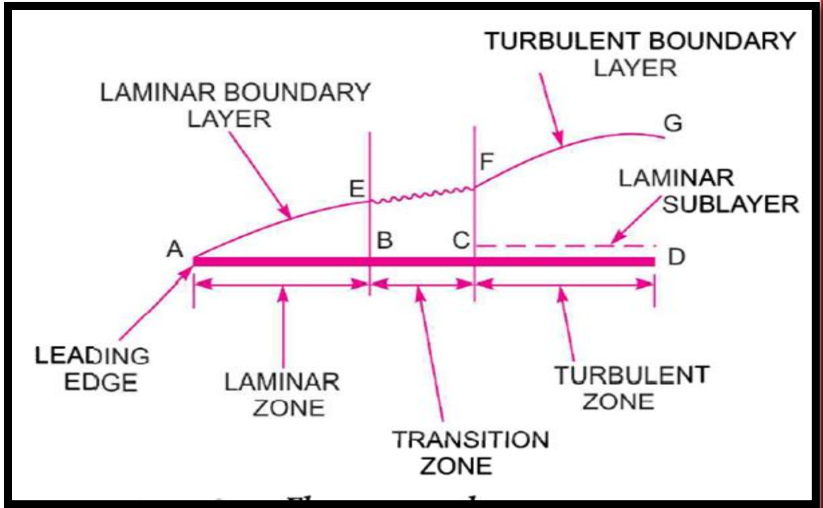
Laminar Boundary Layer(Cont…)

- The distance of B from leading edge (Laminar zone =AB) is obtained from Reynold number($(R_e)_x = \frac{Ux}{v}$) = 5X 10^5 for the plate

Where x= Distance from leading edge

U= Free- stream velocity of fluid

v =Kinematic viscosity of fluid



Turbulent Boundary Layer

- If length of the plate is more than the distance $x((R_e)_x = \frac{Ux}{v}) = 5X\ 10^5)$, the thickness of boundary layer will go on increasing in the downstream direction
- Then laminar boundary layer becomes unstable and motion of fluid within it, is disturbed and irregular which leads to a transition from laminar to turbulent boundary layer

Turbulent Boundary Layer(Cont…)

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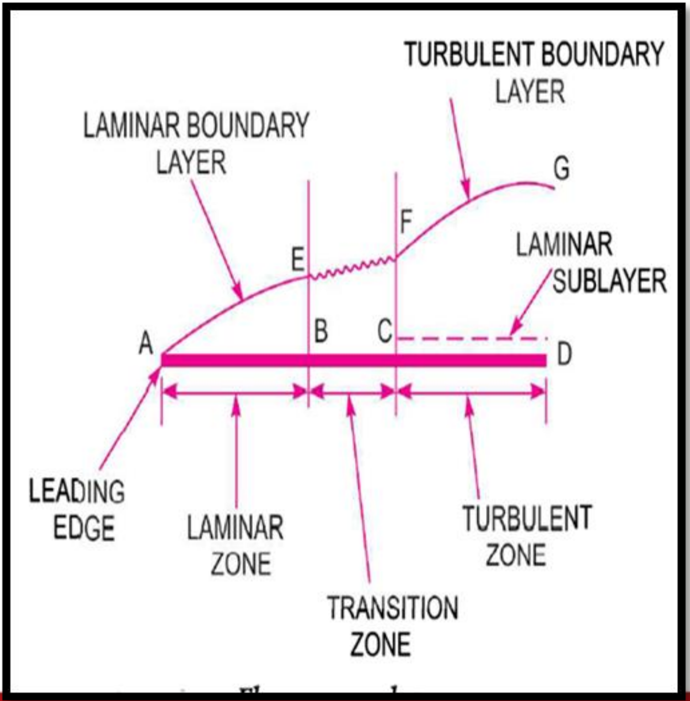
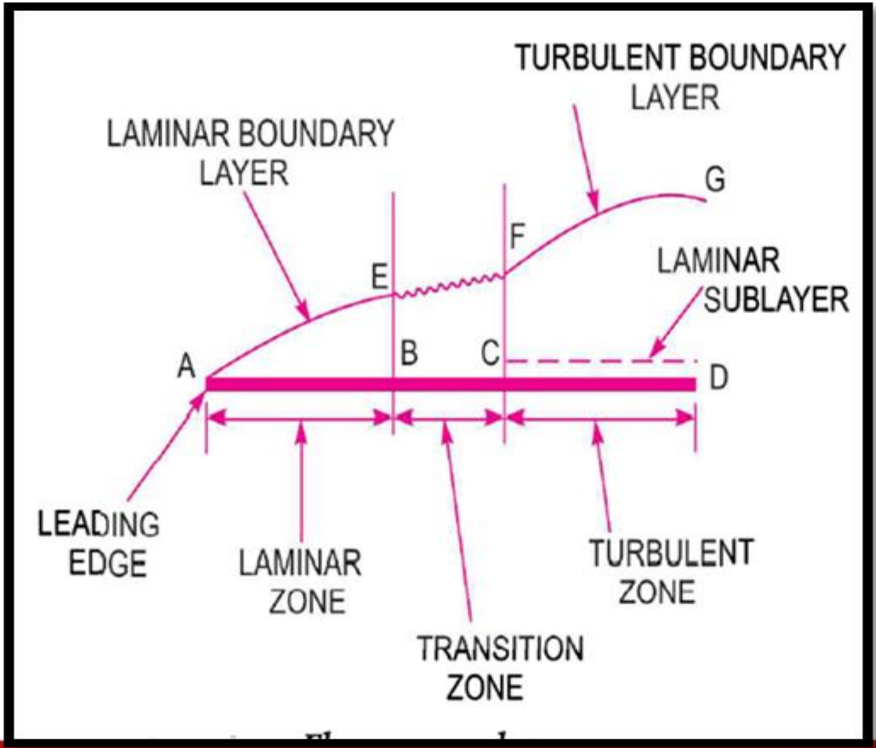
- This short length which the boundary layer flow changes from laminar to turbulent is called transition zone(BC)
- Further downstream the transition zone, the boundary layer is turbulent and continues to grow in thickness
- This layer of boundary is called turbulent boundary layer(FG)



Laminar Sub-layer

- It is adjacent to the solid surface of the plate
- The velocity variation is influenced only by viscous effects



Laminar Sub-layer(Cont...)

- Velocity distribution would be parabolic curve , but in view of the very small thickness we will assume velocity variation is linear and velocity gradient can be considered constant

- So, shear stress would be constant and equal to the boundary shear stress

$$\tau_0 = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu \frac{u}{y} \qquad \left\{For\ linear\ variation\ , \frac{\partial u}{\partial y} = \frac{u}{y}\right\}$$

Summary

- If the boundary is stationary, the velocity of fluid at the boundary will be zero
- The velocity of fluid increases from zero velocity on the stationary boundary to free-stream velocity (U) of the fluid in the direction normal to the boundary
- The theory dealing with boundary layer flows is called boundary layer theory
- Near the leading edge the thickness is small, the flow in the boundary layer is laminar , so this layer of fluid is said to be laminar boundary layer
- For Laminar sub-layer $\tau_0 = \mu \frac{u}{y}$

BOUNDARY LAYER FLOW:PART-2

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Lecture Details:
Unit-III (Boundary layer theory and applications) ,
Boundary Layer Flow: Part-2
FM & HM /Mechanical, I -Semester.

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BOUNDARY LAYER FLOW:PART-2

Contents

- Boundary Layer Thickness(δ)
- Displacement Thickness(δ^*)
- Momentum Thickness(θ)
- Energy Thickness(δ^{**})
- Summary

Boundary Layer Thickness(δ)

- Distance from boundary of the solid body measured in the y- direction to the point, where the velocity of the fluid is approximately equal to 0.99 times the free stream velocity(U) of the fluid
- δ_{lam} =Thickness of laminar boundary layer
- δ_{tur} =Thickness of turbulent boundary layer
- δ' =Thickness of laminar sub-layer

Displacement Thickness(δ^*)

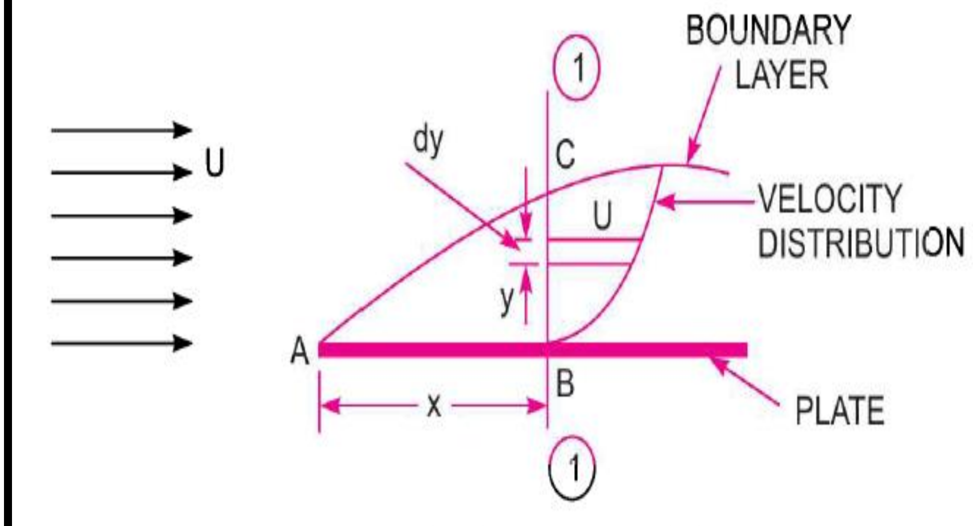
- It is the distance measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in flow rate on account of boundary layer formation

Or

- The distance perpendicular to the boundary, by which the free-stream is displaced due to the formation of boundary layer

Displacement Thickness(δ^*)(Cont…)

- Consider a smooth plate and a section 1-1 at a distance x from the leading edge
- Velocity of the fluid at B is zero
- At C, which is lies on the boundary layer is U
- Thus velocity varies from zero at B to U



Displacement thickness.

Displacement Thickness(δ^*)(Cont...)

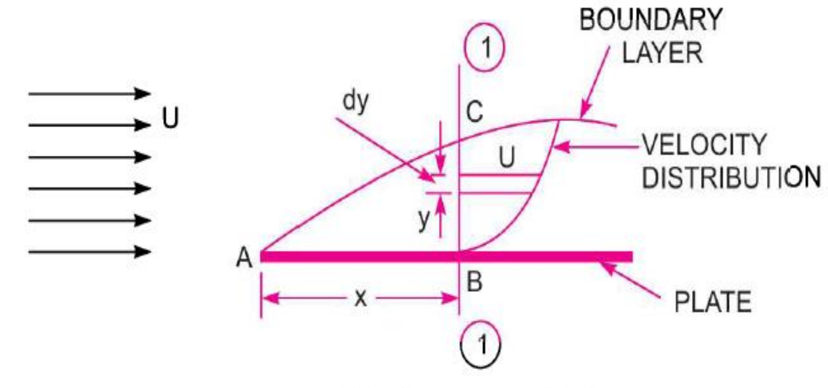
- Distance BC =δ
- At section 1-1, consider an elemental strip
- Let   y= distance of elemental strip from the plate

dy = thickness of the elemental strip

u= velocity of fluid at the elemental strip

b= width of plate

Then area of elemental strip, dA=bX dy



Displacement thickness.

Displacement Thickness(δ^*)(Cont…)

Mass of fluid per second flowing through elemental strip

$$= \rho X\ Velocity\ X\ Area\ of\ elemental\ strip$$

$$= \rho u \mathrm{X\ dA} = \rho u \mathrm{bdy}\text{-------}(1)$$

- If there had been no plate, then the fluid would have been flowing with a constant velocity equal to free-stream velocity(U) at the section 1-1

Then mass of fluid per second flowing through elemental strip = ρ X Velocity X Area= ρ X U X b X dy------(2)

Displacement Thickness(δ^*)(Cont…)

- As $U > u$, due to presence of plate, formation of the boundary layer takes place, there will be a reduction in mass flowing per second through the elemental strip

The reduction in mass/sec flowing through elemental strip = mass/sec given by equation(2) − mass/sec given by equation(1)

= ρUbdy − ρ $u$bdy

= ρb(U-u)dy

Displacement Thickness(δ^*)(Cont…)

- Total reduction in mass of fluid/sec flowing through BC due to plate

$$= \int_0^{\delta} \rho b(U - u)dy$$

- If fluid is incompressible i.e $\rho = constant$

$$= \rho b \int_0^{\delta} (U - u)dy \text{--------}(3)$$

Displacement Thickness(δ^*)(Cont...)

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- Let the plate is displaced by a distance δ^* and velocity of flow for the distance δ^* is equal to the free –stream velocity(i.e U)
- Loss of the mass of the fluid/sec flowing through the distance $\delta^* =$ $\rho X\ Velocity\ X\ Area$

$= \rho X\ U\ X\ (\delta^* X b)$-------(4)

Equating equation (3) and (4) {$\rho b \int_0^{\delta}(U-u)dy$--------(3)}

$\rho b \int_0^{\delta}(U-u)dy = \rho\, U\ \ \delta^* b$

Displacement Thickness(δ^*)(Cont…)

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$$\int_0^{\delta}(U-u)dy = U\ \delta^*$$

$$\delta^* = \frac{1}{U}\int_0^{\delta}(U-u)dy$$

U is constant , therefore

$$\delta^* = \int_0^{\delta}\frac{(U-u)dy}{U}$$

$$\boldsymbol{\delta^* = \int_0^{\delta}\left(1-\frac{u}{U}\right)dy}$$

Momentum Thickness(θ)

- Defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in momentum of the flowing fluid on account of boundary layer formulation

From equation (1) $\{\rho u X\, dA = \rho u bdy$--------$(1)\}$

Momentum of fluid flowing through elemental strip per second= Mass flow rate X Velocity = $\rho u bdy X u$

Momentum Thickness(θ)($Cont\ldots$)

Momentum of fluid in the absence of boundary layer = ρubdyXU

Loss of momentum through elemental strip = ρubdyXU- ρubdyXu

= ρb u(U- u)dy

Total loss of momentum/sec though BC= $\int_0^\delta \rho b\, u(U- u)dy$ ------(5)

- Let θ =distance by which plate is displace when the fluid is flowing with a constant velocity U

Momentum Thickness(θ)(Cont...)

- Loss of momentum/sec of fluid flowing through distance θ with a velocity

U =Mass of fluid through θ X Velocity

$= (\rho X area\ X\ velocity)$ X Velocity

$= (\rho X \theta X b X\ U)$ X U $\{Area = \theta X b\}$

$= \rho \theta b U^2$ -----------(6)

- Equating equation (5) and (6)

$$\rho \theta b U^2 = \int_0^\delta \rho b\, u(U - u) dy$$

Momentum Thickness(θ)(Cont...)

- If the fluid is assumed incompressible flow

$$\rho\theta b U^2 = \rho b \int_0^\delta u(U-u)dy$$

$$\theta U^2 = \int_0^\delta u(U-u)dy$$

$$\theta = \frac{1}{U^2}\int_0^\delta u(U-u)dy = \int_0^\delta \frac{u(U-u)dy}{U^2}$$

$$\boldsymbol{\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy}$$

Energy Thickness(δ^{**})

- Defined as the distance measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in kinetic energy of the flowing fluid on account of boundary layer formation
- We know that mass of fluid per second flowing through elemental strip (m)=$\rho u b d y$
- Kinetic energy of this fluid =$\frac{1}{2} m \; X \; velocity^2 = \frac{1}{2} \rho u b d y \; X \; u^2$

Energy Thickness(δ^{**})(Cont...)

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- Kinetic energy of this fluid in the absence of boundary layer $= \frac{1}{2}(\rho u b d y) U^2$
- Loss of K.E through elemental strip= $\frac{1}{2}(\rho u b d y) U^2 - \frac{1}{2} \rho u b d y \; X \; u^2$

$$= \frac{1}{2} \rho u b (U^2 - u^2) dy$$

- Total loss of K.E of fluid passing through BC= $\int_0^{\delta} \frac{1}{2} \rho u b (U^2 - u^2) dy$

Energy Thickness(δ^{**})(Cont...)

- If the fluid is incompressible

$$= \frac{1}{2}\rho b \int_0^{\delta} u(U^2 - u^2)dy$$

- Let $\boldsymbol{\delta}^{**}$ = distance by which the plate is displaced to compensate for the reduction in K.E
- Loss of K.E through $\boldsymbol{\delta}^{**}$ of fluid flowing with velocity U= $\frac{1}{2}$ $mass\ X\ velocity^2$

Energy Thickness(δ^{**})(Cont…)

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$$=\frac{1}{2}(\rho\ X\ areaX\ veolocity\ )Xvelocity^2$$

$$=\frac{1}{2}(\rho\ X\ bX\delta^{**}\ XU\ )XU^2 \qquad \{Area = bX\delta^{**}\}$$

$$=\frac{1}{2}\rho\ b\ \delta^{**}\ U^3$$

- Equating the two losses of K.E , we get

$$\frac{1}{2}\rho\ b\ \delta^{**}\ U^3 = \frac{1}{2}\rho b\int_0^{\delta} u(U^2 - u^2)dy$$

Energy Thickness(δ^{**})(Cont...)

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$$\delta^{**}\, U^3 = \int_0^\delta u(U^2 - u^2)dy$$

$$\delta^{**} = \frac{1}{U^3}\int_0^\delta u(U^2 - u^2)dy$$

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right)dy$$

Summary

- **Boundary Layer Thickness** is distance from solid body in the y- direction to the point, where the velocity of the fluid is approximately equal to 0.99 times the free stream velocity(U) of the fluid
- **Displacement Thickness**(δ^*) *is expressed as* $\int_0^\delta (1-\frac{u}{U})dy$
- **Momentum Thickness**(θ) *is expressed as* $\int_0^\delta \frac{u}{U}(1-\frac{u}{U})dy$
- **Energy Thickness**(δ^{**}) *is expressed as* $=\int_0^\delta \frac{u}{U}\left(1-\frac{u^2}{U^2}\right)dy$

BOUNDARY LAYER FLOW:PART-3

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Lecture Details:
Unit-III (Boundary layer theory and applications) ,
Boundary Layer Flow: Part-3
FM & HM /Mechanical, I -Semester.

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BOUNDARY LAYER FLOW:PART-3

Contents

- Drag Force on a Flat Plate due to Boundary Layer
- Summary

Problem:1

Find the displacement thickness, the momentum thickness and energy thickness for the velocity distribution in the boundary layer given by $\frac{u}{U} = \frac{y}{\delta}$, *where u is the velocity at a distance y from the plate and* $u = U$ *at* $y = \delta$, *where* δ = *boundary layer thickness. Also calculate the value of* δ^*/θ.

Solution. Given :

Velocity distribution $$\frac{u}{U} = \frac{y}{\delta}$$

Problem:1(Cont...)

(*i*) Displacement thickness δ^* is given by equation (13.2),

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy \qquad \left\{\because \frac{u}{U} = \frac{y}{\delta}\right\}$$

$$= \left[y - \frac{y^2}{2\delta}\right]_0^\delta \qquad \{\delta \text{ is constant across a section}\}$$

$$= \delta - \frac{\delta^2}{2\delta} = \delta - \frac{\delta}{2} = \frac{\delta}{2}. \quad \textbf{Ans.}$$

Problem:1(Cont...)

(*ii*) Momentum thickness, θ is given by equation

$$\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

Substituting the value of $\dfrac{u}{U} = \dfrac{y}{\delta}$,

$$\theta = \int_0^{\delta} \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy = \int_0^{\delta} \left(\frac{y}{\delta} - \frac{y^2}{\delta^2}\right) dy$$

$$= \left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right]_0^{\delta} = \frac{\delta^2}{2\delta} - \frac{\delta^3}{3\delta^2} = \frac{\delta}{2} - \frac{\delta}{3} = \frac{3\delta - 2\delta}{6} = \frac{\delta}{6}. \text{ Ans.}$$

Problem:1(Cont…)

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(*iii*) Energy thickness δ^{**} is given by equation , as

$$\delta^{**} = \int_0^{\delta} \frac{u}{U}\left[1 - \frac{u^2}{U^2}\right] dy = \int_0^{\delta} \frac{y}{\delta}\left[1 - \frac{y^2}{\delta^2}\right] dy \qquad \left\{\because \quad \frac{u}{U} = \frac{y}{\delta}\right\}$$

$$= \int_0^{\delta}\left[\frac{y}{\delta} - \frac{y^3}{\delta^3}\right] dy = \left[\frac{y^2}{2\delta} - \frac{y^4}{4\delta^3}\right]_0^{\delta} = \frac{\delta^2}{2\delta} - \frac{\delta^4}{4\delta^3}$$

$$= \frac{\delta}{2} - \frac{\delta}{4} = \frac{2\delta - \delta}{4} = \frac{\delta}{4}. \textbf{ Ans.}$$

(*iv*)

$$\frac{\delta^*}{\theta} = \frac{\left(\frac{\delta}{2}\right)}{\left(\frac{\delta}{6}\right)} = \frac{\delta}{2} \times \frac{6}{\delta} = 3. \textbf{ Ans.}$$

Problem:2

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Find the displacement thickness, the momentum thickness and energy thickness for the velocity distribution in the boundary layer given by $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$.

Solution. Given :

Velocity distribution $$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

Problem:2(Cont…)

(i) Displacement thickness δ^* is given by equation

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

Substituting the value of $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$, we have

$$\delta^* = \int_0^\delta \left\{1 - \left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]\right\} dy$$

$$= \int_0^\delta \left\{1 - 2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2\right\} dy = \left[y - \frac{2y^2}{2\delta} + \frac{y^3}{3\delta^2}\right]_0^\delta$$

$$= \delta - \frac{\delta^2}{\delta} + \frac{\delta^3}{3\delta^2} = \delta - \delta + \frac{\delta}{3} = \frac{\delta}{3}. \quad \textbf{Ans.}$$

Problem:2(Cont…)

(*ii*) Momentum thickness θ, is given by equation

$$\theta = \int_0^{\delta} \frac{u}{U}\left\{1-\frac{u}{U}\right\}dy = \int_0^{\delta}\left(\frac{2y}{\delta}-\frac{y^2}{\delta^2}\right)\left[1-\left(\frac{2y}{\delta}-\frac{y^2}{\delta^2}\right)\right]dy$$

$$= \int_0^{\delta}\left[\frac{2y}{\delta}-\frac{y^2}{\delta^2}\right]\left[1-\frac{2y}{\delta}+\frac{y^2}{\delta^2}\right]dy$$

$$= \int_0^{\delta}\left[\frac{2y}{\delta}-\frac{4y^2}{\delta^2}+\frac{2y^3}{\delta^3}-\frac{y^2}{\delta^2}+\frac{2y^3}{\delta^3}-\frac{y^4}{\delta^4}\right]dy$$

$$= \int_0^{\delta}\left[\frac{2y}{\delta}-\frac{5y^2}{\delta^2}+\frac{4y^3}{\delta^3}-\frac{y^4}{\delta^4}\right]dy = \left[\frac{2y^2}{2\delta}-\frac{5y^3}{3\delta^2}+\frac{4y^4}{4\delta^3}-\frac{y^5}{5\delta^4}\right]_0^{\delta}$$

$$= \left[\frac{\delta^2}{\delta}-\frac{5\delta^3}{3\delta^2}+\frac{\delta^4}{\delta^3}-\frac{\delta^5}{5\delta^4}\right] = \delta-\frac{5\delta}{3}+\delta-\frac{\delta}{5}$$

$$= \frac{15\delta-25\delta+15\delta-3\delta}{15} = \frac{30\delta-28\delta}{15} = \frac{\mathbf{2\delta}}{\mathbf{15}}. \quad \textbf{Ans.}$$

Problem:2(Cont...)

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(*iii*) Energy thickness δ^{**} is given by equation

$$\delta^{**} = \int_0^{\delta} \frac{u}{U}\left[1 - \frac{u^2}{U^2}\right] dy = \int_0^{\delta} \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \left[\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right]^2\right) dy$$

$$= \int_0^{\delta} \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \left[\frac{4y^2}{\delta^2} + \frac{y^4}{\delta^4} - \frac{4y^3}{\delta^3}\right]\right) dy$$

$$= \int_0^{\delta} \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \frac{4y^2}{\delta^2} - \frac{y^4}{\delta^4} + \frac{4y^3}{\delta^3}\right) dy$$

$$= \int_0^{\delta} \left(\frac{2y}{\delta} - \frac{8y^3}{\delta^3} - \frac{2y^5}{\delta^5} + \frac{8y^4}{\delta^4} - \frac{y^2}{\delta^2} + \frac{4y^4}{\delta^4} + \frac{y^6}{\delta^6} - \frac{4y^5}{\delta^5}\right) dy$$

Problem:2(Cont...)

$$= \int_0^{\delta}\left[\frac{2y}{\delta}-\frac{y^2}{\delta^2}-\frac{8y^3}{\delta^3}+\frac{12y^4}{\delta^4}-\frac{6y^5}{\delta^5}+\frac{y^6}{\delta^6}\right]dy$$

$$= \left[\frac{2y^2}{2\delta}-\frac{y^3}{3\delta^2}-\frac{8y^4}{4\delta^3}+\frac{12y^5}{5\delta^4}-\frac{6y^6}{6\delta^5}+\frac{y^7}{7\delta^6}\right]_0^{\delta}$$

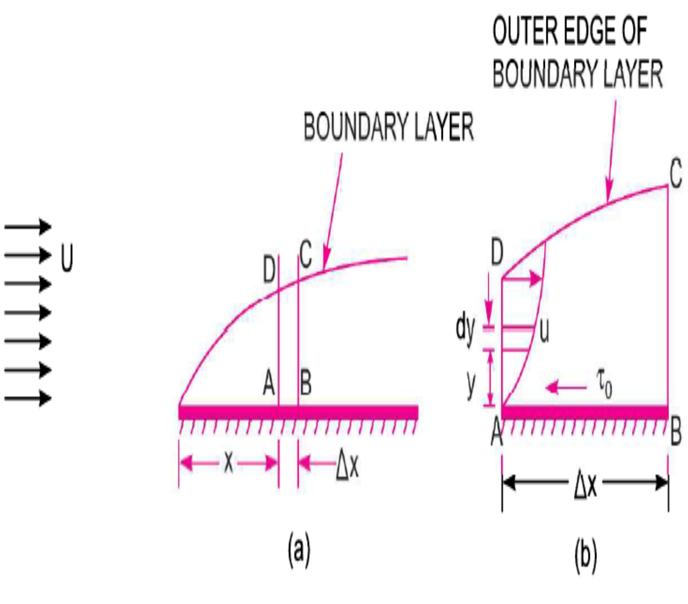
$$= \frac{\delta^2}{\delta}-\frac{\delta^3}{3\delta^2}-\frac{2\delta^4}{\delta^3}+\frac{12\delta^5}{5\delta^4}-\frac{\delta^6}{\delta^5}+\frac{\delta^7}{7\delta^6}=\delta-\frac{\delta}{3}-2\delta+\frac{12}{5}\delta-\delta+\frac{\delta}{7}$$

$$= -2\delta-\frac{\delta}{3}+\frac{12}{5}\delta+\frac{\delta}{7}=\frac{-210\delta-35\delta+252\delta+15\delta}{105}$$

$$= \frac{-245\delta+267\delta}{105}=\frac{22\delta}{105}.\quad \textbf{Ans.}$$

Drag Force on a Flat Plate due to Boundary Layer

- In **fluid dynamics**, **drag** (**fluid resistance**, another type of friction or **fluid** friction) is a **force** acting opposite to the relative motion of any object moving with respect to a surrounding **fluid**
- The drag force on the plate can be determined if the velocity profile near the plate is known
- Consider a small length Δx of the plate at a distance of x from the leading edge



Drag force on a plate due to boundary layer.

Drag Force on a Flat Plate due to Boundary Layer (Cont…)

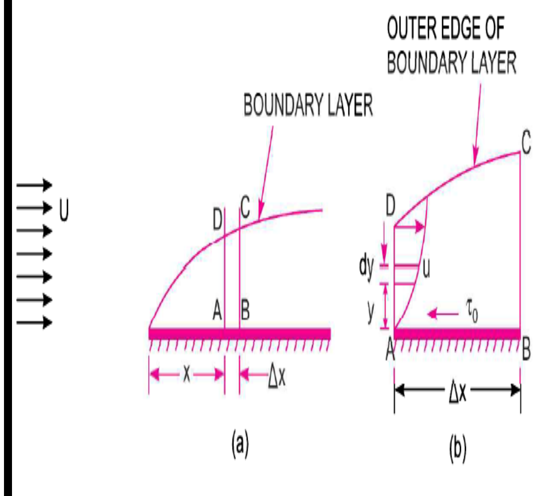
- The shear stress $\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0}$, where $\left(\frac{du}{dy}\right)_{y=0}$ is the velocity distribution near the plate at y=0
- Then drag force or shear force on a small distance Δx

$$\Delta F_D = \text{shear stress X area}$$

$$= \tau_0 X \Delta x X b$$

$$\{Taking\ width\ of\ plate = b\}$$

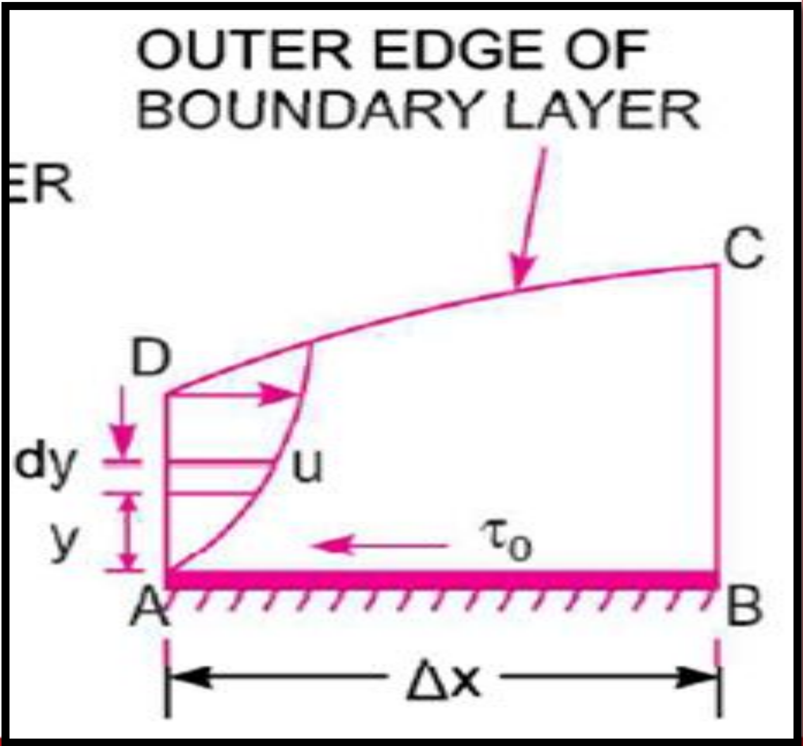
Where ΔF_D = drag force on distance Δx



Drag force on a plate due to boundary layer.

Drag Force on a Flat Plate due to Boundary Layer (Cont…)

- The drag force must also be equal to the rate of change of momentum over the distance Δx
- Let ABCD is the control volume of the fluid over the distance Δx
- The edge DC represents the outer edge of the boundary layer



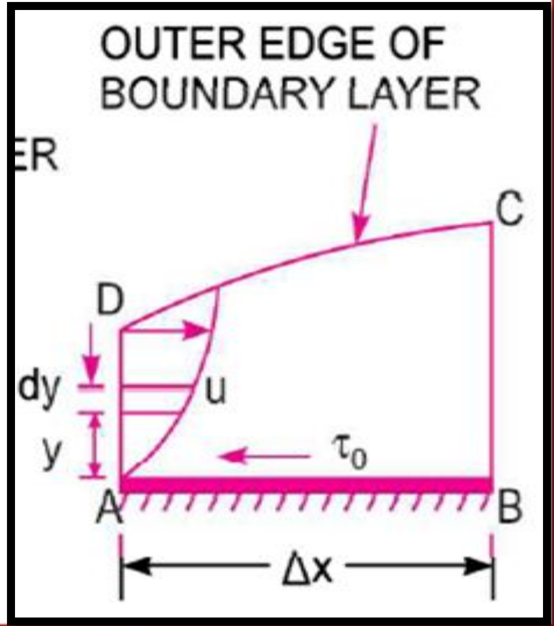
Drag Force on a Flat Plate due to Boundary Layer (Cont...)

- Let u= velocity at any point within the boundary layer

b= width of plate

The mass flow rate entering through the side AD

$$=\int_0^{\delta} \rho X \; velocity \; X \; area \; of \; strip \; of \; thickness \; dy$$



Drag Force on a Flat Plate due to Boundary Layer (Cont…)

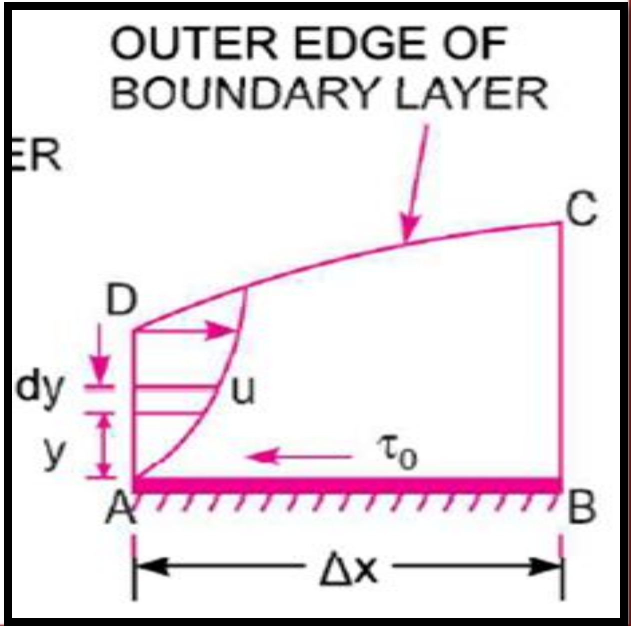
$$= \int_0^{\delta} \rho X\, uXbX\, dy = \int_0^{\delta} \rho ubdy$$

$$\{Area\ of\ strip = bX\ dy\}$$

Mass flow rate leaving the side BC= mass flow rate

through AD + $\frac{\partial}{\partial x}(mass\ flow\ rate\ through\ AD)X\ \Delta x$

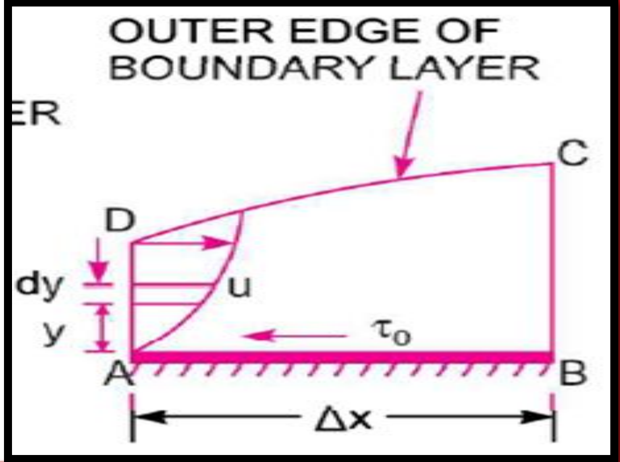
$$= \int_0^{\delta} \rho ubdy + \frac{\partial}{\partial x}\left[\int_0^{\delta} \rho ubdy\right] X\Delta x$$



Drag Force on a Flat Plate due to Boundary Layer (Cont…)

- From continuity equation for a steady incompressible fluid flow
- Mass flow rate entering AD+ Mass flow rate entering DC = Mass flow rate leaving BC
- Mass flow rate entering DC = Mass flow rate through BC – Mass flow rate through AD

$$= \int_0^{\delta} \rho u b dy + \frac{\partial}{\partial x}\left[\int_0^{\delta} \rho u b dy\right] X \Delta x - \int_0^{\delta} \rho u b dy$$



Drag Force on a Flat Plate due to Boundary Layer (Cont…)

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$$= \int_0^{\delta} \rho u b dy + \frac{\partial}{\partial x}\left[\int_0^{\delta} \rho u b dy\right] X \Delta x - \int_0^{\delta} \rho u b dy$$

$$= \frac{\partial}{\partial x}\left[\int_0^{\delta} \rho u b dy\right] X \Delta x$$

- The fluid is entering through side DC with a uniform velocity U
- Now let us calculate momentum flux through control volume

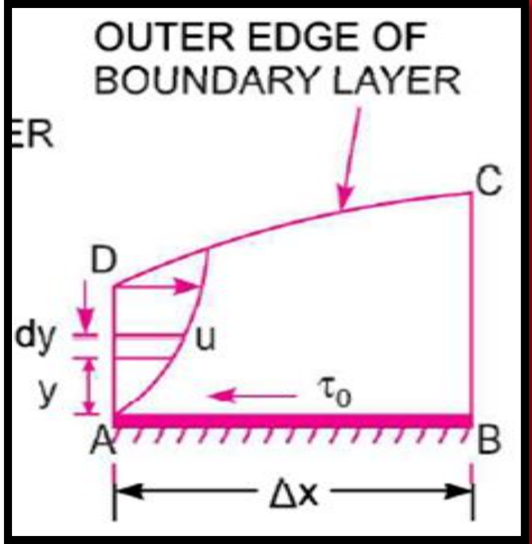
Drag Force on a Flat Plate due to Boundary Layer (Cont…)

Momentum flux entering through AD

$$=\int_0^{\delta} momentum\ flux\ through\ strip\ of\ thickness\ dy$$

$$=\int_0^{\delta} mass\ flow\ rate\ through\ strip\ X\ velocity$$

$$=\int_0^{\delta} (\rho u b dy) X\ u = \int_0^{\delta} \rho\, u^2 b dy$$



Momentum flux leaving the side BC $=\int_0^{\delta} (\rho\, u b dy) u + \frac{\partial}{\partial x}\left[\int_0^{\delta} (\rho u b dy) u\right] X \Delta x$

Drag Force on a Flat Plate due to Boundary Layer (Cont…)

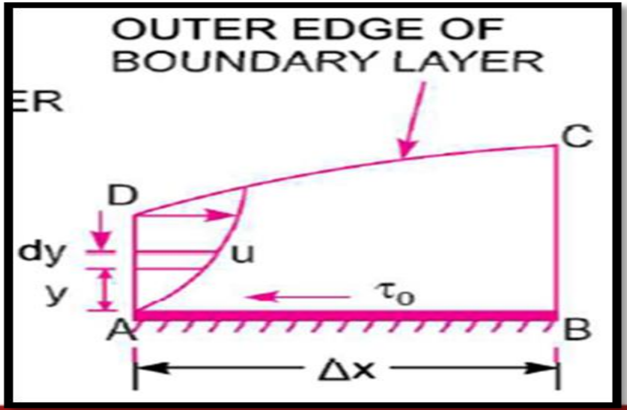
$$= \int_0^{\delta} \rho\, u^2 b dy + \frac{\partial}{\partial x}\left[\int_0^{\delta} \rho u^2 b dy\right] X \Delta x$$

Momentum flux entering the side DC = mass flow rate through DC X velocity

$$\{Velocity = U\ and\ it\ is\ a\ constant\ value\}$$

$$= \frac{\partial}{\partial x}\left[\int_0^{\delta} \rho u b dy\right] X \Delta x X U$$

$$= \frac{\partial}{\partial x}\left[\int_0^{\delta} \rho u U b dy\right] X \Delta x$$



Drag Force on a Flat Plate due to Boundary Layer (Cont…)

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- Rate of change of momentum of the control volume= Momentum flux through BC- Momentum flux through AD – Momentum flux through DC

$$= \int_0^{\delta} \rho\, u^2 b dy + \frac{\partial}{\partial x}\left[\int_0^{\delta} \rho u^2 b dy\right] X \Delta x - \int_0^{\delta} \rho\, u^2 b dy - \frac{\partial}{\partial x}\left[\int_0^{\delta} \rho u U b dy\right] X \Delta x$$

$$= \frac{\partial}{\partial x}\left[\int_0^{\delta} \rho u^2 b dy\right] X \Delta x - \frac{\partial}{\partial x}\left[\int_0^{\delta} \rho u U b dy\right] X \Delta x$$

Drag Force on a Flat Plate due to Boundary Layer (Cont...)

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$$= \frac{\partial}{\partial x}\left[\int_0^{\delta} \rho u^2 b dy - \int_0^{\delta} \rho u U b dy\right] \Delta x$$

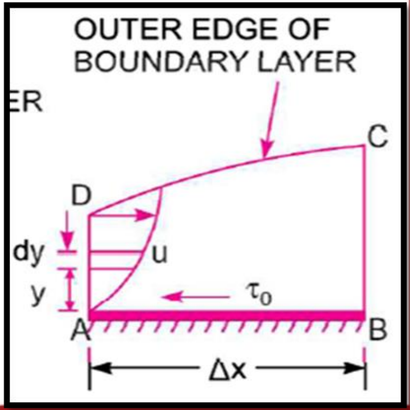
$\{For\ incompressible\ fluid\ \rho = constant\}$

$$= \frac{\partial}{\partial x}\left[\rho b \int_0^{\delta} (u^2 - uU) dy\right] \Delta x$$

$$= \rho b \frac{\partial}{\partial x}\left[\int_0^{\delta} (u^2 - uU) dy\right] \Delta x \text{-------(1)}$$

Drag Force on a Flat Plate due to Boundary Layer (Cont…)

- Now the rate of change of momentum on the control volume ABCD must be equal to the total force on the control volume in the same direction according to the momentum principle
- But for a flat plate $\frac{\partial p}{\partial x} = 0$, which means there is no external pressure force on the control volume
- Also the force on the side DC is negligible as the velocity is constant and velocity gradient is zero approximately



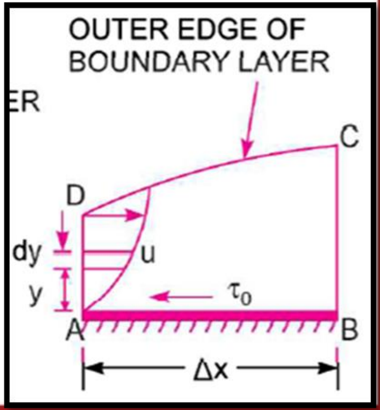
Drag Force on a Flat Plate due to Boundary Layer (Cont…)

- The only external force acting on the control volume is the shear force acting on the side AB in the direction from B to A

$$\Delta F_D = \tau_0 X\ \Delta\mathrm{x}\ \mathrm{X}\ \mathrm{b}$$

The external force in the direction of rate of change of momentum= $-\tau_0 X\ \Delta\mathrm{x}\ \mathrm{X}\ \mathrm{b}$-----(2)

- According to momentum principle, equating equation (1) and (2)



Drag Force on a Flat Plate due to Boundary Layer (Cont…)

$$-\tau_0 X\ \Delta\mathrm{x}\ \mathrm{X}\ \mathrm{b} = \rho b \frac{\partial}{\partial x}\left[\int_0^{\delta}(u^2 - uU)dy\right]\Delta x$$

$$-\tau_0 = \rho \frac{\partial}{\partial x}\left[\int_0^{\delta}(u^2 - uU)dy\right]$$

$$\tau_0 = -\rho \frac{\partial}{\partial x}\left[\int_0^{\delta}(u^2 - uU)dy\right] = \rho \frac{\partial}{\partial x}\left[\int_0^{\delta}(uU - u^2)dy\right]$$

$$= \rho \frac{\partial}{\partial x}\left[\int_0^{\delta} U^2\left(\frac{u}{U} - \frac{u^2}{U^2}\right)dy\right]$$

Drag Force on a Flat Plate due to Boundary Layer (Cont...)

$$= \rho U^2 \frac{\partial}{\partial x}\left[\int_0^{\delta} \frac{u}{U}(1-\frac{u}{U})dy\right]$$

$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\left[\int_0^{\delta} \frac{u}{U}(1-\frac{u}{U})dy\right]$$

$\int_0^{\delta} \frac{u}{U}(1-\frac{u}{U})dy$ is equal to momentum thickness (θ)

$$\frac{\tau_0}{\rho U^2} = \frac{\partial \theta}{\partial x} \text{--------(3)}$$

- Equation (3) is known as **Von Karman momentum integral equation** for boundary layer Flows

Drag Force on a Flat Plate due to Boundary Layer (Cont…)

- **Von Karman momentum integral equation** is applied to

(1) Laminar boundary layers

(2) Transition boundary layers

(3) Turbulent boundary layer flows

- The drag force on a small distance Δx of the plate is obtained as $\Delta F_D = \tau_0 \Delta x b$
- Then the total drag on the plate of length L on one side is

$$F_D = \int \Delta F_D = \int_0^L \tau_0 b dx \qquad \{Change \Delta x = dx\}$$

Summary

- The drag force on the plate can be determined if the velocity profile near the plate is known
- **Von Karman momentum integral equation** is given by $\frac{\tau_0}{\rho U^2} = \frac{\partial \theta}{\partial x}$

BOUNDARY LAYER FLOW:PART-5

Presented By:
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Lecture Details:
Unit-III (Boundary layer theory and applications) ,
Boundary Layer Flow: Part-5
FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
- FLUID DYNAMICS
- CLOSED CONDUIT FLOW
- BOUNDARY LAYER THEORY AND APPLICATIONS
- BASICS OF TURBO MACHINERY
- HYDRAULIC TURBINES
- PERFORMANCE OF HYDRAULIC TURBINES
- CENTRIFUGAL PUMPS
- RECIPROCATING PUMPS

BOUNDARY LAYER FLOW:PART-5

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Contents

- Boundary Layer Separation
- Effect of Pressure Gradient on Boundary Layer Separation
- Location of Separation Point
- Methods of Preventing the Separation of Boundary Layer
- Summary

Boundary Layer Separation

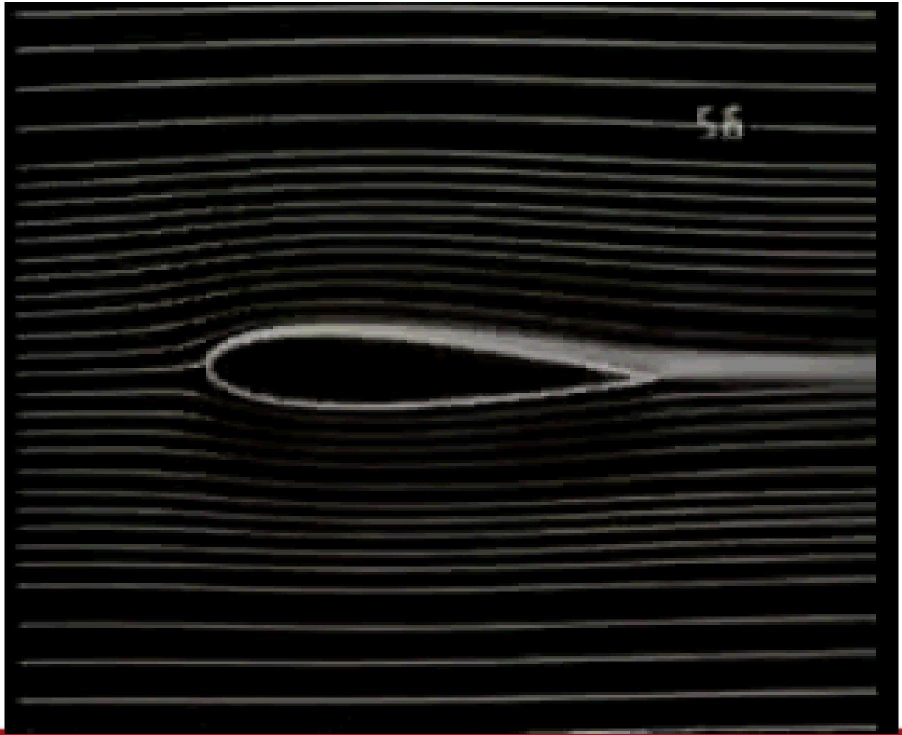
- When a solid body is immersed in flowing fluid, along the length of the solid body, the thickness of the boundary layer increases
- The fluid layer adjacent to the solid surface has to do work against surface friction at the expense of its kinetic energy
- This loss of the kinetic energy is recovered from the immediate fluid layer in contact with the layer adjacent to solid surface through momentum exchange process

Boundary Layer Separation(Cont…)

- Thus the velocity of the layer goes on decreasing
- Along the length of the solid body, at a certain point a stage may come when the boundary layer may not be able to keep sticking to the solid body
- In other words, the boundary layer will be separated from the surface

Boundary Layer Separation(Cont…)

- This phenomenon is called the boundary layer separation
- The point on the body at which the boundary layer is on the verge of separation from the surface is called point of separation



Effect of Pressure Gradient on Boundary Layer Separation

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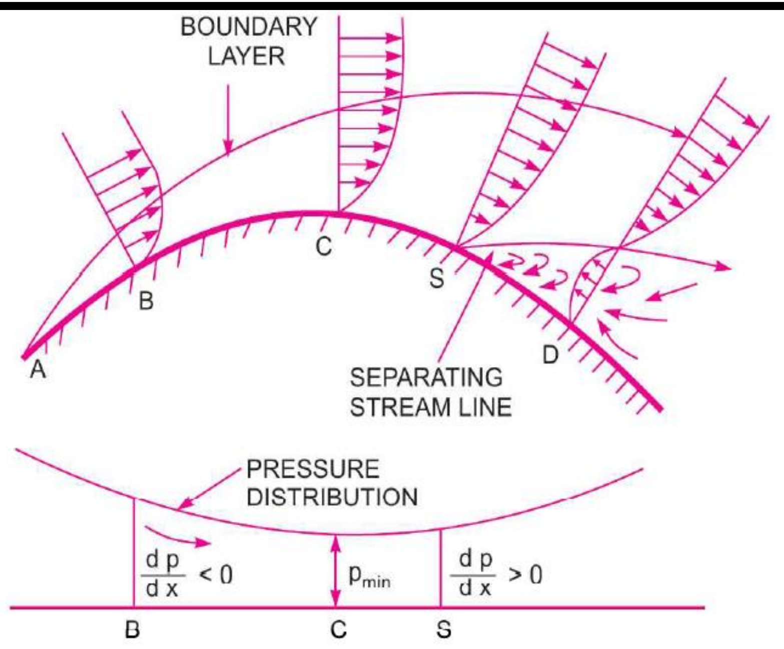
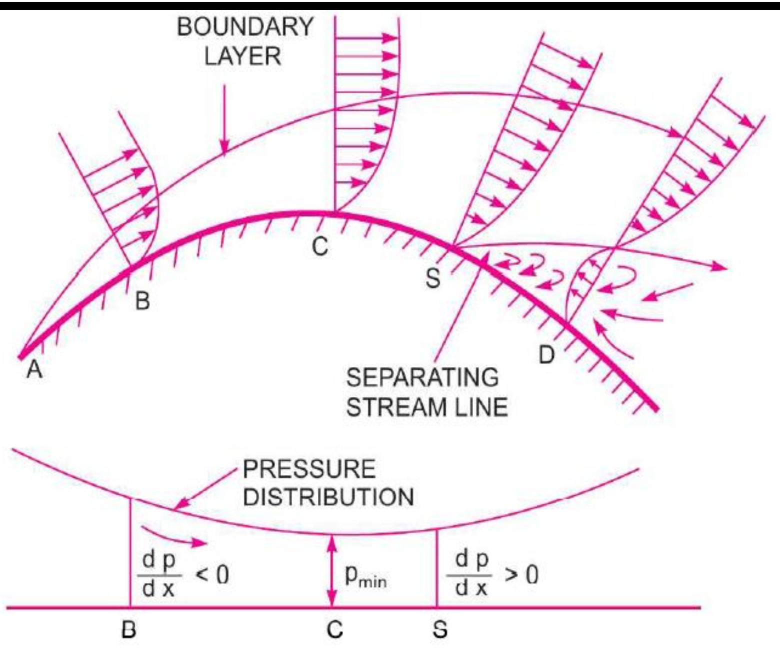
- The effect of pressure gradient $\left(\frac{dp}{dx}\right)$ on boundary layer separation can be explained by considering the flow over a curved surface ABCSD



Effect of pressure gradient on boundary layer separation.

Effect of Pressure Gradient on Boundary Layer Separation(Cont…)

- In the region ABC of the curved surface, the area of flow decreases and hence velocity increases
- This means flow gets accelerated in this region

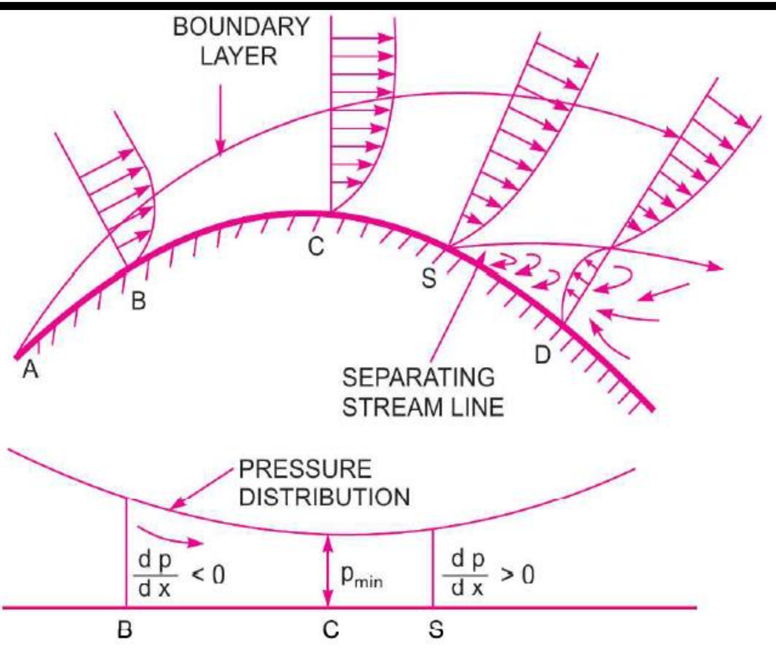


Effect of pressure gradient on boundary layer separation.

Effect of Pressure Gradient on Boundary Layer Separation(Cont...)

➢ The pressure decreases in the direction of the flow and hence pressure gradient $\frac{dp}{dx}$ is negative in this region

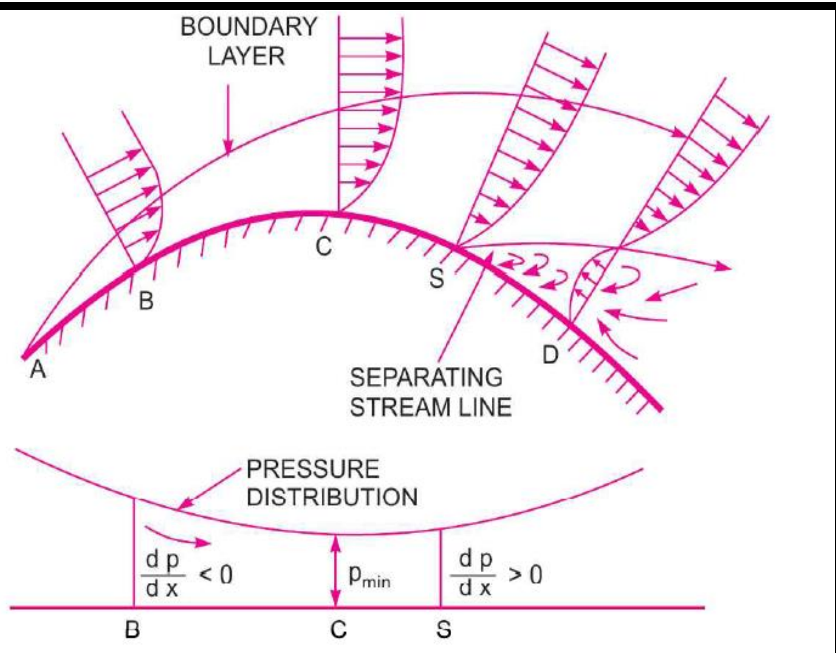
➢ As long as $\frac{dp}{dx} < 0$, the entire boundary layer moves forward



Effect of pressure gradient on boundary layer separation.

Effect of Pressure Gradient on Boundary Layer Separation(Cont...)

- The pressure is minimum at the point C
- Along the region CSD of the curved surface, the area of flow increases and hence velocity of flow along the direction of fluid decreases



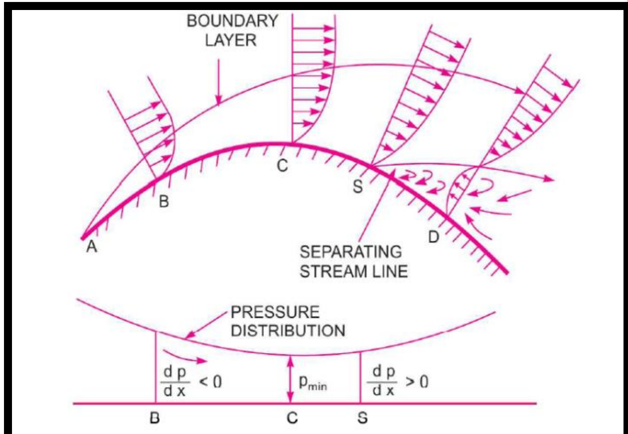
Effect of pressure gradient on boundary layer separation.

Effect of Pressure Gradient on Boundary Layer Separation(Cont…)

- Due to decrease of velocity, the pressure increases in the direction of flow and hence $\frac{dp}{dx}$ is positive or $\frac{dp}{dx} > 0$
- As the kinetic energy of the layer is used to overcome the frictional resistance of the surface
- Combined effect of positive pressure gradient and surface resistance reduce the momentum of the fluid

Effect of Pressure Gradient on Boundary Layer Separation(Cont…)

- A stage may come where momentum of the fluid is unable to overcome the surface resistance and the boundary layer starts separating from the surface at the point S
- Downstream the point S, the flow is taking place in reverse direction and the velocity gradient becomes negative
- Thus the positive pressure gradient helps in separating the boundary layer



Effect of pressure gradient on boundary layer separation.

Location of Separation Point

- The separation point S is determined from the condition $\left\{\frac{\partial u}{\partial y}\right\}_{y=0} = 0$

1. If $\left\{\frac{\partial u}{\partial y}\right\}_{y=0} < 0$, then the flow has separated
2. If $\left\{\frac{\partial u}{\partial y}\right\}_{y=0} = 0$, then the flow is on the verge of separated
3. If $\left\{\frac{\partial u}{\partial y}\right\}_{y=0} > 0$, then the flow will not separate or flow will remain attached with the surface

Methods of Preventing the Separation of Boundary Layer

1. Suction of the slow moving fluid by a suction slot
2. Supplying additional energy from a blower
3. Providing a bypass in the slotted wing
4. Rotating boundary in the direction of flow

Methods of Preventing the Separation of Boundary Layer (Cont...)

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5. Providing small divergence in a diffuser
6. Providing guide-blades in a bend
7. Providing a trip-wire ring in the laminar region for the flow over a sphere

Summary

- When a solid body is immersed in flowing fluid, along the length of the solid body, the thickness of the boundary layer increases
- The point on the body at which the boundary layer is on the verge of separation from the surface is called point of separation
- The boundary layer separation point S is determined from the condition

$$\left\{\frac{\partial u}{\partial y}\right\}_{y=0} = 0$$

BOUNDARY LAYER FLOW:PART-6

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Lecture Details:
Unit-III (Boundary layer theory and applications) ,
Boundary Layer Flow: Part-6
FM & HM /Mechanical, I -Semester.

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BOUNDARY LAYER FLOW:PART-6

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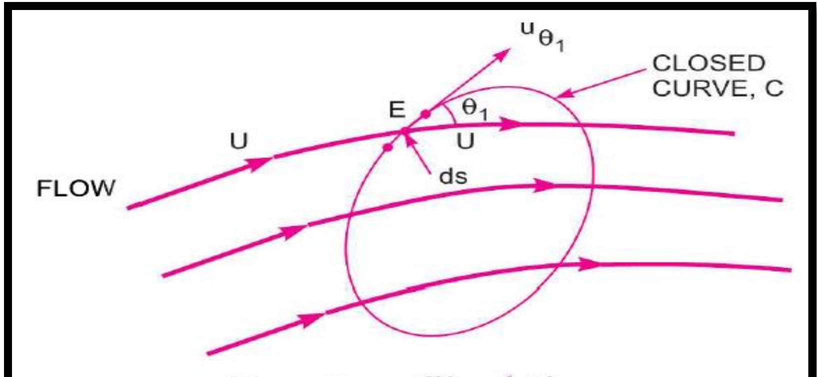
Contents

- Circulation on submerged Bodies
- Force Exerted by a Flowing Fluid on a Stationary Body
- Expression for Drag and Lift
- Magnus Effect
- Summary

Circulation on submerged Bodies

- Circulation is defined as the flow along a closed curve
- Mathematically, the circulation is obtained if the product of the velocity component along the curve at any point and the length of the small element containing that point is integrated around the curve
- Let E is any point on the closed curve and 'dS' is a small length of the closed curve containing point E



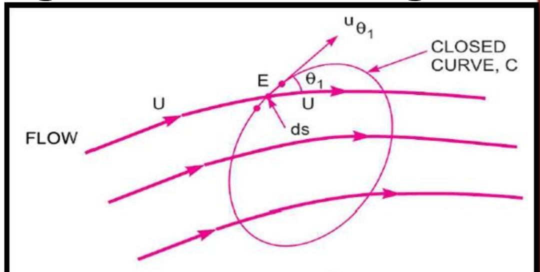
Circulation.

Circulation on submerged Bodies(Cont…)

- Let θ_1 =Angle made by the tangent at E with the direction of flow

u_{θ_1} = Component of free stream velocity along the tangent at E and is given as=U $\cos\theta_1$

- By definition, circulation along the closed curve is



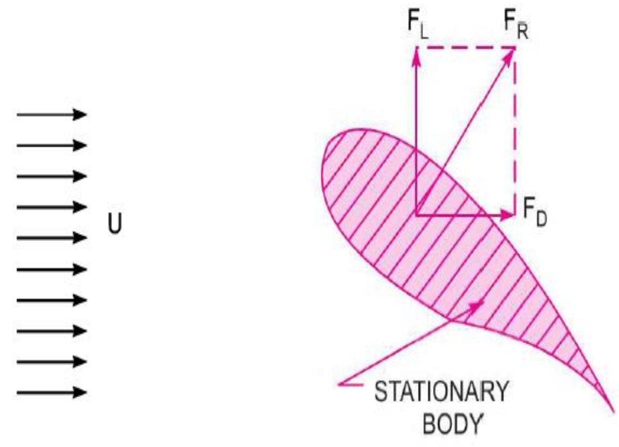
Circulation.

$$\Gamma = \oint velocity\ component\ along\ curve\ X\ Length\ of\ element$$

$= \oint U \cos\theta_1$X dS , where $\oint$=Integral for the complete closed curve

Force Exerted by a Flowing Fluid on a Stationary Body

- Consider a body held stationary in a real fluid
- The fluid will exert a force on the stationary body
- The total force (F_R) exerted by the fluid on the body is perpendicular to the surface of the body
- The total force can be resolved into two components, one in direction of motion and other perpendicular to direction of motion



Force on a stationary body.

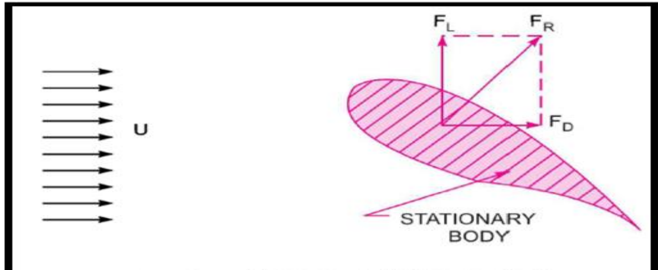
Force Exerted by a Flowing Fluid on a Stationary Body(Cont...)

Drag:

- The component of the total force(F_R) in the direction of motion is called 'drag'. This component is denoted by F_D

Lift:

- The component of the total force(F_R) in the direction perpendicular to the direction of motion is known as 'lift'
- This is denoted by F_L



Force on a stationary body.

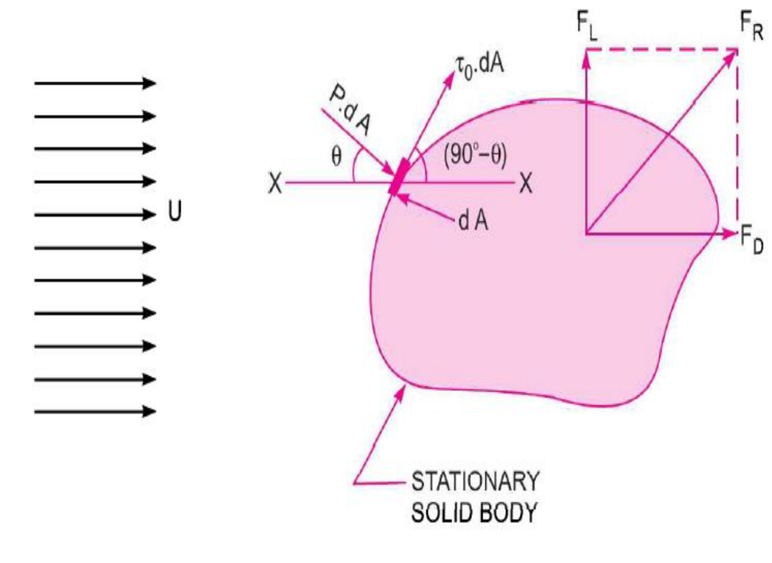
Force Exerted by a Flowing Fluid on a Stationary Body(Cont...)

- Lift force occurs only when the axis of the body is inclined to the direction of fluid flow
- If the axis of the body is parallel to the direction of fluid flow ,lift force is zero
- If the fluid is assumed ideal and the body is symmetrical such as a sphere or cylinder, both the drag and lift will be zero

Expression for Drag and Lift

- Consider a small elemental area dA on the surface of the body
- The forces acting on the surface area dA are:

1. Pressure force equal to pXdA, acting perpendicular to the surface
2. Shear force equal to $\tau_0 X\, dA$, acting along the tangential direction to the surface



Drag and lift.

Expression for Drag and Lift(Cont...)

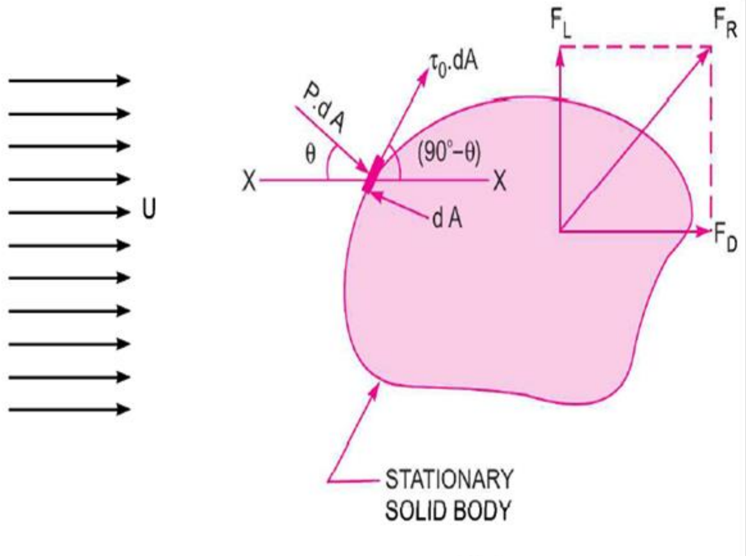
- Let θ =Angle made by pressure force with horizontal direction

(a) Drag Force(F_D)

The drag force on the elemental area= Force due to pressure in the direction of fluid motion+ Force due to shear stress in the direction of fluid motion

$$= pdA\, cos\theta + \tau_0 dA \cos(90^0 - \theta)$$

$$= pdA\, cos\theta + \tau_0 dA\, sin\theta$$



Drag and lift.

Expression for Drag and Lift(Cont…)

- Total drag F_D = Summation of pdA $\cos\theta$ + Summation of $\tau_0 dA \sin\theta$

$$= \int \mathrm{p} \cos\theta \, \mathrm{dA} + \int \tau_0 \sin\theta \, dA$$

- The term $\int \mathrm{p} \cos\theta \, \mathrm{dA}$ is called the pressure drag or form drag while the term $\int \tau_0 \sin\theta \, dA$ is called the friction drag or skin drag or shear drag

(b) Lift Force(F_L)

- The lift force on elemental area= Force due to pressure in the direction perpendicular to the direction of motion + Force due to shear stress in the direction perpendicular to the direction of motion

Expression for Drag and Lift(Cont...)

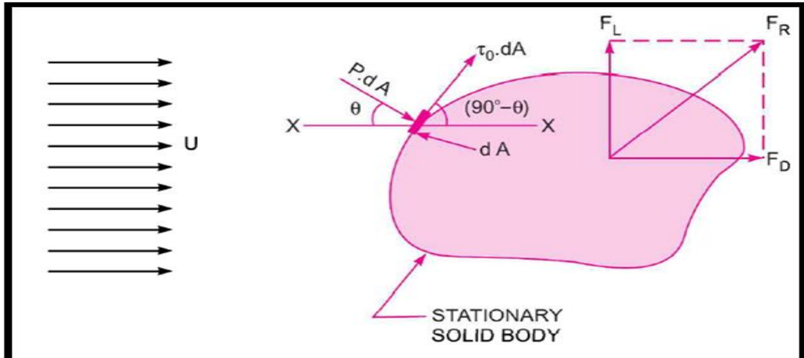
$$= -pdA\sin\theta + \tau_0 dA\sin(90^0 - \theta)$$

$$= -pdA\sin\theta + \tau_0 dA\cos\theta$$

- Negative sign is taken with pressure force as it is acting in the downward direction while shear force is acting vertically up

Total lift, $F_L = \int \tau_0 dA\cos\theta - \int pdA\sin\theta$

$$= \int \tau_0 \cos\theta\, dA - \int p\sin\theta\, dA$$



Drag and lift.

Expression for Drag and Lift(Cont...)

- The drag and lift for a body moving in a fluid of density ρ, at a uniform velocity U are calculated mathematically as

$$F_D = C_D A \frac{\rho U^2}{2}$$

$$F_L = C_L A \frac{\rho U^2}{2}$$

Where C_D = Co-efficient of drag, C_L = Co-efficient of lift,

Expression for Drag and Lift(Cont...)

A= Area of the body which is the projected area of the body perpendicular to the direction of flow or largest projected area of the immersed body

- Then resultant force on the body, $F_R = \sqrt{{F_D}^2 + {F_L}^2}$

Problem:1

A flat plate 1.5 m × 1.5 m moves at 50 km/hour in stationary air of density 1.15 kg/m³. If the co-efficients of drag and lift are 0.15 and 0.75 respectively, determine :

(i) The lift force, *(ii) The drag force,*

(iii) The resultant force, and

(iv) The power required to keep the plate in motion.

Solution. Given :

Area of the plate, $A = 1.5 \times 1.5 = 2.25\ \text{m}^2$

Velocity of the plate, $U = 50\ \text{km/hr} = \dfrac{50 \times 1000}{60 \times 60}\ \text{m/s} = 13.89\ \text{m/s}$

Density of air $\rho = 1.15\ \text{kg/m}^3$

Co-efficient of drag, $C_D = 0.15$

Co-efficient of lift, $C_L = 0.75$

Problem:1(Cont...)

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(*i*) **Lift Force (F_L).** Using equation

$$F_L = C_L A \times \frac{\rho U^2}{2} = 0.75 \times 2.25 \times \frac{1.15 \times 13.89^2}{2} \text{ N} = \mathbf{187.20\ N.\ Ans.}$$

(*ii*) **Drag Force (F_D).** Using equation

$$F_D = C_D \times A \times \frac{\rho U^2}{2} = 0.15 \times 2.25 \times \frac{1.15 \times 13.89^2}{2} \text{ N} = \mathbf{37.44.\ N.\ Ans.}$$

Problem:1(Cont...)

(*iii*) **Resultant Force (F_R).** Using equation

$$F_R = \sqrt{F_D^2 + F_L^2} = \sqrt{37.44^2 + 187.20^2} \text{ N}$$

$$= \sqrt{1400 + 35025} = \mathbf{190.85\ N.\ Ans.}$$

(*iv*) **Power Required to keep the Plate in Motion**

$$P = \frac{\text{Force in the direction of motion} \times \text{Velocity}}{1000} \text{ kW}$$

$$= \frac{F_D \times U}{1000} = \frac{37.425 \times 13.89}{1000} \text{ kW} = \mathbf{0.519\ kW.\ Ans.}$$

Magnus Effect

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- When a cylinder is rotated in a uniform flow , a lift force is produced on the cylinder
- This phenomenon of the lift force produced by a rotating cylinder in a uniform flow is known as Magnus Effect

Summary

- Circulation is obtained if the product of the velocity component along the curve at any point and the length of the small element containing that point is integrated around the curve $= \oint U\, cos\, \theta_1 \mathrm{X}\, \mathrm{dS}$
- The component of the total force(F_R) in the direction of motion is called 'drag'= $= \int \mathrm{p}\, \mathrm{cos}\theta\, \mathrm{dA} + \int \tau_0\, sin\theta\, dA$
- The component of the total force(F_R) in the direction perpendicular to the direction of motion is known as 'lift' $= \int \tau_0 cos\theta dA - \int \mathrm{p}\, \mathrm{sin}\theta\, \mathrm{dA}$
- The phenomenon of the lift force produced by a rotating cylinder in a uniform flow is known as Magnus Effect

IMPACT OF JETS ON A STATIONARY PLATE: PART-1

Presented By:

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Lecture Details:

Unit-IV (Basics of Turbo Machinery) , IMPACT OF JETS ON A STATIONARY PLATE: PART-1FM & HM /Mechanical, I - Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
- FLUID DYNAMICS
- CLOSED CONDUIT FLOW
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- RECIPROCATING PUMPS

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IMPACT OF JETS ON A STATIONARY PLATE: PART-1

Contents

- Introduction
- Force Exerted by the jet on a Stationary Vertical Plate
- Force Exerted by the jet on a Stationary Inclined Flat Plate
- Summary

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Introduction

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- The liquid comes out in the form of a jet from the outlet of a nozzle, which is fitted to a pipe through which the liquid is flowing under pressure
- If some plate, which may be fixed or moving is placed in the path of the jet, a force is exerted by the jet on the plate
- This force is obtained from Newton's second law of motion or from impulse- momentum equation

Introduction(Cont...)

- Thus impact of jet means the force exerted by the jet on a plate which may be stationary or moving. The cases of the impact of jet are

- **On a Stationary Plate, when**
  - Plate is Vertical to the Jet
  - Plate is Inclined to the Jet
  - Plate is Curved
- **On a Moving Plate, when**
  - Plate is Vertical to the Jet
  - Plate is Inclined to the Jet
  - Plate is Curved

Force Exerted by the jet on a Stationary Vertical Plate

- Consider a jet of water coming out from the nozzle, strikes a flat vertical plate as shown in fig
- Let V= velocity of the jet,

d= diameter of the jet

a= area of cross section of the jet

$=\frac{\pi}{4} d^2$

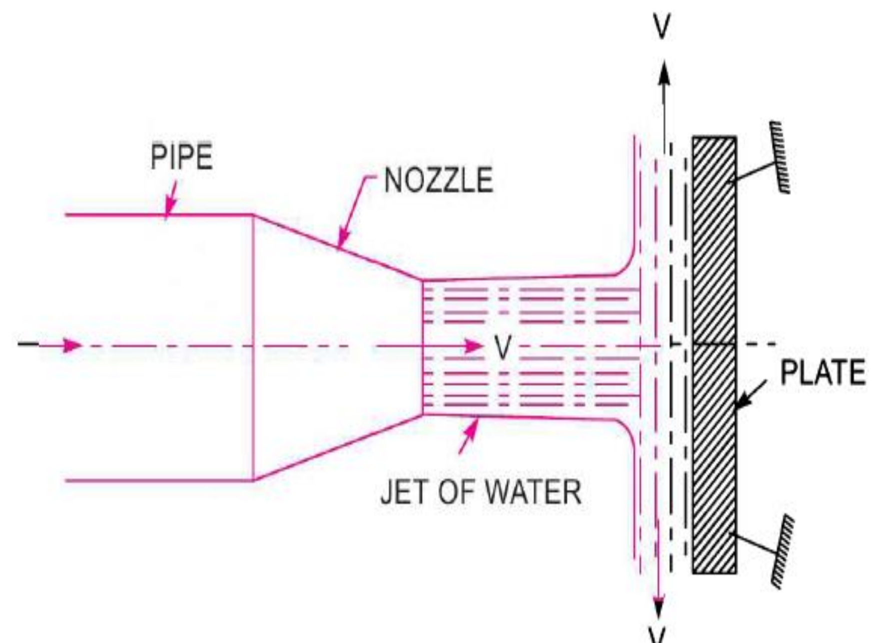

Fig. *Force exerted by jet on vertical plate.*

Force Exerted by the jet on a Stationary Vertical Plate(Cont…)

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- The jet after striking the plate, will move along the plate
- But the plate is at right angles to the jet
- Hence the jet after striking, will get deflected through 90^0
- Hence the component of the velocity of jet in the direction of jet, after striking will be zero

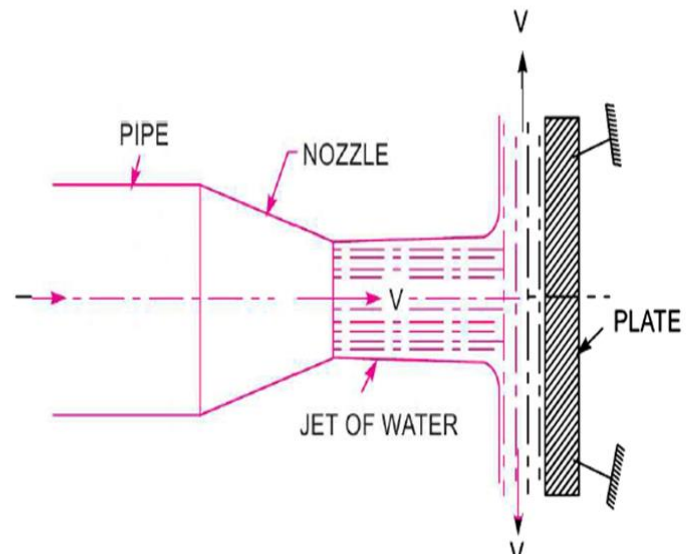

Fig. *Force exerted by jet on vertical plate.*

Force Exerted by the jet on a Stationary Vertical Plate(Cont…)

- The force exerted by the jet on the plate in the direction of jet

F_x =Rate of change of momentum in the direction of force

$$= \frac{Initial\ momentum\ - Final\ momentum}{Time}$$

$$= \frac{(Mass\ X\ Initial\ velocity)\ - (Mass\ X\ Final\ velocity)}{Time}$$

$$= \frac{Mass}{Time}[Initial\ velocity - Final\ velocity]$$

Force Exerted by the jet on a Stationary Vertical Plate(Cont…)

$$= \frac{Mass}{Time}[Velocity\ of\ jet\ before\ striking - Velocity\ of\ jet\ after\ striking]$$

$$= \rho a V[V - 0] \qquad \left\{\frac{mass}{sec} = \rho a V\right\}$$

$$F_x = \rho a V^2$$

- ➢ If the force exerted on the jet is to be calculated then final velocity minus initial velocity is taken
- ➢ If the force exerted by the jet on the plate is to be calculated then initial velocity minus final velocity is taken

Problem:1

Find the force exerted by a jet of water of diameter 75 mm on a stationary flat plate, when the jet strikes the plate normally with velocity of 20 m/s.

Solution. Given :

Diameter of jet, $d = 75 \text{ mm} = 0.075 \text{ m}$

$\therefore$ Area, $a = \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4} (.075)^2 = .004417 \text{ m}^2$

Velocity of jet, $V = 20 \text{ m/s.}$

Problem:1(Cont…)

The force exerted by the jet of water on a stationary vertical plate is given by equation

$$F = \rho a V^2 \quad \text{where } \rho = 1000 \text{ kg/m}^3$$

$$\therefore \quad F = 1000 \times .004417 \times 20^2 \text{ N} = \mathbf{1766.8\ N.\ Ans.}$$

Problem:2

Water is flowing through a pipe at the end of which a nozzle is fitted. The diameter of the nozzle is 100 mm and the head of water at the centre nozzle is 100 m. Find the force exerted by the jet of water on a fixed vertical plate. The co-efficient of velocity is given as 0.95.

Solution. Given :

Diameter of nozzle, $d = 100 \text{ mm} = 0.1 \text{ m}$

Head of water, $H = 100 \text{ m}$

Co-efficient of velocity, $C_v = 0.95$

Area of nozzle, $a = \frac{\pi}{4}(.1)^2 = .007854 \text{ m}^2$

Problem:2(Cont...)

Theoretical velocity of jet of water is given as

$$V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 100} = 44.294 \text{ m/s}$$

From Flow through nozzles ,We know that actual velocity

$$=V_{act} = V = v = \sqrt{\frac{2gH}{1+\frac{4fLa^2}{DA^2}}}$$

$$V_{th} = v = \sqrt{\frac{2gH}{1+\frac{4(0)La^2}{DA^2}}} = \sqrt{2gH}$$

Problem:2(Cont...)

But $$C_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}}$$

$\therefore$ Actual velocity of jet of water, $V = C_v \times V_{th} = 0.95 \times 44.294 = 42.08$ m/s.

Force on a fixed vertical plate is given by equation

$$F = \rho a V^2 = 1000 \times .007854 \times 42.08^2 \quad (\because \text{ In S.I. units } \rho \text{ for water} = 1000 \text{ kg/m}^3)$$

$$= 13907.2 \text{ N} = \mathbf{13.9 \text{ kN. Ans.}}$$

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Force Exerted by the jet on a Stationary Inclined Flat Plate

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- Let a jet of water, coming out from the nozzle, strikes an inclined flat plate as shown in fig

- Let V= Velocity of jet in the direction of x
- θ =Angle between the jet and plate
- a= Area of cross –section of the jet

Mass of water per sec striking the plate=ρaV

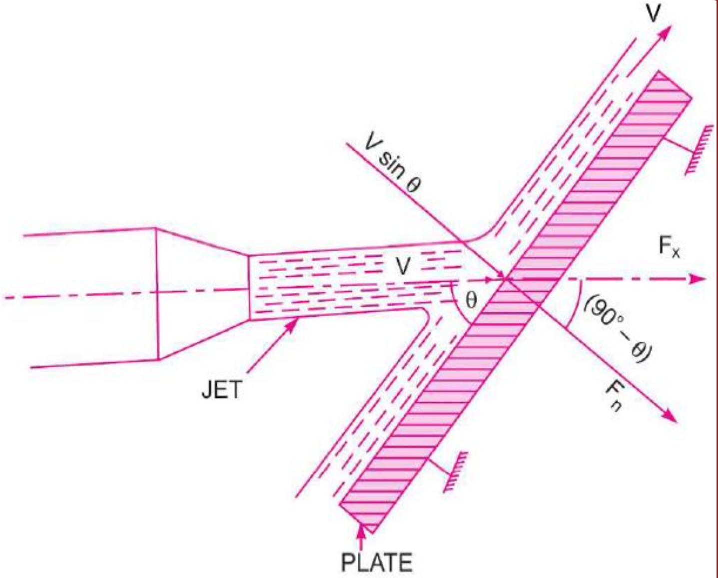

Fig. *Jet striking stationary inclined plate.*

Force Exerted by the jet on a Stationary Inclined Flat Plate(Cont…)

- If the plate is smooth and assumed that there is no loss of energy due to impact of jet
- Then jet will move over the plate after striking with a velocity equal to initial velocity i.e with a velocity V

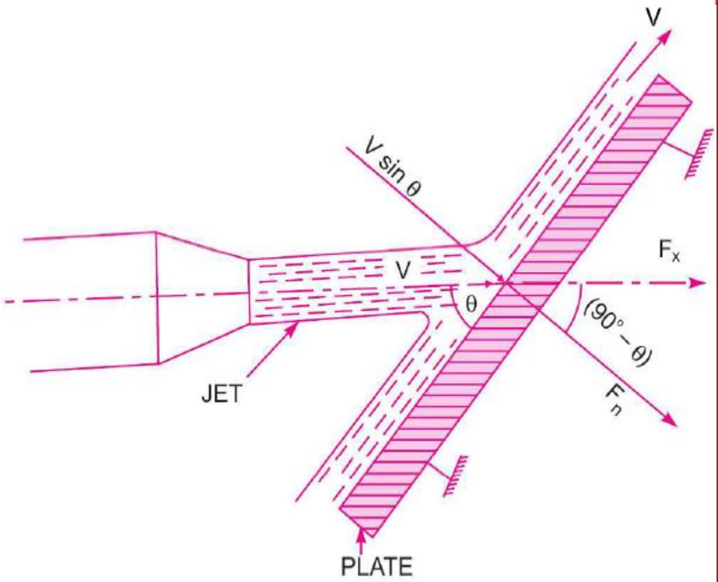

Fig. *Jet striking stationary inclined plate.*

Force Exerted by the jet on a Stationary Inclined Flat Plate(Cont...)

- Force exerted by the jet on the plate in the direction normal to the plate(F_n)

F_n

= mass of jet striking per second X [Initial velocity of jet before striking in the direction of n−Final velocity of jet after striking in the direction of n]

$$= \rho a V [V sin\theta - 0]$$

$$F_n = \rho a V^2 sin\theta$$

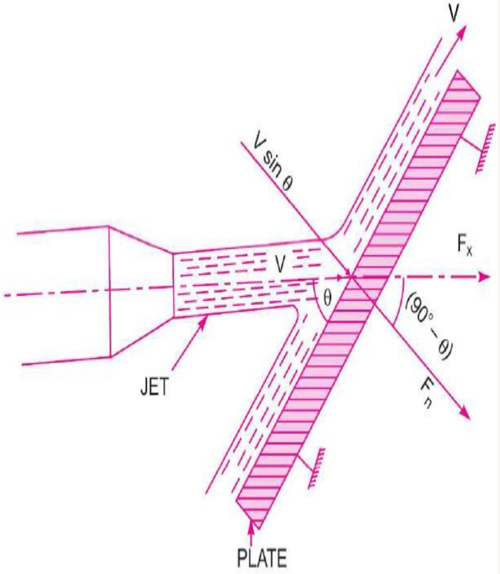

Fig. Jet striking stationary inclined plate.

Force Exerted by the jet on a Stationary Inclined Flat Plate(Cont...)

- This force can be resolved into two components, one in the direction of the jet and other perpendicular to the direction of flow
- F_x =component of F_n in the direction of flow

$$= F_n \cos(90^0 - \theta)$$

$$= F_n \sin\theta$$

But $\quad F_n = \rho a V^2 sin\theta$

$$= \rho a V^2 sin\theta \sin\theta$$

$$F_x = \rho a V^2 sin^2\theta$$

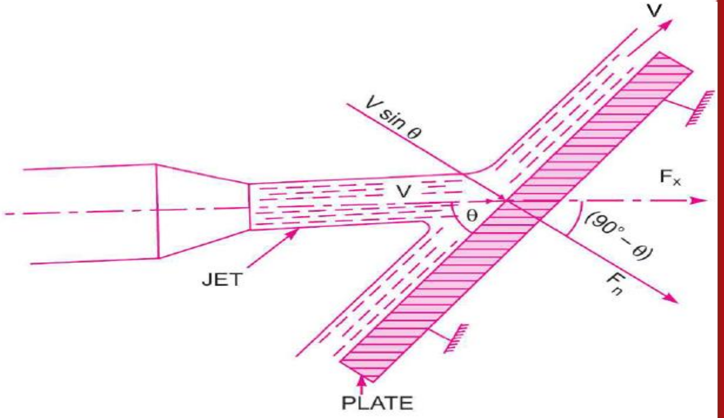

Fig. Jet striking stationary inclined plate.

Force Exerted by the jet on a Stationary Inclined Flat Plate(Cont…)

- F_y =component of F_n, perpendicular to flow

$$F_y = F_n \sin(90^0 - \theta)$$

$$F_y = F_n \cos\theta$$

$$F_y = \rho A V^2 sin\theta\ cos\theta$$

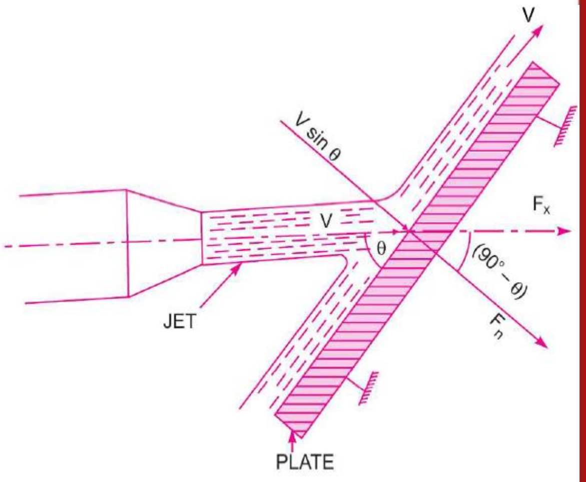

Fig. *Jet striking stationary inclined plate.*

Problem:3

A jet of water of diameter 75 mm moving with a velocity of 25 m/s strikes a fixed plate in such a way that the angle between the jet and plate is 60°. Find the force exerted by the jet on the plate (i) in the direction normal to the plate and (ii) in the direction of the jet.

Solution. Given :

Diameter of jet, $d = 75 \text{ mm} = 0.075 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.075)^2 = 0.004417 \text{ m}^2$

Velocity of jet, $V = 25 \text{ m/s.}$

Angle between jet and plate $\theta = 60°$

Problem:3(Cont...)

(*i*) The force exerted by the jet of water in the direction normal to the plate is given by equation

$$F_n = \rho a V^2 \sin\theta$$

$$= 1000 \times .004417 \times 25^2 \times \sin 60° = \mathbf{2390.7\ N.\ Ans.}$$

(*ii*) The force in the direction of the jet is given by equation

$$F_x = \rho a V^2 \sin^2\theta$$

$$= 1000 \times .004417 \times 25^2 \times \sin^2 60° = \mathbf{2070.4\ N.\ Ans.}$$

Summary

- Impact of jet means the force exerted by the jet on a plate which may be stationary or moving
- The force exerted by the jet on the plate in the direction of jet for Stationary Vertical Plate $F_x = \rho a V^2$
- Force exerted by the jet on the plate in the direction normal to the plate $for\ stationary\ inclined\ plate\ F_n = \rho a V^2 sin\theta$
- The component of F_n in the direction of flow $for\ stationary\ inclined\ plate\ F_x = \rho a V^2 sin^2\theta$
- The component of F_n, perpendicular to flow $for\ stationary\ inclined\ plate\ F_y = \rho A V^2 sin\theta\ cos\theta$

IMPACT OF JETS ON A STATIONARY PLATE: PART-2

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Lecture Details:
Unit-IV (Basics of Turbo Machinery) , Impact of jets on a stationary plate: part-2
FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
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IMPACT OF JETS ON A STATIONARY PLATE: PART-2

Contents

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- Force Exerted by a Jet on Stationary curved Plate
- Jet Strikes the Curved Plate at the Centre
- Jet Strikes the Curved Plate at One end Tangentially when the plate is Symmetrical
- Jet Strikes the Curved Plate at One end Tangentially when the plate is Unsymmetrical
- Summary

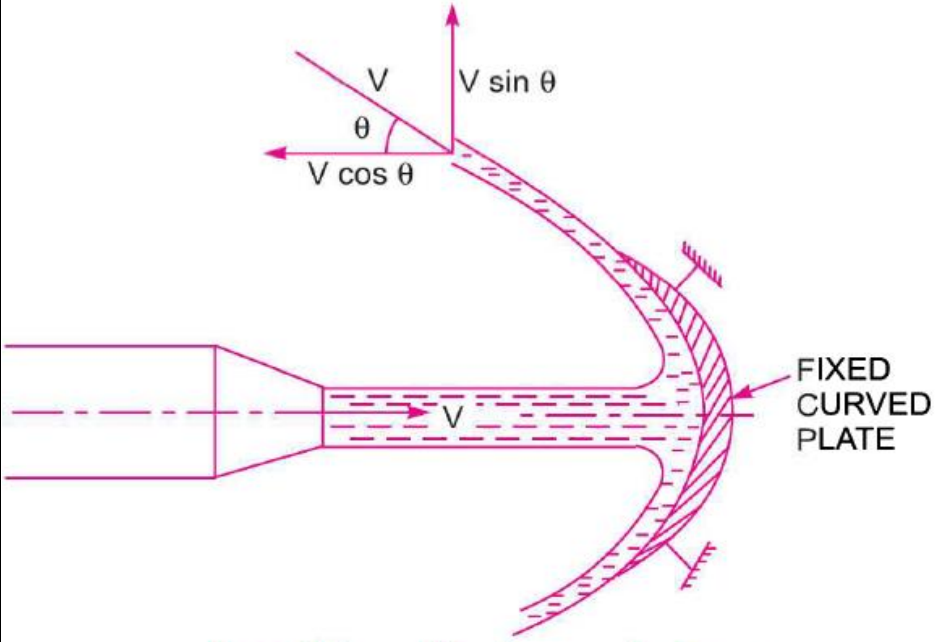
Force Exerted by a Jet on Stationary Curved Plate

Force exerted by a jet on stationary curved plate

- Jet strikes the curved plate at the centre
- Jet strikes the curved plate at one end tangentially when plate is symmetrical
- Jet strikes the curved plate at one end tangentially when the plate is unsymmetrical

Jet Strikes the Curved Plate at the Centre

- The jet after striking the plate, comes out with the same velocity if the plate is smooth and there is no loss of energy due to impact of the jet, in the tangential direction of the curved plate

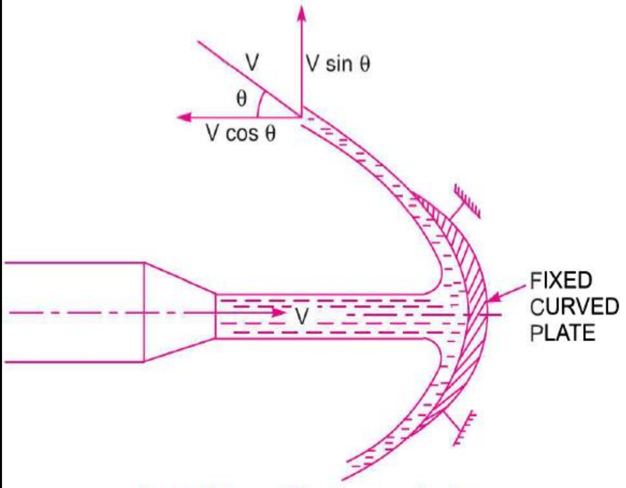


Jet striking a fixed curved plate at centre.

Jet Strikes the Curved Plate at the Centre(Cont...)

- The velocity at outlet of the plate can be resolved into two components
- One in the direction of jet and other perpendicular to the direction of the jet
- Force exerted by the jet in the direction of jet

  F_x =mass per sec X $[V_{1x} - V_{2x}]$
- Where V_{1x} =Initial velocity in the direction of jet=V



Jet striking a fixed curved plate at centre.

Jet Strikes the Curved Plate at the Centre(Cont…)

- V_{2x}=Final velocity in the direction of jet=$-V cos\theta$

$F_x = \rho a V$ [V$-(-V cos\theta)$] $= \rho a V$ [V $+ V cos\theta$]

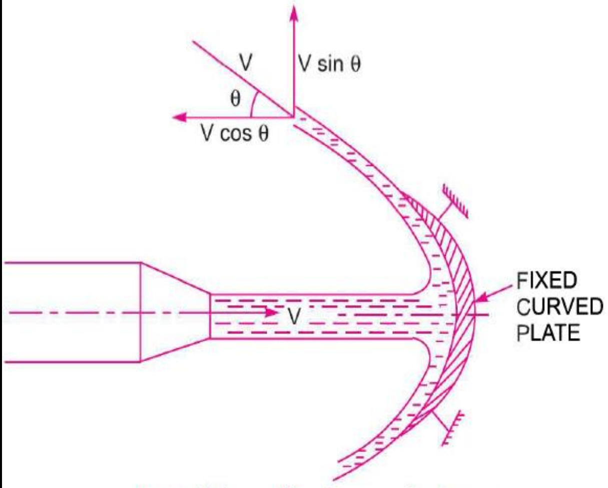
$F_x = \rho a V^2$ [1+$cos\theta$]

Similarly,

F_y =mass per sec X [$V_{1y} - V_{2y}$]

Where V_{1y} =Initial velocity in the direction of y=0

V_{2x}=Final velocity in the direction of y=$V sin\theta$



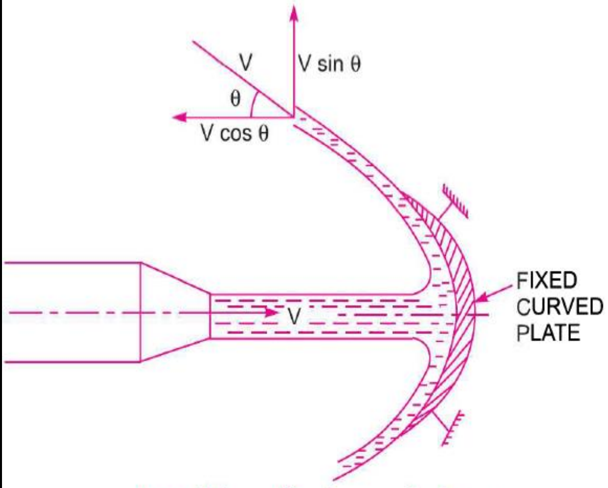
Jet striking a fixed curved plate at centre.

Jet Strikes the Curved Plate at the Centre(Cont…)

$F_y = \rho a V\ [0 - V sin\theta]$

$F_y = -\rho a V^2 sin\theta$

- -ve sign means that force is acting in the downward direction
- In this case the angle of deflection of the jet= $(180^0 - \theta)$



Jet striking a fixed curved plate at centre.

Problem:1

A jet of water of diameter 50 mm moving with a velocity of 40 m/s, strikes a curved fixed symmetrical plate at the centre. Find the force exerted by the jet of water in the direction of the jet, if the jet is deflected through an angle of 120° at the outlet of the curved plate.

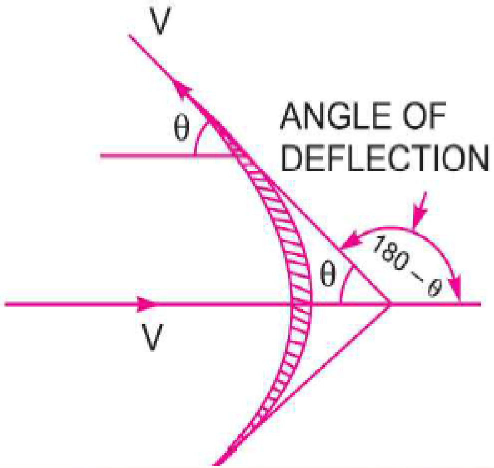
Solution. Given :

Diameter of the jet, $d = 50 \text{ mm} = 0.05 \text{ m}$

$\therefore$ Area, $a = \dfrac{\pi}{4}(.05)^2 = 0.001963 \text{ m}^2$

Velocity of jet, $V = 40 \text{ m/s}$

Angle of deflection $= 120°$



Problem:1(Cont...)

From equation the angle of deflection $= 180° - \theta$

$\therefore$ $180° - \theta = 120°$ or $\theta = 180° - 120° = 60°$

Force exerted by the jet on the curved plate in the direction of the jet is given by equation

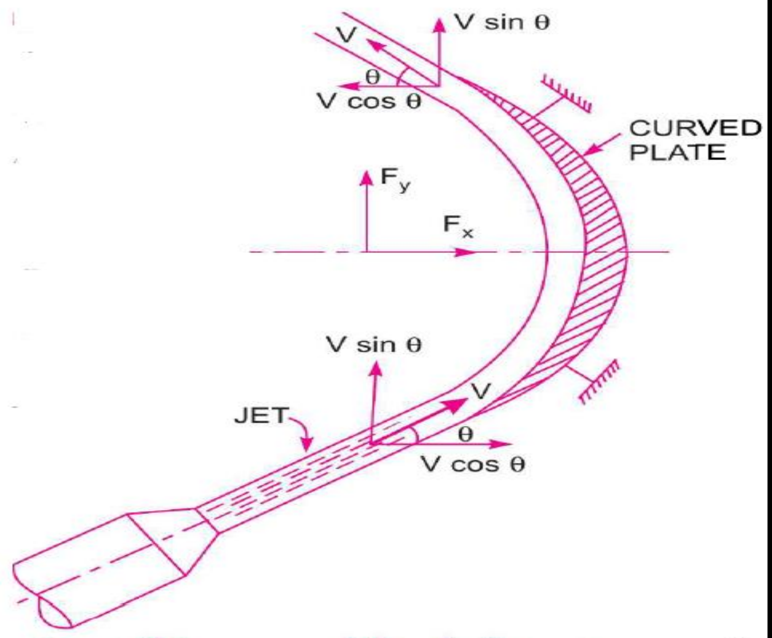
$$F_x = \rho a V^2 [1 + \cos\theta]$$

$$= 1000 \times .001963 \times 40^2 \times [1 + \cos 60°] = \mathbf{4711.15\ N.\ Ans.}$$

Jet Strikes the Curved Plate at One end Tangentially when the plate is Symmetrical

- Jet strikes the curved fixed plate at one end tangentially
- Let the curved plate is symmetrical about x axis
- Let V= Velocity of jet of water

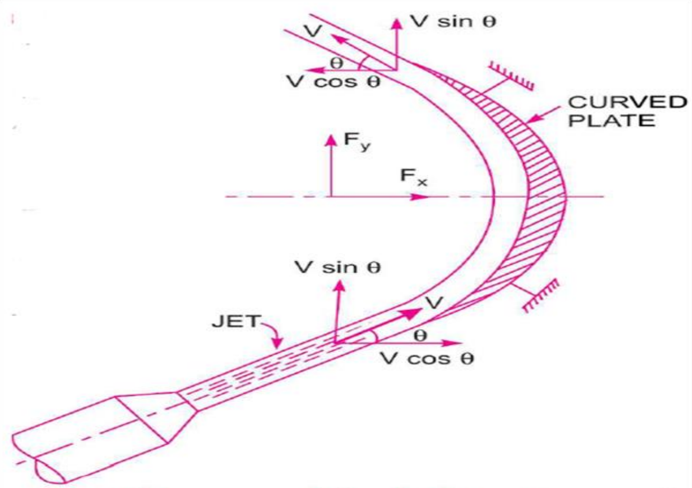
θ= Angle made by jet with x axis at inlet tip of the curved plate



Jet striking curved fixed plate at one end.

Jet Strikes the Curved Plate at One end Tangentially when the plate is Symmetrical

- If the plate is smooth and loss of energy due to impact is zero
- Then the velocity of water at the outlet tip of the curved plate will be equal to V
- The forces exerted by the jet of water in the direction of x and y are



Jet striking curved fixed plate at one end.

F_x =(mass/sec)X $[V_{1x} - V_{2x}]$

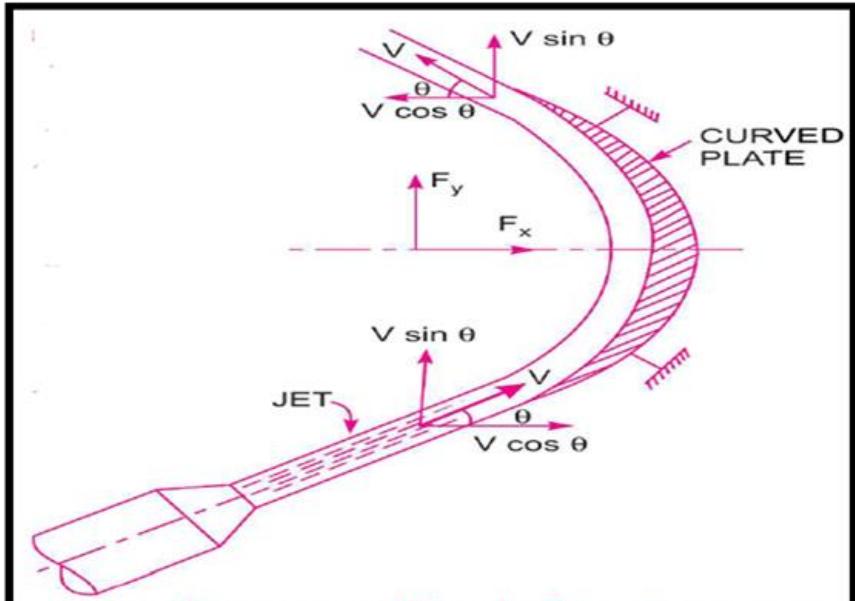
$= \rho a V\ [V cos\theta - (-V cos\theta)]$

Jet Strikes the Curved Plate at One end Tangentially when the plate is Symmetrical(Cont…)

$= 2\rho a V^2 \cos\theta$

$F_y = \rho a V \left[V_{1y} - V_{2y}\right]$

$F_y = \rho a V \left[V \sin\theta - V \sin\theta\right] = 0$



Jet striking curved fixed plate at one end.

Jet Strikes the Curved Plate at One end Tangentially when the plate is Unsymmetrical

- Angle made by the tangents drawn at the inlet and outlet tips of the plate with x axis will be different
- Let θ = angle made by tangent at inlet tip with x axis, ϕ =angle made by tangent at outlet tip with x axis
- The two components of the velocity at inlet are

V_{1x} =V$\cos\theta$ and V_{1y} =V$\sin\theta$

- The two components of the velocity at outlet are

$V_{2x} = -$V$\cos\phi$ and V_{2y} =V$\sin\phi$

Jet Strikes the Curved Plate at One end Tangentially when the plate is Unsymmetrical(Cont...)

- The forces exerted by the jet of water in the directions of x and y are

$$F_x = \rho a V[V_{1x} - V_{2x}] = \rho a V[V cos\theta - (-V cos\phi)]$$

$$= \rho a V[V cos\theta + V cos\phi]$$

$$= \rho a V^2[cos\theta + cos\phi]$$

$$F_y = \rho a V[V_{1y} - V_{2y}] = \rho a V[V sin\theta - (V sin\phi)]$$

$$= \rho a V^2[sin\theta - sin\phi]$$

Problem:2

A jet of water of diameter 75 mm moving with a velocity of 30 m/s, strikes a curved fixed plate tangentially at one end at an angle of 30° to the horizontal. The jet leaves the plate at an angle of 20° to the horizontal. Find the force exerted by the jet on the plate in the horizontal and vertical direction.

Solution. Given :

Diameter of the jet, $d = 75 \text{ mm} = 0.075 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4}(.075)^2 = .004417 \text{ m}^2$

Velocity of jet, $V = 30 \text{ m/s}$

Problem:2(Cont...)

Angle made by the jet at inlet tip with horizontal, $\theta = 30°$

Angle made by the jet at outlet tip with horizontal, $\phi = 20°$

The force exerted by the jet of water in the direction of x is given by equation and in the direction of y by equation

$$F_x = \rho a V^2 [\cos\theta + \cos\phi]$$

$$= 1000 \times .004417 [\cos 30° + \cos 20°] \times 30^2 = \textbf{7178.2 N. Ans.}$$

$$F_y = \rho a V^2 [\sin\theta - \sin\phi]$$

$$= 1000 \times .004417 [\sin 30° - \sin 20°] \times 30^2 = \textbf{628.13 N. Ans.}$$

Summary

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- For the case Jet Strikes the Curved Plate at the Centre , $F_x = \rho aV^2\ [1+cos\theta]$ and $F_y = -\rho aV^2 sin\theta$
- For the case Jet Strikes the Curved Plate at One end Tangentially when the plate is Symmetrical, $F_y = \rho aV\ [V_{1y} - V_{2y}]\ and\ \ F_y = 0$
- For the case Jet Strikes the Curved Plate at One end Tangentially when the plate is Unsymmetrical $F_x = \rho aV^2[cos\theta + cos\phi\ ]$ and $F_y = \rho aV^2[sin\theta - sin\phi\ ]$

IMPACT OF JETS ON MOVING PLATE: PART-1

Presented By:
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Lecture Details:
Unit-IV (Basics of Turbo Machinery) , Impact of jets on moving plate: part-1
FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
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- BASICS OF TURBO MACHINERY
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- PERFORMANCE OF HYDRAULIC TURBINES
- CENTRIFUGAL PUMPS
- RECIPROCATING PUMPS

IMPACT OF JETS ON A MOVING PLATE: PART-1

Contents

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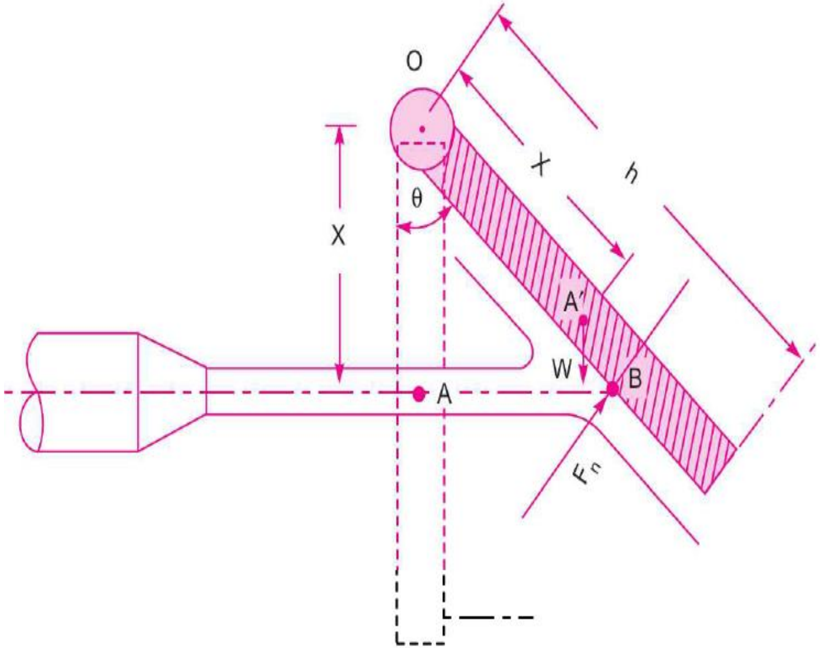
- Force exerted by a Jet on a Hinged Plate
- Force on Flat Vertical Plate Moving in the Direction of Jet
- Summary

Force exerted by a Jet on a Hinged Plate

- Consider a jet of water striking a vertical plate at the centre which is hinged at O
- Due to force exerted by the jet on the plate, the plate will swing through some angle about the hinge



Force on a hinged plate.

Force exerted by a Jet on a Hinged Plate(Cont...)

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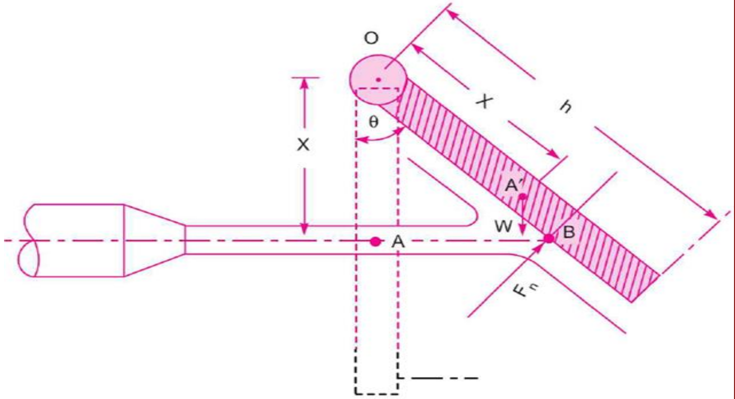


Force on a hinged plate.

Let x= distance of the centre jet from hinge O

θ = angle of swing about hinge

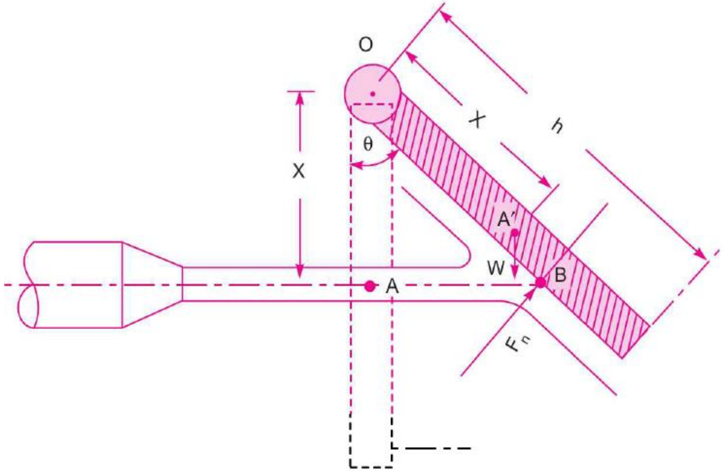
W= weight of plate acting at C.G of the plate

- The point A on the plate will be at A' after the jet strikes the plate
- The distance OA=O A' =x
- Let the weight of the plate is acting at A'

Force exerted by a Jet on a Hinged Plate(Cont...)

- When the plate is in equilibrium after the jet strikes the plate, the moment of all the forces about the hinge must be zero
- Two forces are acting on the plate

1. Force due to jet of water , normal to the plate
2. Weight of the plate, W



Force on a hinged plate.

Force exerted by a Jet on a Hinged Plate(Cont...)

1. Force due to jet of water , normal to the plate

$F_n = \rho a V^2 sin\theta'$

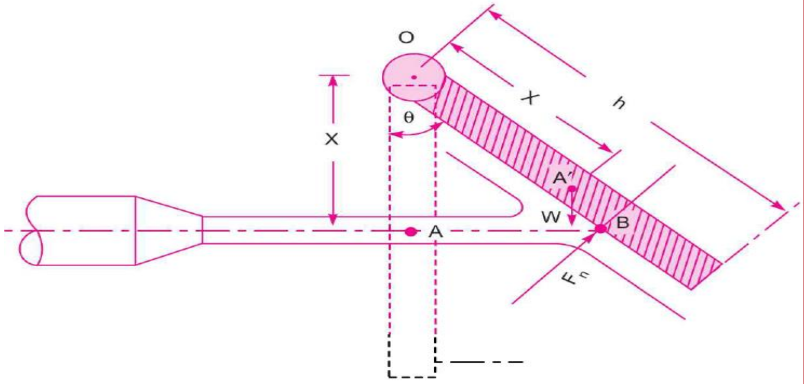
Where θ' =Angle between jet and plate=$(90^0 - \theta)$

2. Weight of the plate, W

Moment of force F_n about hinge= F_n X OB

$= \rho a V^2 \sin(90^0 - \theta)$XOB

$= \rho a V^2 \cos\theta$XOB



Force on a hinged plate.

Force exerted by a Jet on a Hinged Plate(Cont...)

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$= \rho a V^2 \cos\theta \mathrm{X} \frac{OA}{cos\theta}$

$= \rho a V^2 \mathrm{OA} = \rho a V^2 \mathrm{x}$

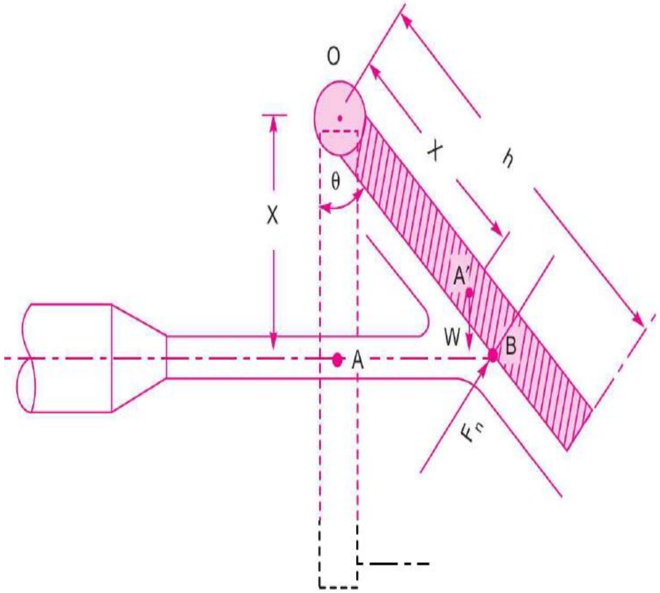
Moment of weight W about hinge=W XOA'sin θ

=W XxXsin θ

For equilibrium of the plate,

$\rho a V^2 X x = W X x X sin\theta$

sin θ= $\frac{\rho a V^2}{W}$



Force on a hinged plate.

Problem:1

A jet of water of 2.5 cm diameter, moving with a velocity of 10 m/s, strikes a hinged square plate of weight 98.1 N at the centre of the plate. The plate is of uniform thickness. Find the angle through which the plate will swing.

Solution. Given :

Diameter of jet, $d = 2.5 \text{ cm} = 0.025 \text{ m}$

Velocity of jet, $V = 10 \text{ m/s}$

Weight of plate, $W = 98.1 \text{ N}$

Area of jet, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (.025)^2 = .00049 \text{ m}^2$

Problem:1(Cont…)

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The angle through which the plate will swing is given by equation

$$\sin\theta = \frac{\rho a V^2}{W} = 1000 \times \frac{.00049 \times 10^2}{98.1}$$

$$= .499$$

$$\therefore \qquad \theta = \mathbf{29.96^\circ}. \ \textbf{Ans.}$$

Problem:2

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A jet of water of 30 mm diameter strikes a hinged square plate at its centre with a velocity of 20 m/s. The plate is deflected through an angle of 20°. Find the weight of the plate.

If the plate is not allowed, to swing, what will be the force required at the lower edge of the plate to keep the plate in vertical position.

Solution. Given :

Diameter of the jet, $d = 30 \text{ mm} = 3 \text{ cm} = 0.03 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(.03)^2 = .0007068 \text{ m}^2$

Velocity of jet, $V = 20 \text{ m/s}$

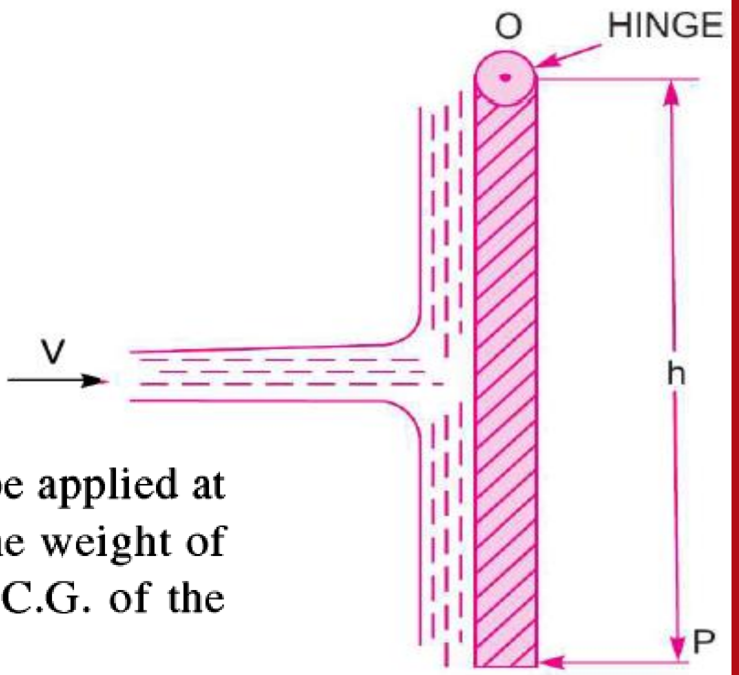
Angle of swing, $\theta = 20°$

Problem:2(Cont...)

$$\sin \theta = \frac{\rho a V^2}{W}$$

$$\sin 20^\circ = 1000 \times \frac{.0007068 \times 20^2}{W} = \frac{282.72}{W}$$

$$W = \frac{282.72}{\sin 20^\circ} = 826.6 \text{ N}$$



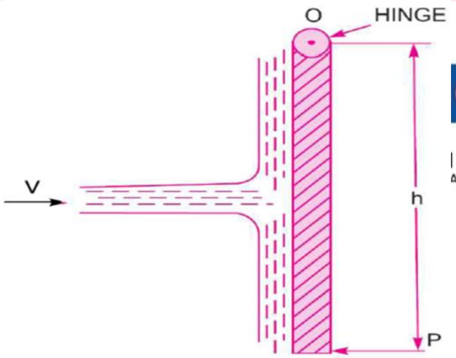
If the plate is not allowed to swing, a force P will be applied at the lower edge of the plate as shown in Fig. The weight of the plate is acting vertically downward through the C.G. of the plate.

Problem:2(Cont...)

Let F = Force exerted by jet of water

h = Height of plate

= Distance of P from the hinge.



The jet strikes at the centre of the plate and hence distance of the centre of the jet from hinge = $\frac{h}{2}$.

Taking moments* about the hinge, O, $P \times h = F \times \frac{h}{2}$.

$$\therefore \qquad P = \frac{F \times h}{2 \times h} = \frac{F}{2} = \frac{\rho a V^2}{2} \qquad (\because \quad F = \rho a V^2)$$

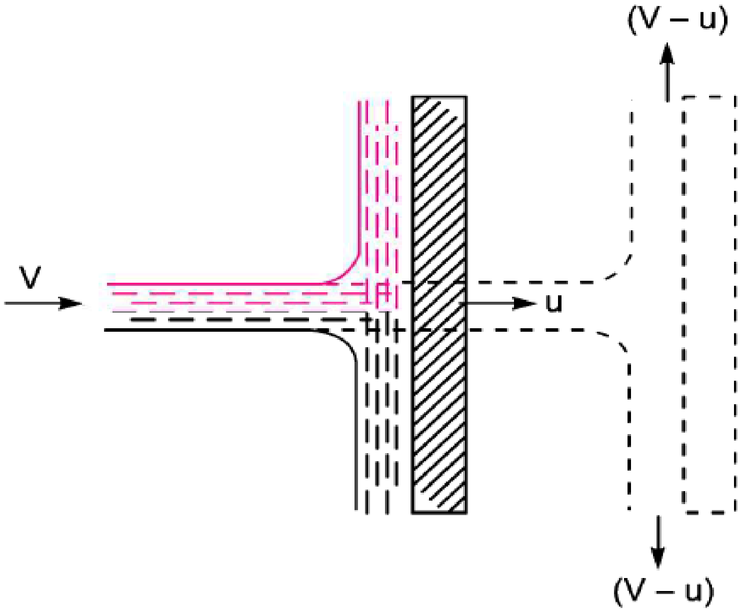
$$= 1000 \times \frac{.0007068 \times 20^2}{2} = \mathbf{141.36\ N.\ Ans.}$$

Force on Flat Vertical Plate Moving in the Direction of Jet

- Fig shows a jet of water striking a flat vertical plate moving with a uniform velocity away from the jet
- Let V=velocity of the jet(absolute)

a= area of cross section of the jet

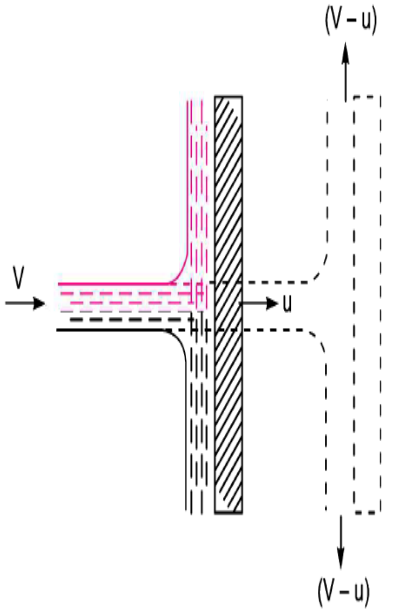
u= velocity of the flat plate



Jet striking a flat vertical moving plate.

Force on Flat Vertical Plate Moving in the Direction of Jet(Cont…)

- In this case, the jet does not strike the plate with a velocity V, but it strikes with a relative velocity, which is equal to the absolute velocity of jet of water minus the velocity of the plate
- Hence relative velocity of the jet with respect to plate= $(V-u)$
- Mass of water striking the plate per sec=ρX Area of jet X Velocity with which jet strikes the plate= ρaX[V − u]



Jet striking a flat vertical moving plate.

Force on Flat Vertical Plate Moving in the Direction of Jet(Cont…)

- Force exerted by the jet on the moving plate in the direction of the jet

F_x =Mass of water striking per sec X [Initial velocity with which water strikes- Final velocity]

$$F_x = \rho a(V-u)[(V-u)-0]$$

(Final velocity in the direction of jet is zero)

$$F_x = \rho a(V-u)^2$$

Force on Flat Vertical Plate Moving in the Direction of Jet(Cont…)

- For the stationary plates, the work done is zero
- In this case, the work be done by the jet on the plate, as plate is moving

Work done per second by the jet on the plate= Force X$\frac{Distance\ in\ the\ direction\ of\ force}{Time}$

$$= F_x X u$$

$$= \rho a (V-u)^2 X u$$

Problem:3

A jet of water of diameter 10 cm strikes a flat plate normally with a velocity of 15 m/s. The plate is moving with a velocity of 6 m/s in the direction of the jet and away from the jet. Find:

(i) the force exerted by the jet on the plate

(ii) work done by the jet on the plate per second.

Solution. Given :

Diameter of the jet, $d = 10 \text{ cm} = 0.1 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.1)^2 = .007854 \text{ m}^2$

Velocity of jet, $V = 15 \text{ m/s}$

Velocity of the plate, $u = 6 \text{ m/s}$.

Problem:3

(*i*) The force exerted by the jet on a moving flat vertical plate is given by equation

$$F_x = \rho a\,(V - u)^2$$

$$= 1000 \times .007854 \times (15 - 6)^2 \text{ N} = \mathbf{636.17\ N.\ Ans.}$$

(*ii*) Work done per second by the jet

$$= F_x \times u = 636.17 \times 6 = \mathbf{3817.02\ Nm/s.\ Ans.}$$

Summary

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- Due to force exerted by the jet on the plate, the plate will swing through some angle about the hinge
- Force due to jet of water on a Hinged Plate, normal to the plate $F_n = \rho a V^2 sin\theta'$
- Force exerted by the jet on Flat Vertical Plate Moving in the Direction of Jet $F_x = \rho a(V-u)^2$
- Work done per second by the jet on Flat Vertical Plate Moving in the Direction of Jet $= \rho a(V-u)^2 X u$

IMPACT OF JETS ON MOVING PLATE: PART-2

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Lecture Details:
Unit-IV (Basics of Turbo Machinery) , Impact of jets on moving plate: part-2
FM & HM /Mechanical, I -Semester.

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IMPACT OF JETS ON A MOVING PLATE: PART-2

Contents

- Force on the Inclined Plate Moving in the Direction of the Jet
- Force on the Curved Plate when the Plate is Moving in the direction of jet
- Summary

Force on the Inclined Plate Moving in the Direction of the Jet

Let V= Absolute velocity of jet of water

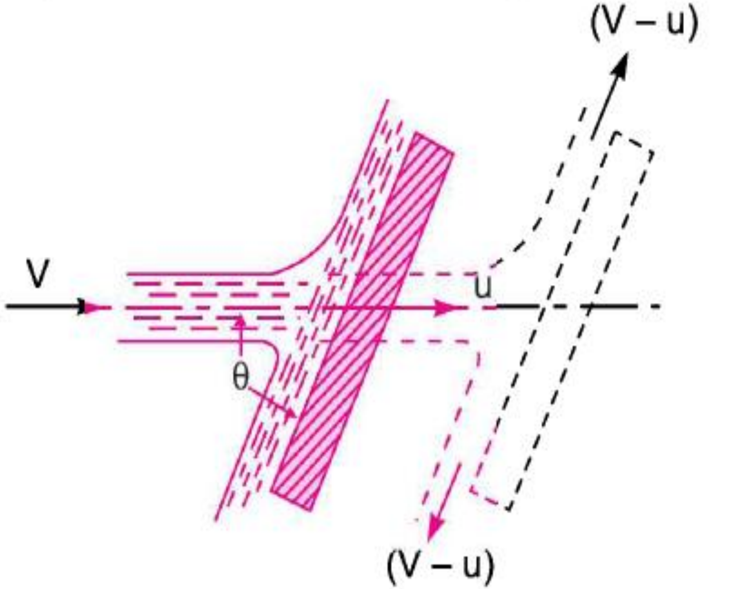
u= velocity of the plate in the direction of jet

a= cross sectional area of jet

θ =angle between jet and plate

Relative velocity of jet of water=(V-u)

The velocity with which jet strikes=(V-u)



Jet striking an inclined moving plate.

Force on the Inclined Plate Moving in the Direction of the Jet(Cont...)

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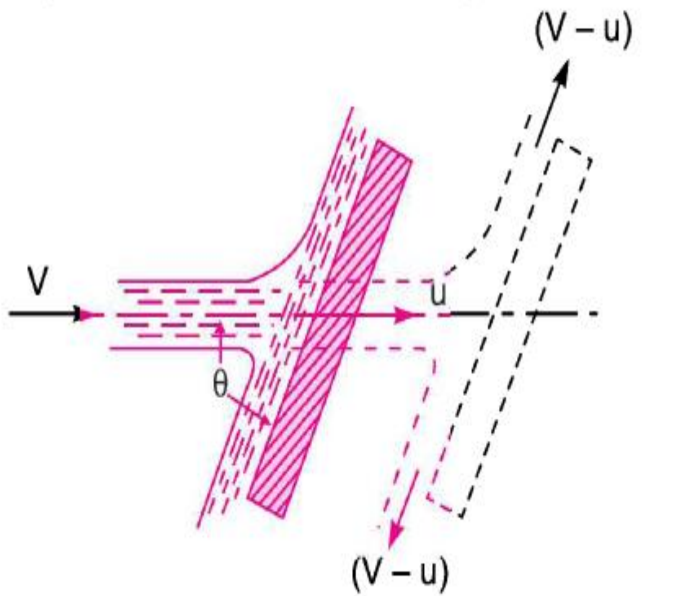
Mass of water striking per second

$= \rho X a X (V - u)$

- If the plate is smooth and loss of energy due to impact of the jet is assumed to be zero
- The jet of water will leave the inclined plate with a velocity =(V-u)



Jet striking an inclined moving plate.

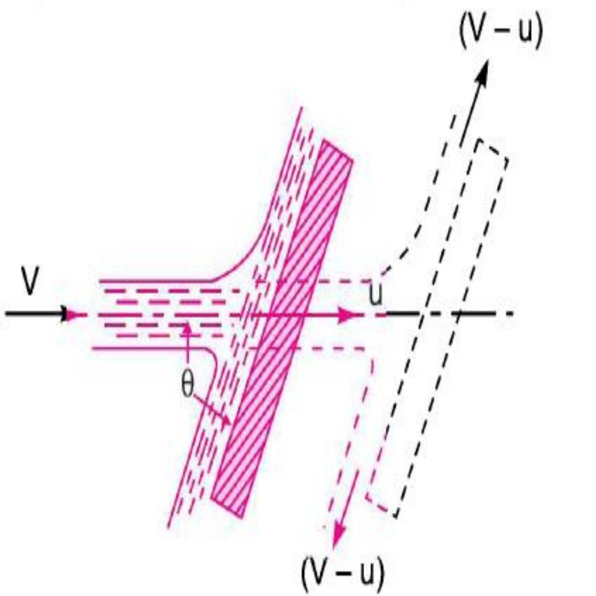
Force on the Inclined Plate Moving in the Direction of the Jet(Cont...)

The force exerted by the jet of water on the plate in the direction normal to the plate

F_n =Mass striking per second X [Initial velocity in the normal direction with which jet strikes- Final velocity]

$$F_n = \rho a(V-u)[(V-u)sin\theta - 0]$$

$$F_n = \rho a(V-u)^2 sin\theta$$



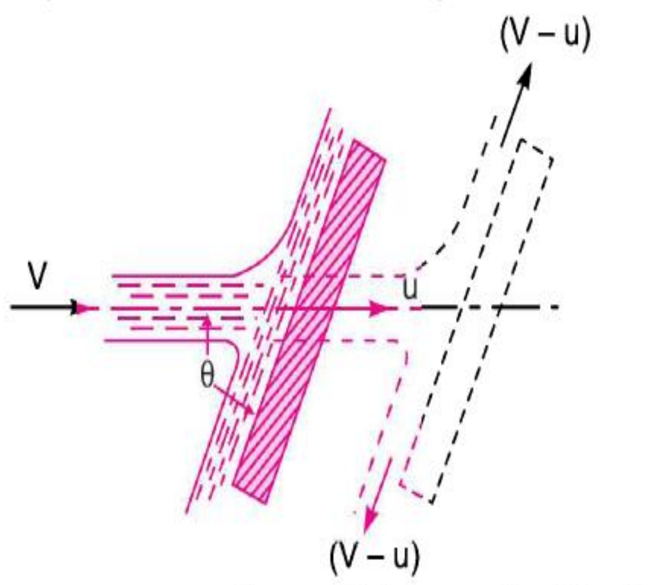
Jet striking an inclined moving plate.

Force on the Inclined Plate Moving in the Direction of the Jet(Cont...)

- This normal force F_n is resolved into F_x and F_y in the direction of the jet and perpendicular to the direction of the jet respectively

$$F_x = F_n sin\theta = \rho a(V-u)^2 sin\theta^2$$

$$F_y = F_n cos\theta = \rho a(V-u)^2 sin\theta\ cos\theta$$



Jet striking an inclined moving plate.

Force on the Inclined Plate Moving in the Direction of the Jet(Cont…)

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- Work done per second by the jet on the plate $= F_x X$ Distance per second in the direction of x

$$= F_x X u$$

$$= \rho a(V-u)^2 sin\theta^2 X\, u$$

$$= \rho a(V-u)^2 u sin\theta^2$$

Problem:1

A 7.5 cm diameter jet having a velocity of 30 m/s strikes a flat plate, the normal of which is inclined at 45° to the axis of the jet. Find the normal pressure on the plate : (i) when the plate is stationary , and (ii) when the plate is moving with a velocity of 15 m/s and away from the jet. Also determine the power and efficiency of the jet when the plate is moving.

Solution. Given :

Diameter of the jet, $d = 7.5\text{ cm} = 0.075\text{ m}$

$\therefore$ Area, $a = \dfrac{\pi}{4}(.075)^2 = .004417\text{ m}^2$

Angle between the jet and plate $\theta = 90° - 45° = 45°$

Velocity of jet, $V = 30\text{ m/s}$.

Problem:1(Cont…)

(*i*) When the plate is stationary, the normal force on the plate is given by equation

$$F_n = \rho a V^2 \sin\theta = 1000 \times .004417 \times 30^2 \times \sin 45° = \mathbf{2810.96\ N.\ Ans.}$$

(*ii*) When the plate is moving with a velocity 15 m/s and away from the jet, the normal force on the plate is given by equation

$$F_n = \rho a (V - u)^2 \sin\theta \qquad \text{where } u = 15 \text{ m/s.}$$

$$= 1000 \times .004417 \times (30 - 15)^2 \times \sin 45° = \mathbf{702.74\ N.\ Ans.}$$

(*iii*) The power and efficiency of the jet when plate is moving is obtained as

Work done per second by the jet

= Force in the direction of jet × Distance moved by the plate in the direction of jet/sec

$= F_x \times u$, where $F_x = F_n \sin\theta = 702.74 \times \sin 45° = 496.9$ N

Problem:1(Cont…)

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Work done/sec = $496.9 \times 15 = 7453.5$ Nm/s

$$\therefore \quad \text{Power in kW} = \frac{\text{Work done per second}}{1000} = \frac{7453.5}{1000} = \mathbf{7.453\ kW.\ Ans.}$$

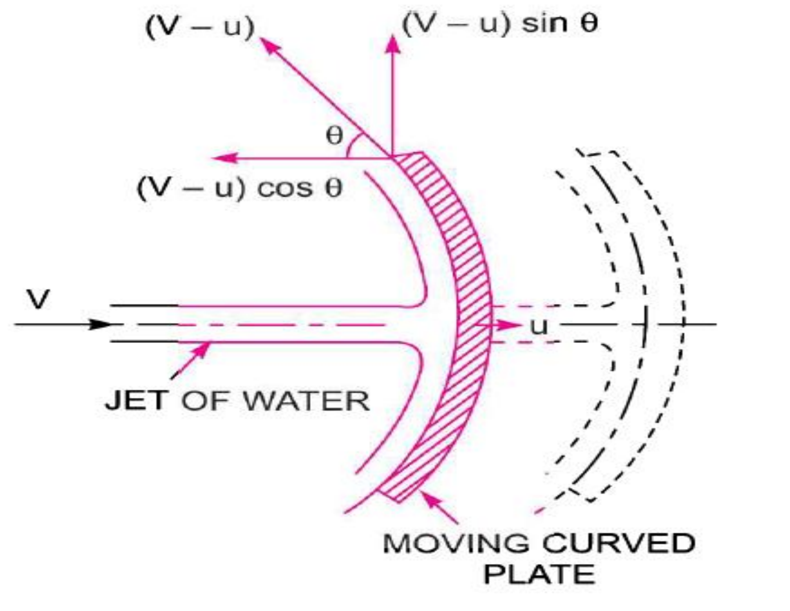
$$\text{Efficiency of the jet} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Work done per second}}{\text{Kinetic energy of the jet}}$$

$$= \frac{7453.5}{\frac{1}{2}(\rho aV) \times V^2} = \frac{7453.5}{\frac{1}{2}\rho aV^3} = \frac{7453.5}{\frac{1}{2} \times 1000 \times .004417 \times 30^3}$$

$$= 0.1249 \simeq 0.125 = \mathbf{12.5\%.\ Ans.}$$

Force on the Curved Plate when the Plate is Moving in the direction of jet

- Let a jet of water strikes a curved plate at the centre of the plate which is moving with a uniform velocity in the direction of the jet as shown in fig

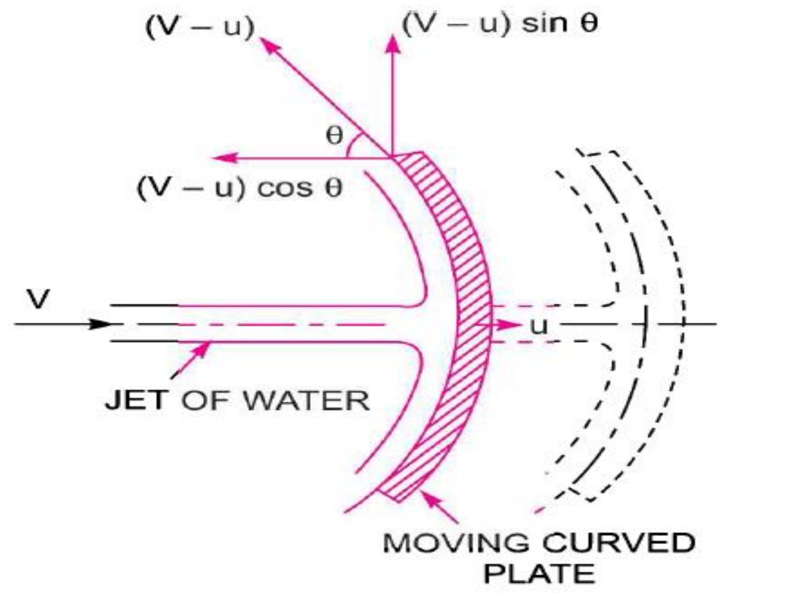


Jet striking a curved moving plate.

Let V=absolute velocity of jet, a= area of jet, u= velocity of the plate in the direction of the jet

Force on the Curved Plate when the Plate is Moving in the direction of jet(Cont….)

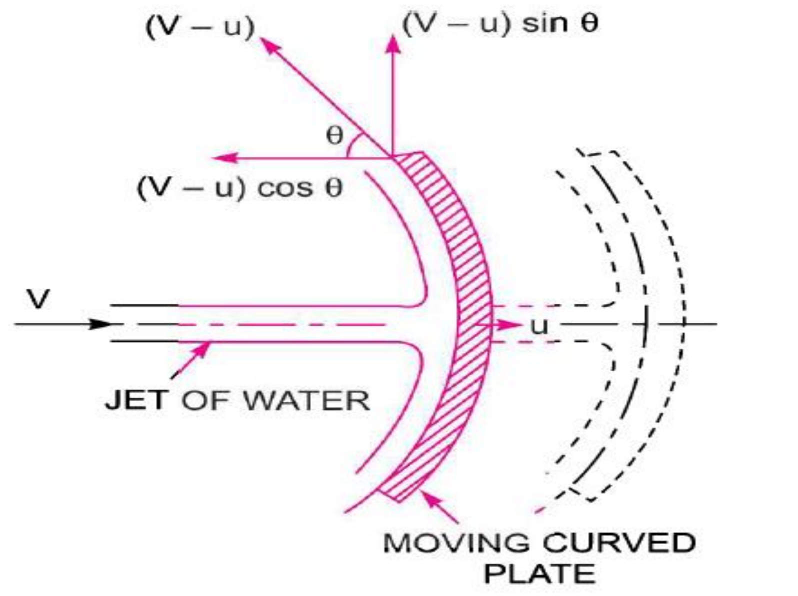
- Relative velocity of the jet of water or the velocity with which jet strikes the curved plate=(V-u)
- If plate is smooth and the loss of energy due to impact of jet is zero
- The velocity with which the jet will be leaving the curved vane=(V-u)



Jet striking a curved moving plate.

Force on the Curved Plate when the Plate is Moving in the direction of jet(Cont....)

- Component of the velocity in the direction of jet$= -(V-u)cos\theta$
- Component of the velocity in the direction perpendicular to the direction of the jet$= (V-u)sin\theta$



Jet striking a curved moving plate.

Force on the Curved Plate when the Plate is Moving in the direction of jet(Cont….)

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- Mass of the water striking the plate $= \rho X a X$velocity with which jet strikes the plate

$$= \rho a(V - u)$$

- Force exerted by the jet of water on the curved plate in the direction of the jet (F_x)
- F_x =Mass striking per sec X [Initial velocity with which jet strikes the plate in the direction of jet−Final velocity]

Force on the Curved Plate when the Plate is Moving in the direction of jet(Cont….)

$$F_x = \rho a(V-u)[(V-u)-(-(V-u)cos\theta)]$$

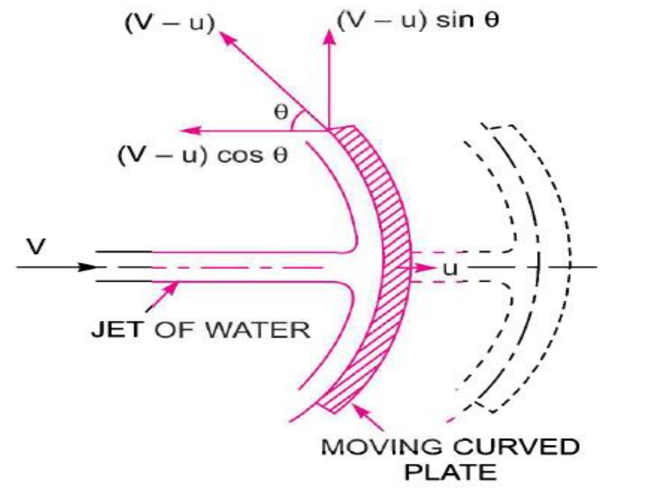
$$F_x = \rho a(V-u)[(V-u)+(V-u)cos\theta]$$

$$F_x = \rho a(V-u)^2\,[1+cos\theta]$$

- Work done by the jet on the plate per second= F_xX Distance travelled per second in the direction of x= F_xXu

$= \rho a(V-u)^2\,[1+cos\theta]$Xu

$= \rho a(V-u)^2\,X$u$[1+cos\theta]$



Jet striking a curved moving plate.

Problem:2

A jet of water of diameter 7.5 cm strikes a curved plate at its centre with a velocity of 20 m/s. The curved plate is moving with a velocity of 8 m/s in the direction of the jet. The jet is deflected through an angle of 165°. Assuming the plate smooth find :

(i) Force exerted on the plate in the direction of jet, (ii) Power of the jet, and (iii) Efficiency of the jet.

Solution. Given :

Diameter of the jet, $d = 7.5 \text{ cm} = 0.075 \text{ m}$

$\therefore$ Area, $a = \dfrac{\pi}{4}(.075)^2 = 0.004417$

Velocity of the jet, $V = 20 \text{ m/s}$

Velocity of the plate, $u = 8 \text{ m/s}$

Problem:2(Cont...)

Angle of deflection of the jet, $= 165°$

$\therefore$ Angle made by the relative velocity at the outlet of the plate,

$$\theta = 180° - 165° = 15°.$$

(*i*) Force exerted by the jet on the plate in the direction of the jet is given by equation

$$= F_x = \rho a(V - u)^2 (1 + \cos\theta)$$

$$= 1000 \times .004417 \times (20 - 8)^2 [1 + \cos 15°] = \mathbf{1250.38\ N.\ Ans.}$$

(*ii*) Work done by the jet on the plate per second

$$= F_x \times u = 1250.38 \times 8 = 10003.04 \text{ N m/s}$$

$\therefore$ Power of the jet $= \dfrac{10003.04}{1000} = \mathbf{10\ kW.\ Ans.}$

Problem:2(Cont...)

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(*iii*) Efficiency of the jet

$$= \frac{\text{Output}}{\text{Input}} = \frac{\text{Work done by jet/sec}}{\text{Kinetic energy of jet/sec}}$$

$$= \frac{1250.38 \times 8}{\frac{1}{2}(\rho a V) \times V^2} = \frac{1250.38 \times 8}{\frac{1}{2} \times 1000 \times .004417 \times V^3}$$

$$= \frac{1250.38 \times 8}{\frac{1}{2} \times 1000 \times .004417 \times 20^3} = 0.564 = \mathbf{56.4\%.\ Ans}$$

Summary

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- The F_n for Inclined Plate Moving in the Direction of the Jet, $F_n = \rho a(V-u)^2 sin\theta$
- The F_x and F_y for Inclined Plate Moving in the Direction of the Jet, $F_x = F_n sin\theta = \rho a(V-u)^2 sin\theta^2, F_y = F_n cos\theta = \rho a(V-u)^2 sin\theta\ \cos\theta$
- Work done per second by the jet on the plate for Inclined Plate Moving in the Direction of the Jet $= \rho a(V-u)^2 usin\theta^2$
- Work done and Force exerted by the jet of water on the Curved Plate when the Plate is Moving in the direction of jet on the curved plate in the direction of the jet $\rho a(V-u)^2\ X\mathrm{u}[1+cos\theta]\ \ and\ F_x = \rho a(V-u)^2\ [1+cos\theta]$

IMPACT OF JETS ON UNSYMMETRICAL MOVING CURVED PLATE

Presented By:
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Lecture Details:
Unit-IV (Basics of Turbo Machinery) , Impact of jets on unsymmetrical moving curved plate , FM & HM /Mechanical, I -Semester.

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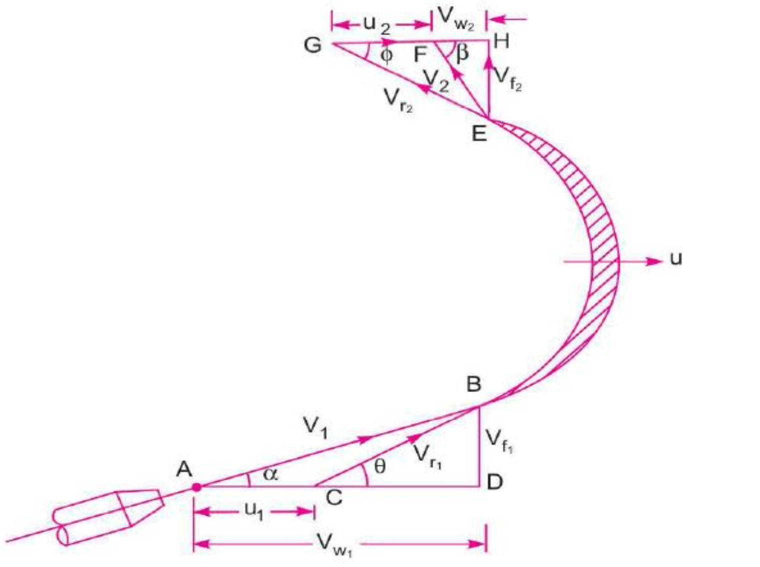
IMPACT OF JETS ON UNSYMMETRICAL MOVING CURVED PLATE

Contents

- Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips
- Summary

Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips

- As the jet strikes tangentially, the loss of energy due to impact of jet will be zero
- The velocity with which jet of water strikes is equal to the relative velocity of the jet with respect to the plate



Jet striking a moving curved vane at one of the tips.

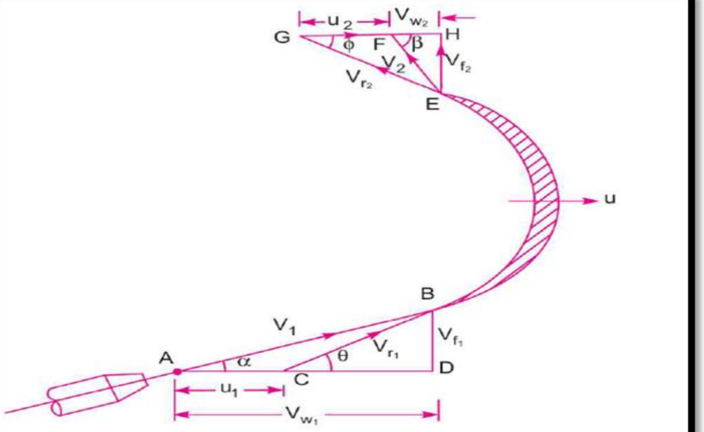
Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)

- Also as the plate is moving in different direction of the jet, the relative velocity at inlet will be equal to the vector difference of the velocity of jet and velocity of the plate at inlet

Let V_1 =Velocity of the jet at inlet

u_1 =Velocity of the plate(vane) at inlet

V_{r_1} =Relative velocity of jet and plate at inlet

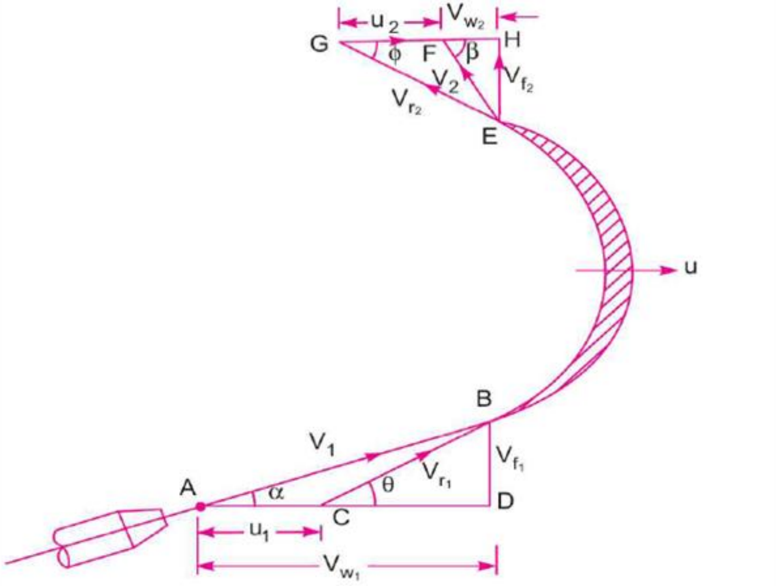


Jet striking a moving curved vane at one of the tips.

Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)

α =Angle between the direction of the jet and direction of motion of the plate, also called guide blade angle

θ = Angle made by the relative velocity(V_{r_2}) with the direction of motion at inlet also called vane angle at inlet



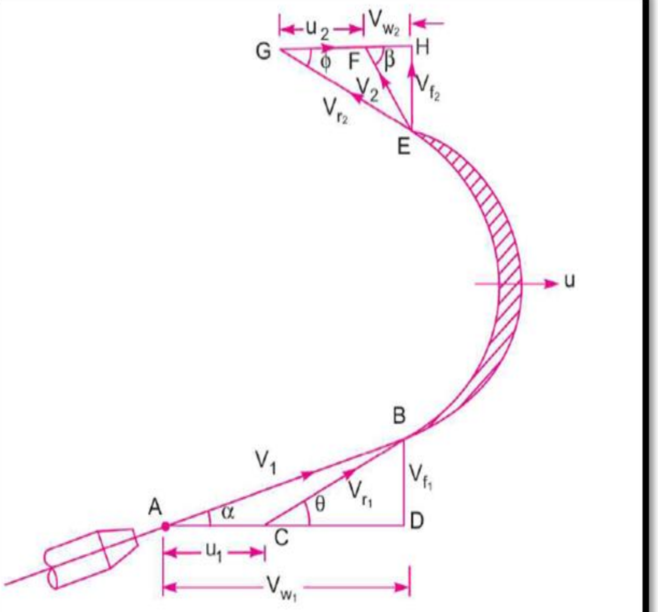
Jet striking a moving curved vane at one of the tips.

Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)

V_{w_1} and V_{f_1} =The components of the velocity of the jet V_1, in the direction of motion and perpendicular to the direction of motion of the vane respectively

V_{w_1} =It is also known as velocity of whirl at inlet

V_{f_1} =It is also known as velocity of flow at inlet



Jet striking a moving curved vane at one of the tips.

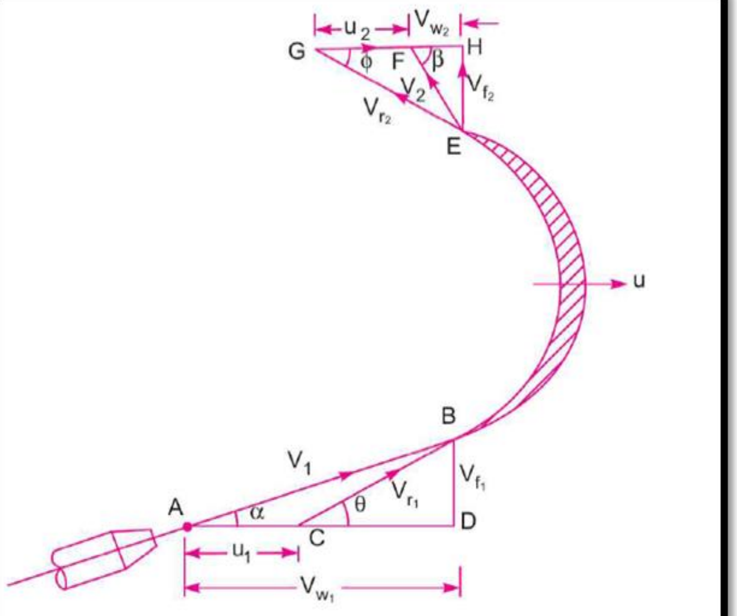
Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)

V_2 = Velocity of the jet, leaving the vane or velocity of jet at outlet of the vane

u_2 =Velocity of the vane at outlet

V_{r_2} = Relative velocity of the jet with respect to the vane at outlet

β =Angle made by the velocity V_2 with the direction of motion of the vane at outlet



Jet striking a moving curved vane at one of the tips.

Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)

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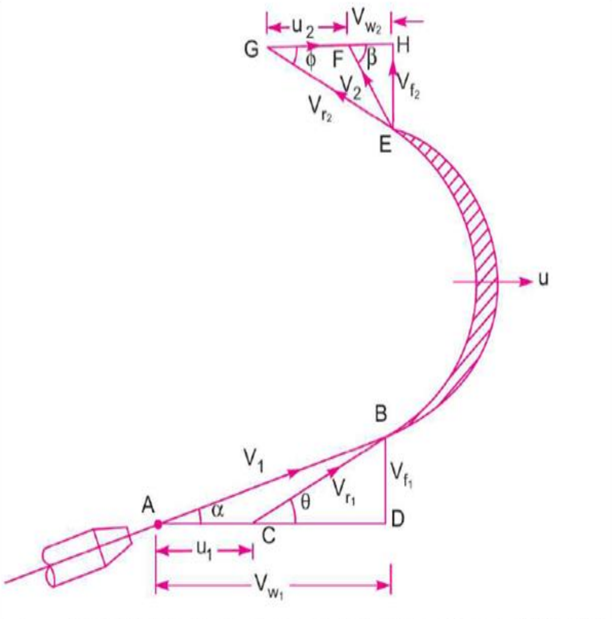
ϕ =Angle made by the relative velocity V_{r_2} with the direction of motion of the vane at outlet and also called vane angle at outlet

V_{w_2} and V_{f_2} = Components of the velocity V_2, in the direction of motion of vane and perpendicular to the direction of motion of vane at outlet



Jet striking a moving curved vane at one of the tips.

Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)

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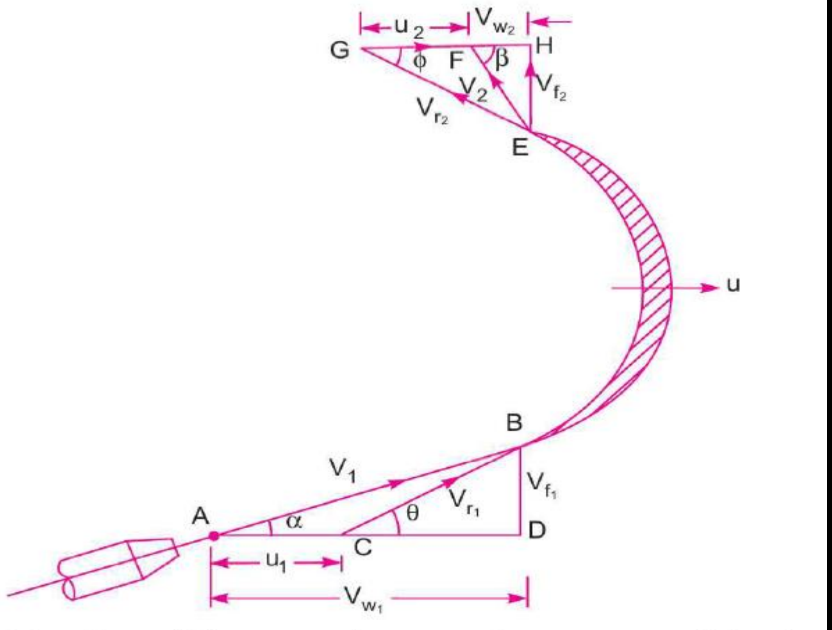
V_{w_2} = It is also called the velocity of whirl at outlet

V_{f_2} = Velocity of flow at outlet

- The triangles ABD and EGH are called the velocity triangles at inlet and outlet



Jet striking a moving curved vane at one of the tips.

Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)

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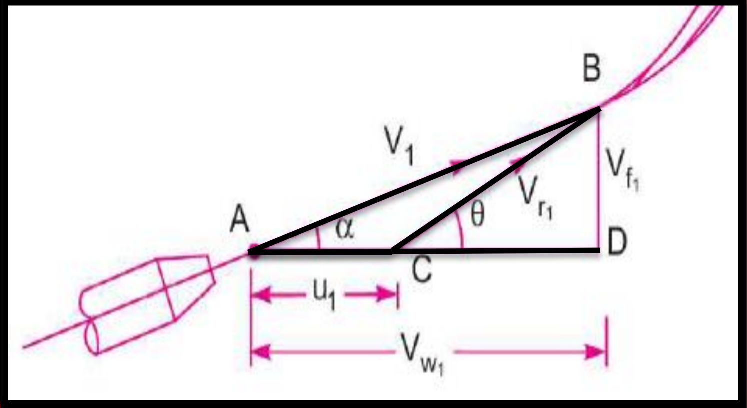
Velocity Triangle at Inlet

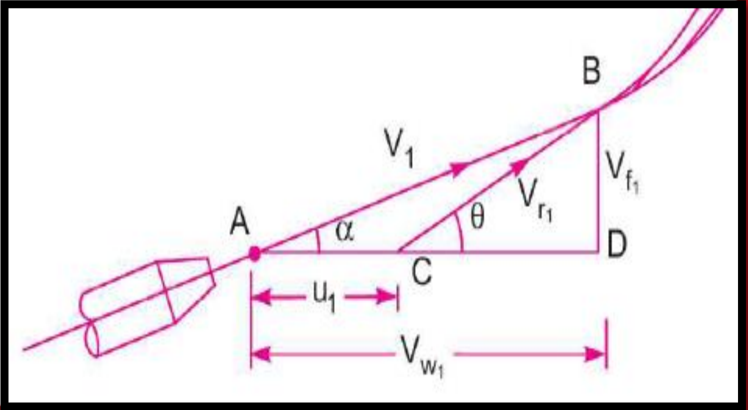
- Take any point A and draw a line AB=V_1 in magnitude and direction which means line AB makes an angle α with the horizontal line AD

- Next draw a line AC= u_1 in magnitude
- Join C to B , CB represents the relative velocity of the jet at inlet

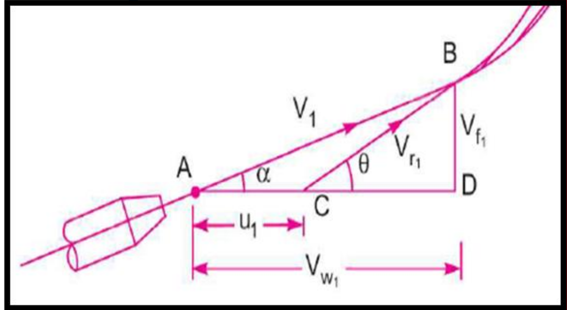


Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)

- If the loss of energy at inlet due to impact is zero, then CB must be in the tangential direction to the vane at inlet
- From B draw a vertical line BD in the direction to meet the horizontal line AC produced at D
- Then BD= Represents the velocity of flow at inlet=V_{f_1}



Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)



AD= Represents the velocity of whirl at inlet $=V_{w_1}$

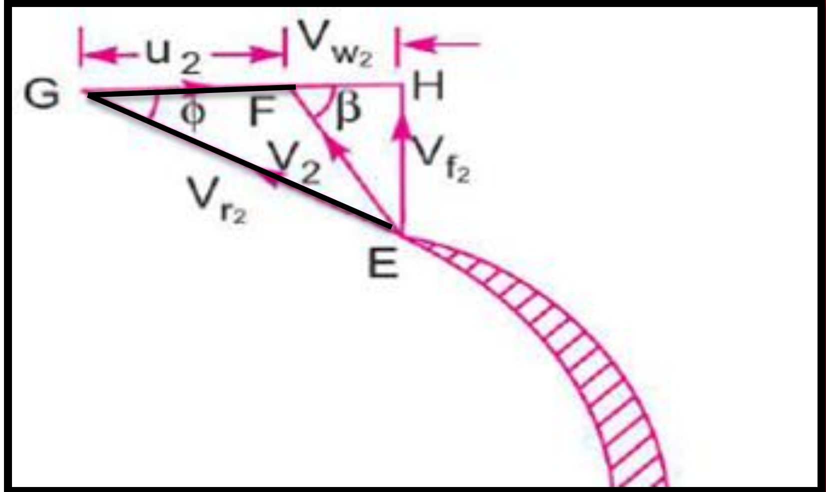
$< BCD$ =Vane angle at inlet=θ

Velocity Triangle at Outlte

- The water will be gliding over the surface of the vane with a relative velocity equal to V_{r_1} and will come out of the vane with a relative velocity V_{r_2}
- The relative velocity at outlet $V_{r_2} = V_{r_1}$

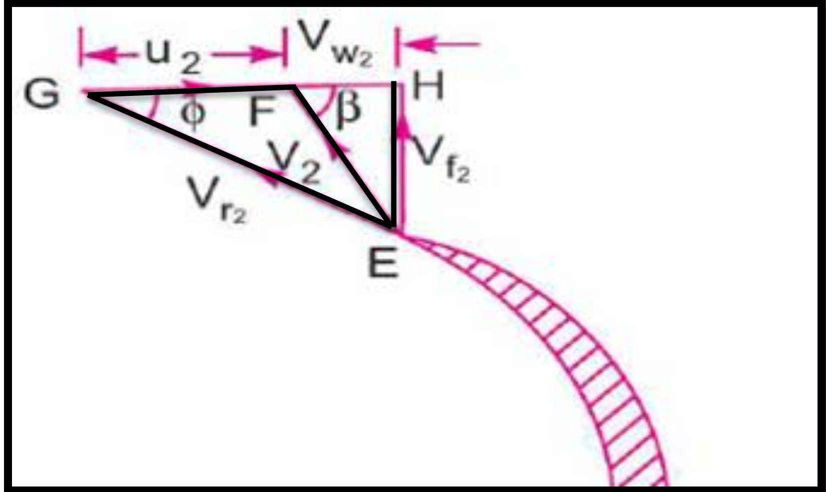
Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)

- The relative velocity at outlet should be in tangential direction to the vane at outlet
- Draw EG in the tangential direction of the vane at outlet and cut EG= V_{r2}
- From G, draw a line GF in the direction of vane at outlet and equal to u_2



Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)

- Join EF represents the absolute velocity of the jet at outlet in magnitude and direction
- From E draw a vertical line EH to meet the line GF produced at H
- If the vane is smooth and is having velocity in the direction of motion at inlet and outlet equal



Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)

- $u_1 = u_2 = u$= Velocity of vane in the direction of motion
- $V_{r_1} = V_{r_2}$
- Now mass of water striking vane per sec= $\rho a \, V_{r_1}$

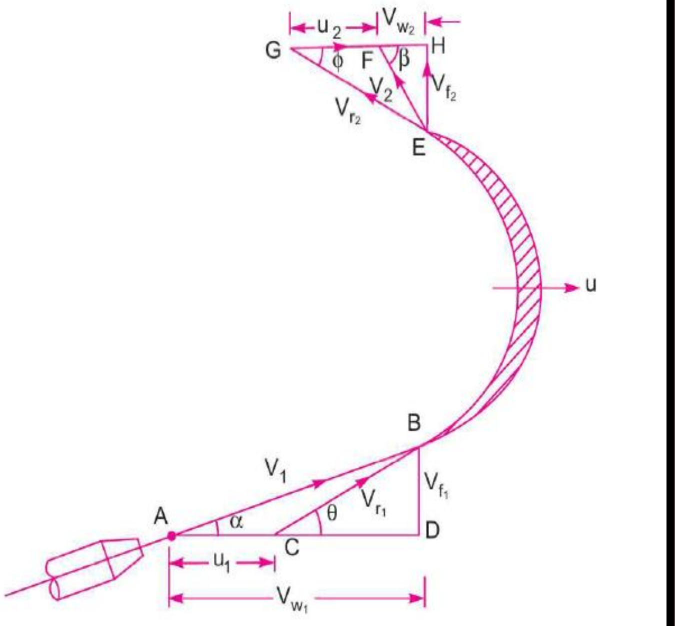
Where a=area of jet of water , V_{r_1}=Relative velocity at inlet

- Force exerted by the jet in the direction of motion

F_x =Mass of water striking per sec X [Initial velocity with which jet strikes in the direction of motion- Final Velocity of jet in the direction of motion]

Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)

- But initial velocity with which jet strikes the vane = V_{r_1}
- The component of this velocity in the direction of motion= $V_{r_1}\cos\theta = (V_{w_1} - u_1)$
- Similarly , the component of the relative velocity at outlet in the direction of motion= $-V_{r_2}\, cos\phi = -[u_2 + V_{w_2}]$



Jet striking a moving curved vane at one of the tips.

Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)

- F_x =Mass of water striking per sec X [Initial velocity with which jet strikes in the direction of motion- Final Velocity of jet in the direction of motion]
- $F_x = \rho a\, V_{r_1} X[(V_{w_1} - u_1) - \{-[u_2 + V_{w_2}]\}$
- $F_x = \rho a\, V_{r_1}[V_{w_1} - u_1 + u_2 + V_{w_2}]$
- But $u_1 = u_2$
- $F_x = \rho a\, V_{r_1}[V_{w_1} + V_{w_2}]$

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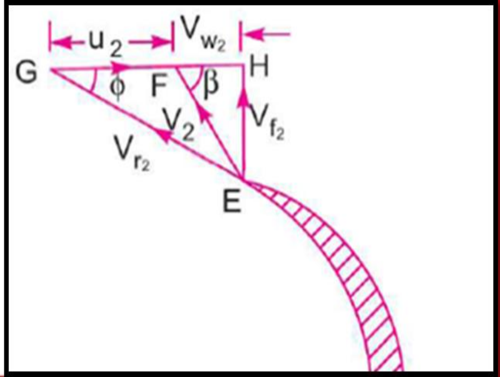
Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)

- If β is an obtuse angle, then $F_x = \rho a\, V_{r_1}[\, V_{w_1} - V_{w_2}]$
- Thus in general , F_x is written as $F_x = \rho a\, V_{r_1}[\, V_{w_1} \pm V_{w_2}]$
- Work done per second on the vane by the jet=Force X Distance per second in the direction of force

$$= F_x\, Xu = \rho a\, V_{r_1}[\, V_{w_1} \pm V_{w_2}]\mathrm{Xu}$$



Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)

- Work done/sec per unit mass of fluid striking per second

$$= \frac{\rho a\, V_{r_1}[\, V_{w_1} \pm V_{w_2}]\mathrm{X}u}{Mass\ of\ fluid\ striking/s}$$

$$= \frac{\rho a\, V_{r_1}[\, V_{w_1} \pm V_{w_2}]\mathrm{X}u}{\rho a\, V_{r_1}}$$

$$= [\, V_{w_1} \pm V_{w_2}]\mathrm{X}u \quad \mathrm{Nm/kg}$$

Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)

- Work done per second per unit weight of fluid striking per second

$$= \frac{\rho a\, V_{r_1}[\, V_{w_1} \pm V_{w_2}]\mathrm{Xu}}{Weight\ of\ fluid\ striking/s}$$

$$= \frac{\rho a\, V_{r_1}[\, V_{w_1} \pm V_{w_2}]\mathrm{Xu}}{gX\rho a\, V_{r_1}}$$

$$= \frac{1}{g}\,[\, V_{w_1} \pm V_{w_2}]\mathrm{Xu}\ \ \mathrm{Nm/N}$$

Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)

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Efficiency of Jet

$$\eta = \frac{Output}{Input} = \frac{Work\ done\ per\ second\ on\ the\ vane}{Initial\ K.E\ per\ second\ of\ the\ jet}$$

$$\eta = \frac{\rho a\, V_{r_1}[\,V_{w_1} \pm V_{w_2}]\mathrm{Xu}}{\frac{1}{2} m {V_1}^2}$$

Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the tips(Cont…)

Where m= mass of the fluid per second in the jet=$\rho a V_1$

V_1 =initial velocity of jet

$$\eta = \frac{\rho a\, V_{r_1}[\, V_{w_1} \pm V_{w_2}]\mathrm{Xu}}{\frac{1}{2}(\rho a V_1)V_1{}^2}$$

Problem:1

A jet of water having a velocity of 20 m/s strikes a curved vane, which is moving with a velocity of 10 m/s. The jet makes an angle of 20° with the direction of motion of vane at inlet and leaves at an angle of 130° to the direction of motion of vane an outlet. Calculate :

(i) Vane angles, so that the water enters and leaves the vane without shock.

(ii) Work done per second per unit weight of water striking (or work done per unit weight of water striking) the vane per second.

Solution. Given :

Velocity of jet, $V_1 = 20$ m/s

Velocity of vane, $u_1 = 10$ m/s

Angle made by jet at inlet, with direction of motion of vane,

$$\alpha = 20°$$

Problem:1(Cont…)

Angle made by the leaving jet, with the direction of motion

$$= 130°$$

$\therefore$ $\beta = 180° - 130° = 50°$

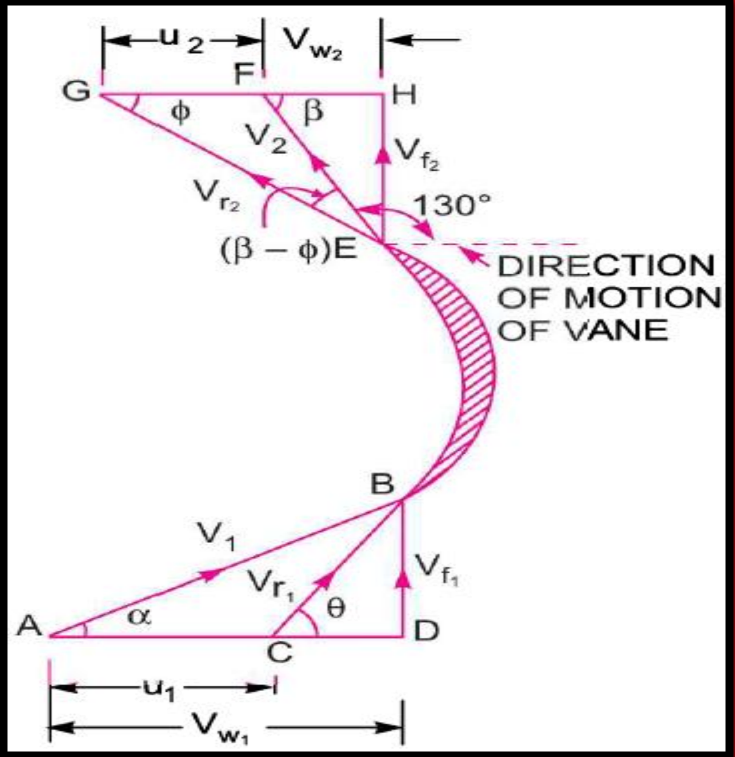
In this problem, $u_1 = u_2 = 10$ m/s

$$V_{r_1} = V_{r_2}$$

(*i*) **Vane Angle** means angle made by the relative velocities at inlet and outlet, *i.e.*, θ and ϕ.

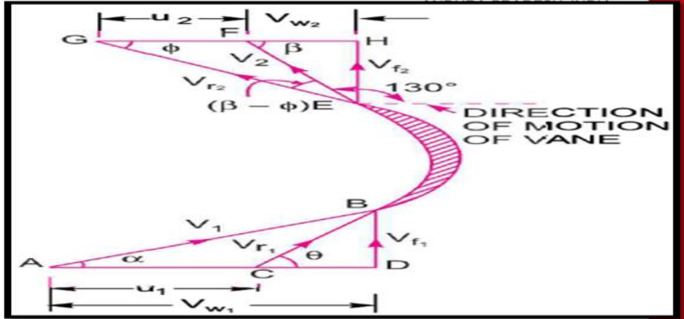
in ΔABD, we have $\tan\theta = \dfrac{BD}{CD}$

$$= \frac{V_{f_1}}{AD - AC} = \frac{V_{f_1}}{V_{w_1} - u_1}$$



Problem:1(Cont…)

where $V_{f_1} = V_1 \sin\alpha = 20 \times \sin 20^\circ = 6.84$ m/s



$V_{w_1} = V_1 \cos\alpha = 20 \times \cos 20^\circ = 18.794$ m/s.

$u_1 = 10$ m/s

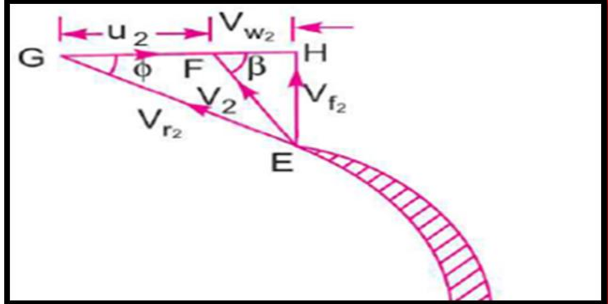
$$\therefore \quad \tan\theta = \frac{6.84}{18.794 - 10} = .7778 \text{ or } \theta = 37.875^\circ$$

$$\therefore \quad \theta = \mathbf{37^\circ\ 52.5'.\ Ans.}$$

From, ΔABC,
$$\sin\theta = \frac{V_{f_1}}{V_{r_1}} \quad \text{or} \quad V_{r_1} = \frac{V_{f_1}}{\sin\theta} = \frac{6.84}{\sin 37.875^\circ} = 11.14$$

$$\therefore \quad V_{r_2} = V_{r_1} = 11.14 \text{ m/s.}$$

Problem:1(Cont…)



From, ΔEFG, applying sine rule, we have

$$\frac{V_{r_2}}{\sin(180^\circ - \beta)} = \frac{u_2}{\sin(\beta - \phi)}$$

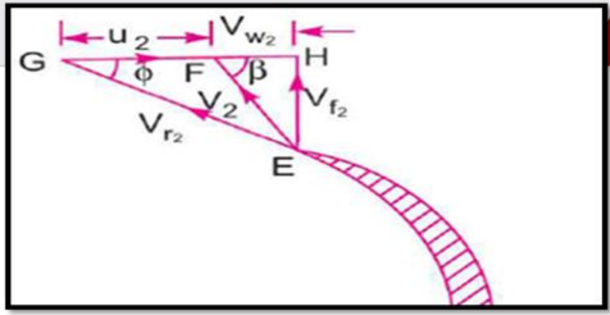
$$\frac{11.14}{\sin\beta} = \frac{10}{\sin[\beta - \phi]} \quad \text{or} \quad \frac{11.14}{\sin 50^\circ} = \frac{10}{\sin[50^\circ - \phi]} \qquad (\because \ \beta = 50^\circ)$$

$$\therefore \qquad \sin(50^\circ - \phi) = \frac{10 \times \sin 50^\circ}{11.14} = 0.6876 = \sin 43.44^\circ$$

$$\therefore \qquad 50^\circ - \phi = 43.44^\circ \quad \text{or} \quad \phi = 50^\circ - 43.44^\circ = 6.56^\circ$$

$$\therefore \qquad \boldsymbol{\phi = 6^\circ\ 33.6'.} \ \textbf{Ans.}$$

Problem:1(Cont…)



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(*ii*) Work done per second per unit weight of the water striking the vane per second is given by equation

$$= \frac{1}{g}[V_{w_1} + V_{w_2}] \times u \text{ Nm/N (+ ve sign is taken as } \beta \text{ is an acute angle)}$$

where V_{w_1} = 18.794 m/s, $V_{w_2} = GH - GF = V_{r_2} \cos\phi - u_2 = 11.14 \times \cos 6.56^\circ - 10 = 1.067$ m/s

$u = u_1 = u_2 = 10$ m/s

∴ Work done per unit weight of water

$$= \frac{1}{9.81}[18.794 + 1.067] \times 10 \text{ Nm/N} = \textbf{20.24 Nm/N. Ans.}$$

Problem:2

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A jet of water having a velocity of 40 m/s strikes a curved vane, which is moving with a velocity of 20 m/s. The jet makes an angle of 30° with the direction of motion of vane at inlet and leaves at an angle of 90° to the direction of motion of vane at outlet. Draw the velocity triangles at inlet and outlet and determine the vane angles at inlet and outlet so that the water enters and leaves the vane without shock.

Solution. Given :

Velocity of jet, $V_1 = 40$ m/s

Velocity of vane, $u_1 = 20$ m/s

Angle made by jet at inlet, $\alpha = 30°$

Angle made by leaving jet $= 90°$

$\therefore$ $\beta = 180° - 90° = 90°$

For this problem, we have

$$u_1 = u_2 = u = 20 \text{ m/s}$$

Problem:2(Cont...)

Vane angles at inlet and outlet are θ and ϕ respectively.

From ΔBCD, we have

$$\tan\theta = \frac{BD}{CD} = \frac{BD}{AD - AC} = \frac{V_{f_1}}{V_{w_1} - u_1}$$

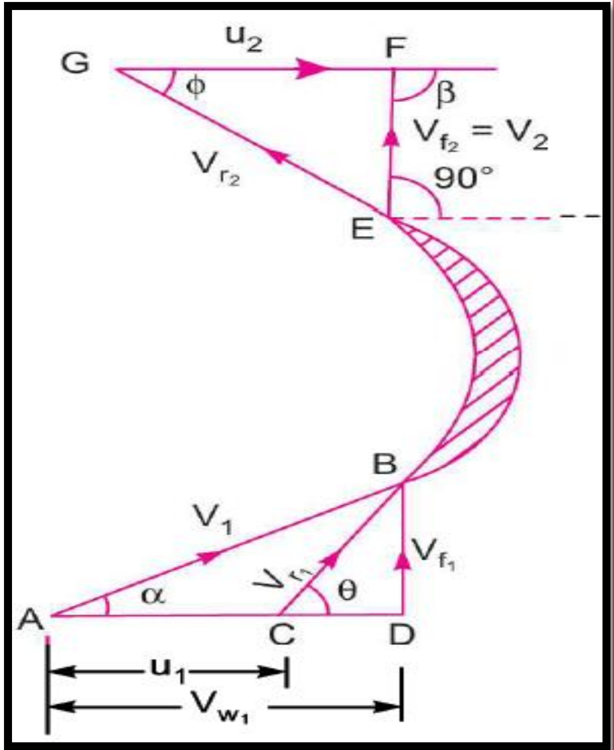
where $V_{f_1} = V_1 \sin\alpha = 40 \times \sin 30^\circ = 20 \text{ m/s}$

$V_{w_1} = V_1 \cos\alpha = 40 \times \cos 30^\circ = 34.64 \text{ m/s}$

$u_1 = 20 \text{ m/s}$

$$\therefore \quad \tan\theta = \frac{20}{34.64 - 20} = \frac{20}{14.64} = 1.366 = \tan 53.79^\circ$$

$$\therefore \quad \theta = 53.79^\circ \text{ or } \mathbf{53^\circ\ 47.4'}.\ \textbf{Ans.}$$

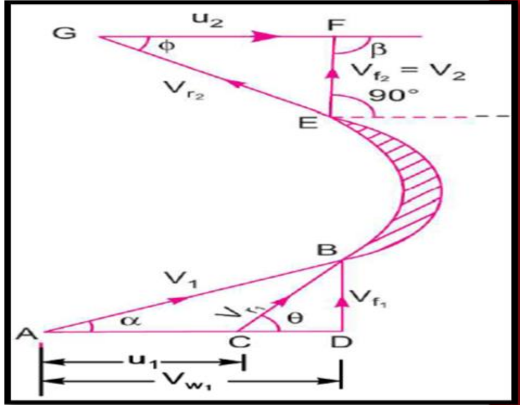


Problem:2(Cont…)

Also from ΔBCD,

$$\sin\theta = \frac{V_{f_1}}{V_{r_1}} \quad \text{or} \quad V_{r_1} = \frac{V_{f_1}}{\sin\theta} = \frac{20}{\sin 53.79^\circ}$$

$$\therefore \qquad V_{r_1} = 24.78$$



But $\qquad V_{r_2} = V_{r_1} = 24.78$

Hence, from ΔEFG, $\cos\phi = \dfrac{u_2}{V_{r_2}} = \dfrac{20}{24.78} = 0.8071 = \cos 36.18^\circ$

$$\therefore \qquad \phi = 36.18^\circ \quad \text{or} \quad \mathbf{36^\circ\ 10.8'.\ Ans.}$$

Summary

- Velocity of the plate(vane) at inlet and outlet are equal, $u_1 = u_2$
- Relative velocity at inlet and outlet are equal, $V_{r_1} = V_{r_2}$
- Force exerted by the jet in the direction of motion, $F_x = \rho a\, V_{r_1}[\, V_{w_1} + V_{w_2}]$
- Work done/sec per unit mass of fluid striking per second $= [\, V_{w_1} \pm V_{w_2}]$Xu Nm/kg
- Work done per second per unit weight of fluid striking per second$= \frac{1}{g}\, [\, V_{w_1} \pm V_{w_2}]$Xu Nm/N
- Efficiency of Jet $\eta = \frac{\rho a\, V_{r_1}[\, V_{w_1} \pm V_{w_2}]Xu}{\frac{1}{2}(\rho a V_1) V_1{}^2}$

IMPACT OF JETS ON SERIES OF VANES

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Mechanical Engineering

GIET(A)

Lecture Details:

Unit-IV (Basics of Turbo Machinery) , IMPACT OF JETS ON SERIES OF VANES, FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
- FLUID DYNAMICS
- CLOSED CONDUIT FLOW
- BOUNDARY LAYER THEORY AND APPLICATIONS
- BASICS OF TURBO MACHINERY
- HYDRAULIC TURBINES
- PERFORMANCE OF HYDRAULIC TURBINES
- CENTRIFUGAL PUMPS
- RECIPROCATING PUMPS

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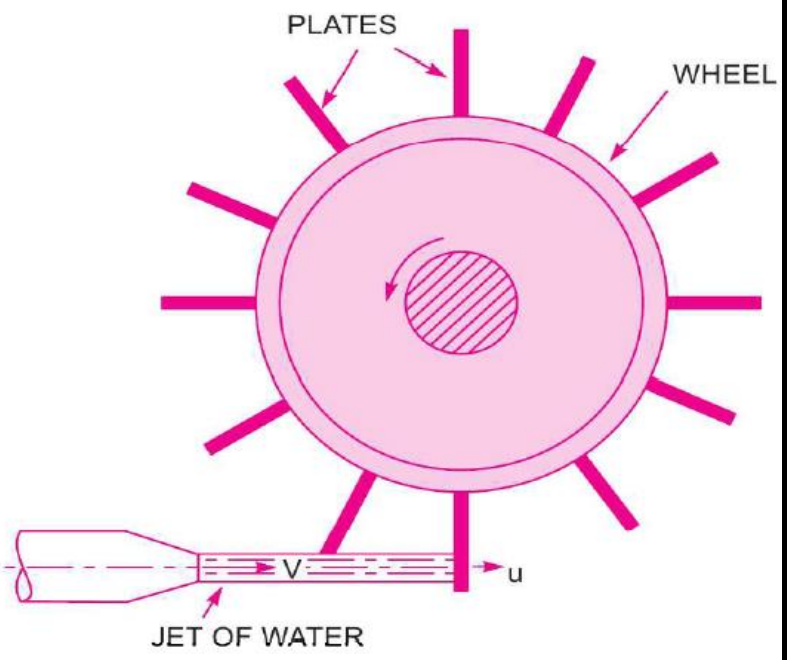
IMPACT OF JETS ON SERIES OF VANES

Contents

- Force Exerted by a Jet of Water on a Series of Vanes
- Force Exerted on a Series of Radial Curved Vanes
- Summary

Force Exerted by a Jet of Water on a Series of Vanes

- The force exerted by a jet of water on a single moving plate(which may be flat or curved) is not practically feasible
- In actual practice, a large number of plates are mounted on the circumference of a wheel at a fixed distance apart



Jet striking a series of vanes.

Force Exerted by a Jet of Water on a Series of Vanes(Cont…)

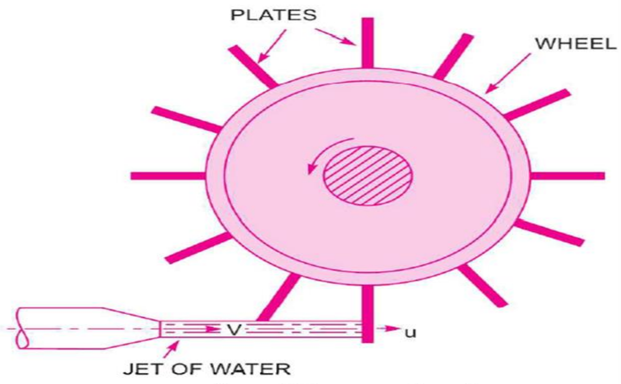
- The jet strikes a plate and due to the force exerted by the jet on the plate, the wheel starts moving and the 2nd plate mounted on the wheel appears before the jet, which again exerts the force on the 2nd plate

Let V= Velocity of jet

d= Diameter of jet

a= Cross sectional area of jet= $\frac{\pi}{4}d^2$

u= Velocity of vane



Jet striking a series of vanes.

Force Exerted by a Jet of Water on a Series of Vanes(Cont…)

- In this case the mass of water coming out from the nozzle per second is always in contact with the plates, when all the plates are considered
- Hence mass of water per second striking the series of plates= $\rho a V$
- The jet strikes the plate with a velocity= $(V - u)$
- After striking, the jet moves tangential to the plate and hence the velocity component in the direction of motion of plate is equal to zero
- The force exerted by the jet in the direction of motion of plate

F_x =Mass per second [Initial velocity−Final velocity]

Force Exerted by a Jet of Water on a Series of Vanes(Cont…)

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$$F_x = \rho aV[(V-u)-0]$$

$$F_x = \rho aV[V-u]$$

- Work done by the jet on the series of plates per second=Force X Distance per second in the direction of force

$$= F_x\ Xu = \rho aV[V-u]Xu$$

- Kinetic energy of the jet per second

$$= \frac{1}{2}mV^2 = \frac{1}{2}(\rho aV)V^2 = \frac{1}{2}\rho aV^3$$

Force Exerted by a Jet of Water on a Series of Vanes(Cont…)

Efficiency, $\eta = \dfrac{Work\ done\ per\ second}{Kinetic\ energy\ per\ second} = \dfrac{\rho aV[V-u]Xu}{\frac{1}{2}\rho aV^3} = \dfrac{2u[V-u]}{V^2}$

<u>Condition for Maximum Efficiency</u>

$$\frac{d\eta}{du} = 0$$

$$\frac{d}{du}\left[\frac{2u[V-u]}{V^2}\right] = 0$$

$$\frac{2V - 2X2u}{V^2} = 0$$

Force Exerted by a Jet of Water on a Series of Vanes(Cont...)

$$2V - 4u = 0$$

$$V = \frac{4u}{2}$$

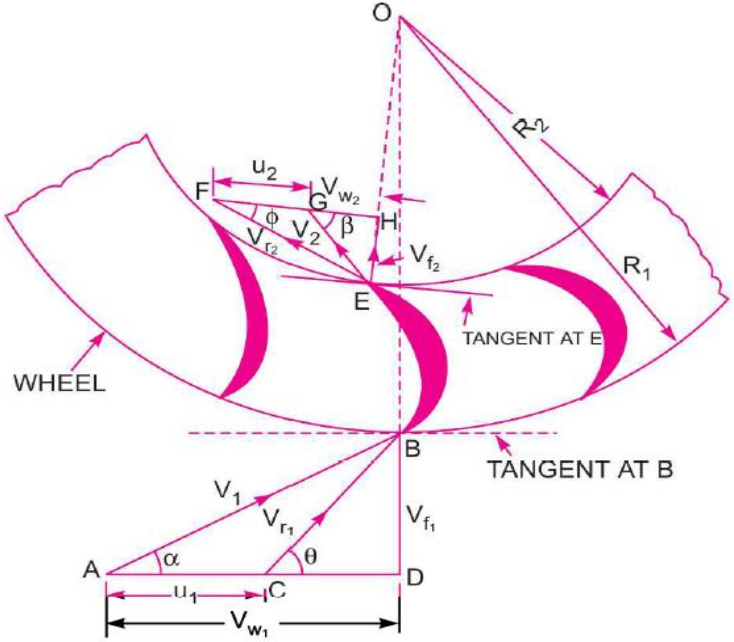
$$u = \frac{V}{2} \text{ or } V=2u \qquad \{\eta = \frac{2u[V-u]}{V^2}\}$$

$$\eta_{max} = \frac{2u[2u-u]}{(2u)^2}$$

$$\eta_{max} = \frac{2uXu}{2uX2u} = \frac{1}{2} = 0.5 \text{ or } 50\%$$

Force Exerted on a Series of Radial Curved Vanes

- For a radial curved vane, the radius of the vane at inlet and outlet is different and hence the tangential velocities of the radial vane at inlet and outlet will not be equal
- Consider a series of radial curved vanes mounted on a wheel

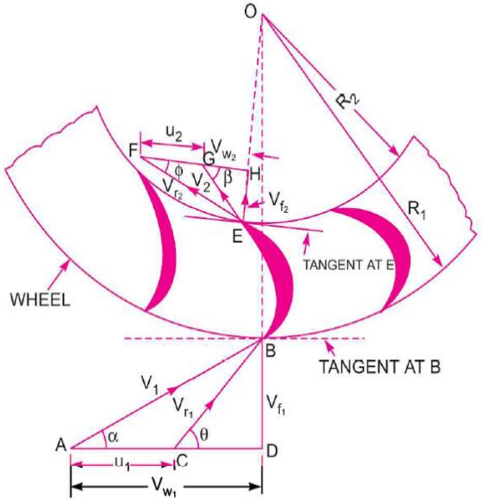


Series of radial curved vanes mounted on a wheel.

Force Exerted on a Series of Radial Curved Vanes(Cont…)

➢ The jet of water strikes the vanes

➢ The wheel starts rotating at constant angular speed



Let R_1 =Radius of wheel at inlet of the vane

R_2 =Radius of wheel at the outlet of the vane

ω =Angular speed of the wheel

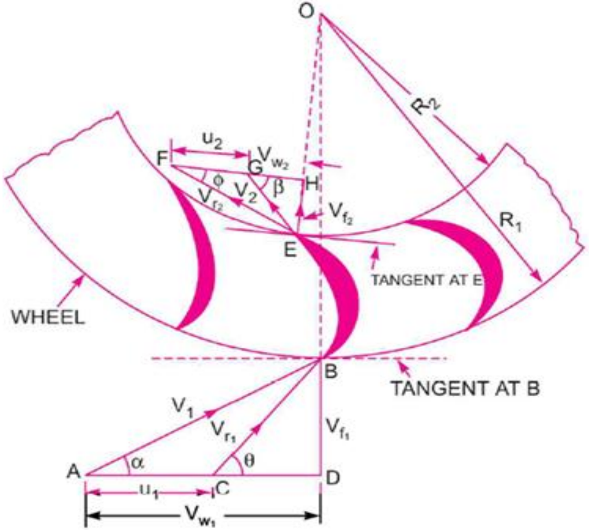
Then , $u_1 = \omega R_1$ and $u_2 = \omega R_2$

➢ The mass of water striking per second for a series of vanes=Mass of water coming out from nozzle per second $= \rho a V_1$

Force Exerted on a Series of Radial Curved Vanes(Cont...)

Where a= area of jet , V_1 = velocity of jet

- Momentum of water striking the vanes in the tangential direction per sec at inlet=Mass of water per second X Component of V_1 in the tangential direction $= \rho a V_1 X V_{w_1}$
- Component of V_1 in tangential direction $=$ $V_1 cos\alpha = V_{w_1}$



Force Exerted on a Series of Radial Curved Vanes(Cont…)

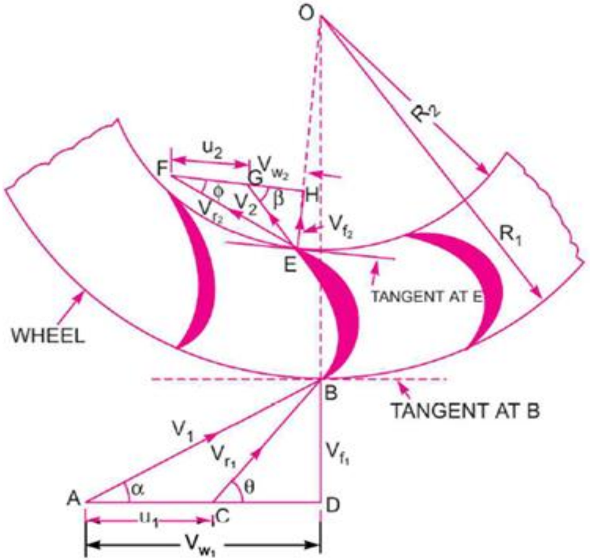
- Similarly, momentum of water at outlet per sec= $\rho a V_1$X Component of V_2 in the tangential direction

$$= \rho a V_1 X\,(-V_2 cos\beta)$$

$$= -\rho a V_1 X V_{w_2} \qquad \{V_2 cos\beta = V_{w_2}\}$$

- Angular momentum per second at inlet=Momentum at inlet X Radius at inlet

$$= \rho a V_1 X V_{w_1} X\, R_1$$



Force Exerted on a Series of Radial Curved Vanes(Cont...)

- Angular momentum per second at outlet=Momentum at outlet X Radius at outlet

$$= -\rho a V_1 X V_{w_2} X\ R_2$$

- Torque exerted by the water on the wheel, T=Rate of change of angular momentum

=[Initial angular momentum per second −Final angular momentum per second]

$$= \rho a V_1 X V_{w_1} X\ R_1 - \left(-\rho a V_1 X V_{w_2} X\ R_2\right) = \rho a V_1 [V_{w_1}\ R_1 + V_{w_2} R_2]$$

Force Exerted on a Series of Radial Curved Vanes(Cont…)

Work done per second on the wheel =Torque X Angular velocity= $TX\omega$

$$= \rho a V_1 [V_{w_1} X R_1 + V_{w_2} R_2]\, X\omega$$

$$= \rho a V_1 [V_{w_1} X R_1 X\omega + V_{w_2} R_2 X\omega]$$

- But $u_1 = \omega R_1$ and $u_2 = \omega R_2$

$$= \rho a V_1 [V_{w_1} u_1 + V_{w_2} u_2]$$

- If the angle β is an obtuse angle, then work done per second will be

$$= \rho a V_1 [V_{w_1} u_1 - V_{w_2} u_2]$$

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Force Exerted on a Series of Radial Curved Vanes(Cont…)

- The general expression for the work done per second on the wheel

$$= \rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2]$$

- If the discharge is radial at outlet, then $\beta = 90^0$ and work done becomes

$$= \rho a V_1 [V_{w_1} u_1]$$

<u>Efficiency of the Radial Curved Vane</u>

$$\eta = \frac{Work\ done\ per\ second}{Kinetic\ energy\ per\ second}$$

Force Exerted on a Series of Radial Curved Vanes(Cont...)

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$$\eta = \frac{Work\ done\ per\ second}{Kinetic\ energy\ per\ second} = \frac{\rho a V_1 [V_{w_1}\ u_1 \pm V_{w_2} u_2]}{\frac{1}{2}\ \left(\frac{mass}{sec}\right) X V_1{}^2}$$

$$= \frac{\rho a V_1 [V_{w_1}\ u_1 \pm V_{w_2} u_2]}{\frac{1}{2}\ (\rho a V_1) X V_1{}^2} = \frac{2\ [V_{w_1}\ u_1 \pm V_{w_2} u_2]}{V_1{}^2}$$

- If there is no loss of energy when water is flowing over the vanes, the work done on the wheel per second is also =Change in K.E of the jet per second

Force Exerted on a Series of Radial Curved Vanes(Cont…)

- Work done per second on the wheel $=$Change of K.E per second of the jet

$=$(Initial K.E per second$-$Final K.E per second) of the jet

$$= \left(\frac{1}{2}mV_1{}^2 - \frac{1}{2}mV_2{}^2\right)$$

$$= \frac{1}{2}m\left(V_1{}^2 - V_2{}^2\right)$$

$$= \frac{1}{2}\rho a V_1\left(V_1{}^2 - V_2{}^2\right)$$

Force Exerted on a Series of Radial Curved Vanes(Cont…)

- Hence efficiency, $\eta = \dfrac{Work\ done\ per\ second\ on\ the\ wheel}{Initial\ Kinetic\ energy\ per\ second\ of\ the\ jet}$

$$= \frac{\frac{1}{2}\rho a V_1 \left({V_1}^2 - {V_2}^2\right)}{\frac{1}{2}(\rho a {V_1}^2){V_1}^2}$$

$$= \frac{\left({V_1}^2 - {V_2}^2\right)}{{V_1}^2}$$

$$\eta = \left(1 - \frac{{V_2}^2}{{V_1}^2}\right)$$

Problem:1

A jet of water having a velocity of 30 m/s strikes a series of radial curved vanes mounted on a wheel which is rotating at 200 r.p.m. The jet makes an angle of 20° with the tangent to the wheel at inlet and leaves the wheel with a velocity of 5 m/s at an angle of 130° to the tangent to the wheel at outlet. Water is flowing from outward in a radial direction. The outer and inner radii of the wheel are 0.5 m and 0.25 m respectively. Determine :

(i) Vane angles at inlet and outlet, (ii) Work done per unit weight of water, and

(iii) Efficiency of the wheel.

Solution. Given :

Velocity of jet, $V_1 = 30$ m/s

Speed of wheel, $N = 200$ r.p.m.

$\therefore$ Angular speed, $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 200}{60} = 20.94$ rad/s

Problem:1(Cont…)

Angle of jet at inlet, $\alpha = 20°$

Velocity of jet at outlet, $V_2 = 5$ m/s

Angle made by the jet at outlet with the tangent to wheel = 130°

$\therefore$ Angle, $\beta = 180° - 130° = 50°$

Outer radius, $R_1 = 0.5$ m

Inner radius, $R_2 = 0.25$ m

$\therefore$ Velocity $u_1 = \omega \times R_1 = 20.94 \times 0.5 = 10.47$ m/s

And $u_2 = \omega \times R_2 = 20.94 \times 0.25 = 5.235$ m/s.

Problem:1(Cont…)

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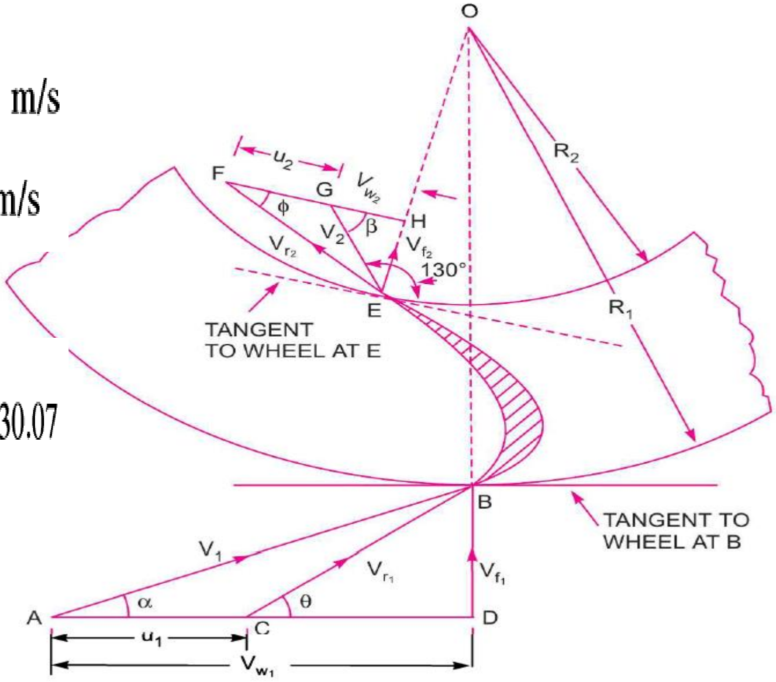
(*i*) **Vane angles** at inlet and outlet means the angle made by the relative velocities V_{r_1} and V_{r_2}, *i.e.*, angle θ and ϕ.



From ΔABD,

$$V_{w_1} = V_1 \cos\alpha = 30 \times \cos 20° = 28.19 \text{ m/s}$$

$$V_{f_1} = V_1 \sin\alpha = 30 \times \sin 20° = 10.26 \text{ m/s}$$

In ΔCBD,

$$\tan\theta = \frac{BD}{CD} = \frac{V_{f_1}}{AD - AC} = \frac{10.26}{V_{w_1} - u_1} = \frac{10.26}{28.19 - 10.47} = 0.579 = \tan 30.07$$

$$\therefore \qquad \theta = 30.07° \quad \text{or} \quad \mathbf{30° \; 4.2'.\ Ans.}$$

Problem:1(Cont…)

From outlet velocity Δ, $V_{w_2} = V_2 \cos\beta = 5 \times \cos 50° = 3.214$ m/s

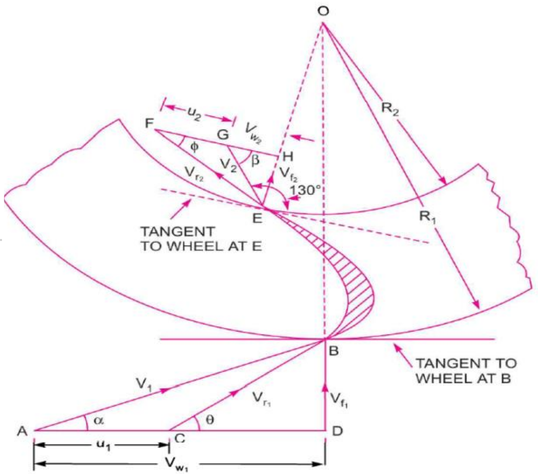
$$V_{f_2} = V_2 \times \sin\beta = 5 \sin 50° = 3.83 \text{ m/s}$$

In ΔEFH, $$\tan\phi = \frac{V_{f_2}}{u_2 + V_{w_2}} = \frac{3.83}{5.235 + 3.214} = 0.453 = \tan 24.385°$$

∴ $\phi = 24.385°$ or **24° 23.1′. Ans.**

(*ii*) Work done per second by water is given by equation

$$= \rho a V_1 \left[V_{w_1} u_1 + V_{w_2} u_2\right]$$



Problem:1(Cont…)

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(+ ve sign is taken as β is acute angle

∴ Work done* per second per unit weight of water striking per second

$$= \frac{\rho a V_1 \left[V_{w_1} u_1 + V_{w_2} u_2 \right]}{\text{Weight of water/s}} = \frac{\rho a V_1 \left[V_{w_1} u_1 + V_{w_2} u_2 \right]}{\rho a V_1 \times g}$$

$$= \frac{1}{g} \left[V_{w_1} u_1 + V_{w_2} u_2 \right] \text{Nm/N} = \frac{1}{9.81} [28.19 \times 10.47 + 3.214 \times 5.235]$$

$$= \frac{1}{9.81} [295.15 + 16.82] = \mathbf{31.8\ Nm/N.\ Ans.}$$

Problem:1(Cont…)

(*iii*) Efficiency, η is given by equation

$$\eta = \frac{2\left[V_{w_1} u_1 + V_{w_2} u_2\right]}{V_1^2} = \frac{2[28.19 \times 10.47 + 3.214 \times 5.235]}{30^2}$$

$$= \frac{2[295.15 + 16.82]}{30 \times 30} = 0.6932 \text{ or } \mathbf{69.32\%.\ Ans.}$$

Summary

- The force exerted by a jet of water on a single moving plate is not practically feasible
- For Jet of Water on a Series of Vanes, $\eta = \frac{2u[V-u]}{V^2}$ and $\eta_{max} = 50^0/_0$
- For a radial curved vane, the radius of the vane at inlet and outlet is different and hence the tangential velocities of the radial vane at inlet and outlet will not be equal
- For a Series of Radial Curved Vanes ,$\eta = \frac{2\,[V_{w_1}\, u_1 \pm V_{w_2} u_2]}{{V_1}^2}$

HYDRAULIC MACHINES

Presented By:

Shaik Nayeem

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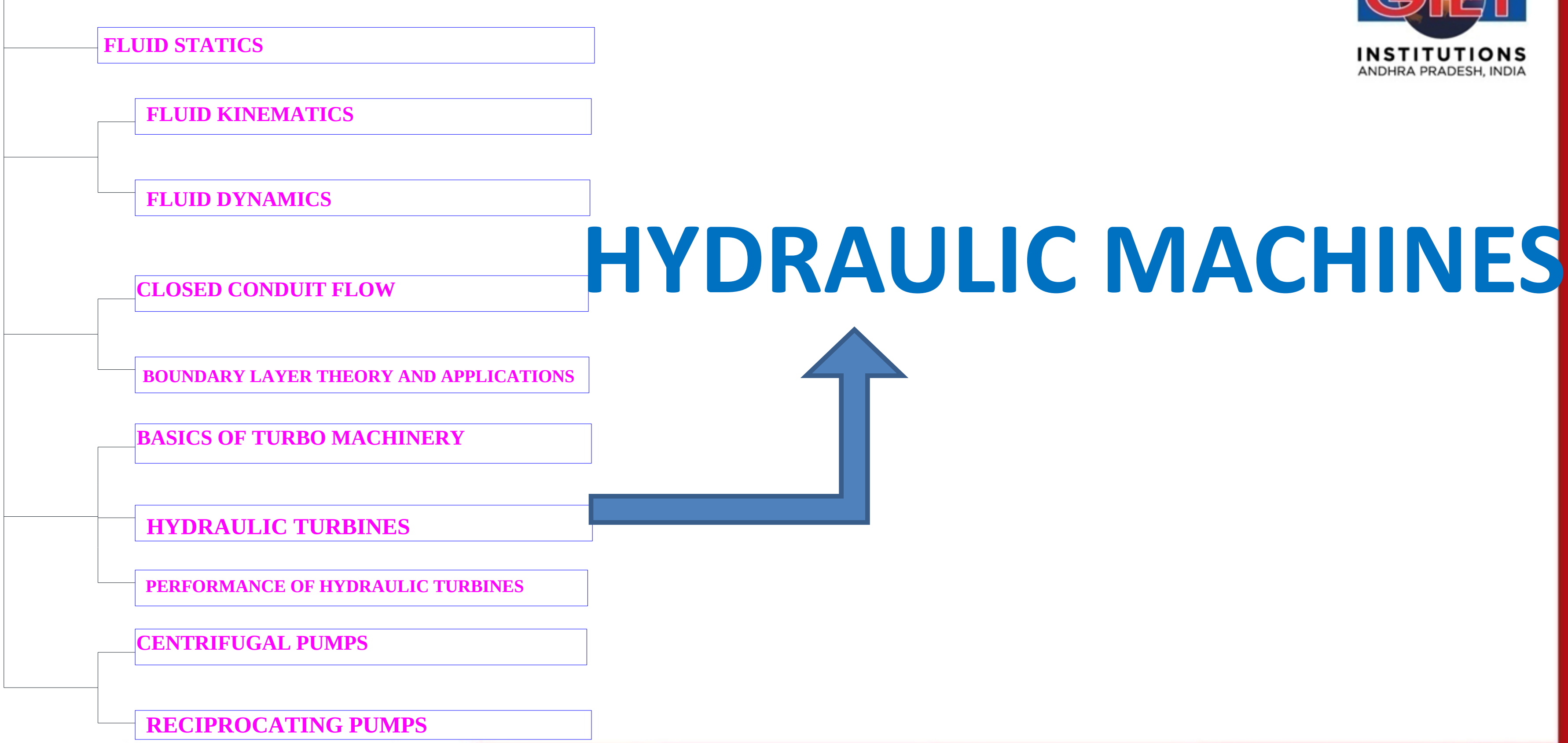
Mechanical Engineering

GIET(A)

Lecture Details:

Unit-IV (Hydraulic Turbines) , Hydraulic Machines, FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery



Contents

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Introduction to Hydraulic Machines

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- Hydraulic machines are those machines which convert either hydraulic energy (energy possessed by water) into mechanical energy(which is further converted into electrical energy) or mechanical energy into hydraulic energy
- The hydraulic machines, which convert the hydraulic energy into mechanical energy, are called turbines , while the hydraulic machines which convert the mechanical energy into hydraulic energy are called pumps

Turbines

- Turbines are defined as the hydraulic machines which convert hydraulic energy into mechanical energy
- This mechanical energy is used in running an electric generator which is directly coupled to the shaft of the turbine
- Thus the mechanical energy is converted into electrical energy
- The electric power which is obtained from the hydraulic energy(energy of water) is known as Hydroelectric Power

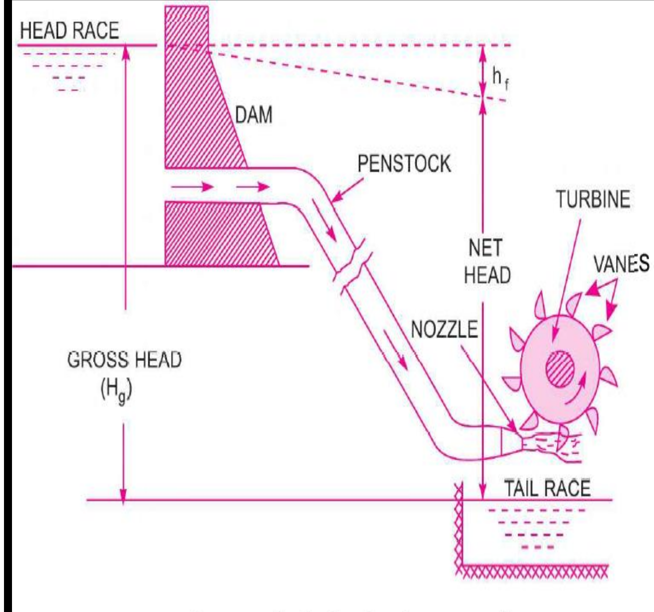
Gross Head and Net Head

Gross Head(H_g)

- The difference between the head race level and tail race level when no water is flowing

Net Head or Effective Head

- The head available at the inlet of the turbine
- When water is flowing from head race to the turbine, a loss of head(h_f) due to friction between the water and penstocks occurs



Layout of a hydroelectric power plant.

Gross Head and Net Head(Cont…)

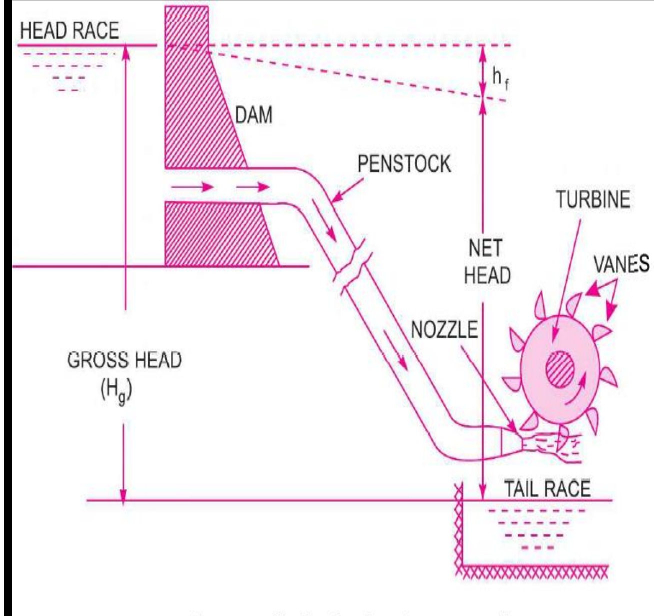
- Net head on turbine is H= $H_g - h_f$

- $h_f = \dfrac{4fLV^2}{DX2g}$

Where, V= Velocity of flow in penstock

L= Length of penstock

D= Diameter of penstock



Layout of a hydroelectric power plant.

Efficiencies of a Turbine

- The following are the important efficiencies of a turbine

Hydraulic Efficiency, η_h

Mechanical Efficiency, η_m

Volumetric Efficiency, η_v

Overall Efficiency, η_o

Hydraulic Efficiency, η_h

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$$\eta_h = \frac{Power\ delivered\ to\ runner}{Power\ supplied\ at\ inlet} = \frac{R.P}{W.P}$$

- The power at the inlet of the turbine is more and this power goes on decreasing as the water flows over the vanes of the turbine due to hydraulic losses as the vanes are not smooth

R.P= Power delivered to runner (Runner Power)

W.P= Power supplied at inlet of the turbine(Water Power)= $\frac{WXH}{1000}$kW

Hydraulic Efficiency, η_h (Cont…)

Where, W= Weight of water striking the vanes of the turbine per second=$\rho g X Q$

$$W.P = \frac{\rho g X Q \, X H}{1000} kW$$

$$\eta_h = \frac{W}{g}\frac{\left[V_{w_1} \pm V_{w_2}\right] X u}{1000} kW$$ ……..For Pelton Turbine

$$\eta_h = \frac{W}{g}\frac{\left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{1000} kW$$ ……..For a Radial flow Turbine

Mechanical Efficiency, η_{m}

$$\eta_{\mathrm{m}} = \frac{Power\ at\ the\ shaft\ of\ the\ turbine(S.P\ or\ B.P)}{Power\ delivered\ by\ water\ to\ the\ runner(R.P)}$$

- Due to mechanical losses, the power available at the shaft of the turbine is less than the power delivered to the runner of a turbine

Where, S.P= Shaft Power

B.P= Brake Power

Volumetric Efficiency, η_v

- The volume of the water striking the runner of a turbine is slightly less than the volume of the water supplied to the turbine
- Some of the volume of the water is discharged to the tail race without striking the runner of the turbine

$$\eta_v = \frac{Volume\ of\ water\ actually\ striking\ the\ runner}{Volume\ of\ water\ supplied\ to\ the\ turbine}$$

Overall Efficiency, η_o

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$$\eta_o = \frac{Power\ available\ at\ the\ shaft\ of\ the\ turbine(Shaft\ Power)}{Power\ supplied\ at\ the\ inlet\ of\ the\ turbine(Water\ Power)}$$

$$\eta_o = \frac{S.P}{W.P} = \frac{S.P}{W.P} X \frac{R.P}{R.P}$$

$$\eta_o = \frac{S.P}{R.P} X \frac{R.P}{W.P} = \eta_m X \eta_h$$

Classification of Hydraulic Turbines

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According to the type of energy at inlet

- Impulse Turbine
- Reaction Turbine

Classification of Hydraulic Turbines(Cont...)

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According to the direction of flow through runner

- Tangential Flow Turbine
- Radial Flow Turbine
- Axial Flow Turbine
- Mixed Flow Turbine

Classification of Hydraulic Turbines(Cont..)

According to the head at the inlet of turbine

- High Head Turbine
- Medium Head Turbine
- Low Head Turbine

Classification of Hydraulic Turbines(Cont…)

According to the specific speed of the turbine

- Low Specific Speed Turbine
- Medium Specific Speed Turbine
- High Specific Speed Turbine

Classification of Hydraulic Turbines(Cont…)

- If at the inlet of the turbine , the energy available is only K.E , the turbine is known as **Impulse Turbine**
- As the water flows over the vanes, the pressure is atmospheric from inlet to outlet of the turbine
- If at the inlet of the turbine , the water possesses K.E as well as pressure energy, the turbine is known as **Reaction Turbine**
- As the water flows through the runner, the water is under pressure and pressure energy goes on changing into K.E

Classification of Hydraulic Turbines(Cont...)

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- If the water flows along the tangent of the runner, the turbine is known as **Tangential Flow Turbine**
- If the water flows in the radial direction through the runner, the turbine is called **Radial Flow Turbine**
- If the water flows from outwards to inwards radially, the turbine is known as **Inward Radial Flow Turbine**
- If the water flows radially from inwards to outwards, the turbine is known as **Outward Radial Flow Turbine**

Classification of Hydraulic Turbines(Cont...)

- If the water flows through the runner along the direction parallel to the axis of rotation of the runner, the turbine is called **Axial Flow Turbine**
- If the water flows through the runner in the radial direction but leaves in the direction parallel to axis of rotation of the runner, the turbine is called **Mixed Flow Turbine**

Summary

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- Hydraulic machines are defined as those machines which convert either hydraulic energy into mechanical energy or mechanical energy into hydraulic energy
- Turbines are defined as the hydraulic machines which convert hydraulic energy into mechanical energy
- Gross Head is difference between the head race level and tail race level when no water is flowing
- Net Head or Effective Head is the head available at the inlet of the turbine

PELTON WHEEL OR TURBINE

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Lecture Details:
Unit-IV (Hydraulic Turbines) , Pelton wheel or turbine,
FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
- FLUID DYNAMICS
- CLOSED CONDUIT FLOW
- BOUNDARY LAYER THEORY AND APPLICATIONS
- BASICS OF TURBO MACHINERY
- HYDRAULIC TURBINES
- PERFORMANCE OF HYDRAULIC TURBINES
- CENTRIFUGAL PUMPS
- RECIPROCATING PUMPS

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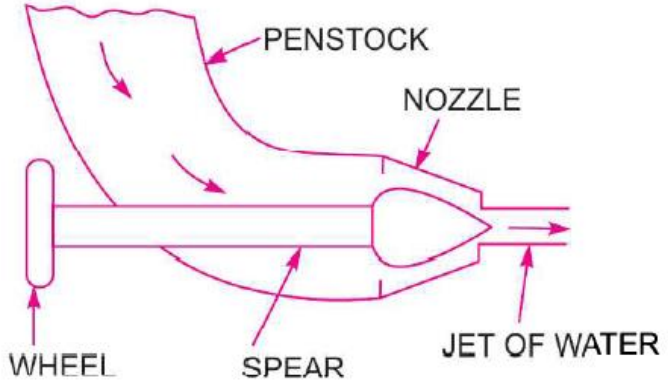
PELTON WHEEL OR TURBINE

Contents

- Pelton Wheel or Turbine
- Velocity Triangles and Work done for Pelton Wheel
- Points to be Remembered for Pelton Wheel
- Design of Pelton Wheel
- Summary

Pelton Wheel or Turbine

- The pelton turbine is a tangential flow impulse turbine
- The water strikes the bucket along the tangent of the runner
- The energy available at the inlet of the turbine is only K.E
- The pressure at the inlet and outlet of the turbine is atmospheric
- This turbine is used for high heads
- The water from the reservoir flows through the penstocks at the outlet of which a nozzle is fitted

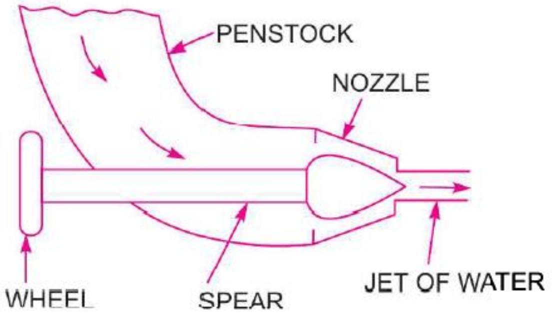


Nozzle with a spear to regulate flow.

Pelton Wheel or Turbine(Cont…)

- At the outlet of the nozzle, the water comes out in the form of a jet and strikes the buckets(Vanes) of the runner
- The main parts of the pelton turbine are:

1. Nozzle and flow regulating arrangement(Spear)
2. Runner and buckets
3. Casing
4. Breaking Jet

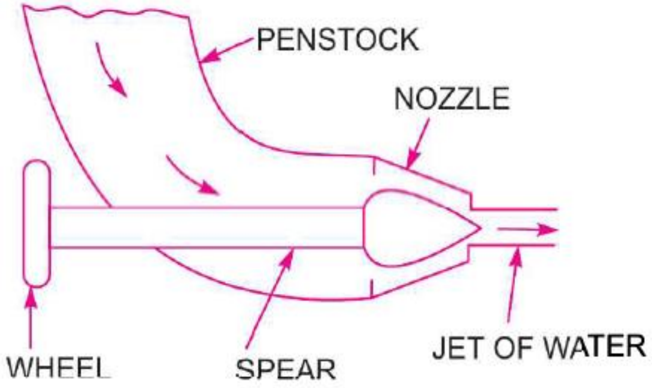


Nozzle with a spear to regulate flow.

Pelton Wheel or Turbine(Cont...)

Nozzle and flow regulating arrangement

- The amount of water striking the buckets(Vanes) of the runner is controlled by providing a spear in the nozzle
- The spear is a conical needle which is operated either by a hand wheel or automatically in the axial direction
- When spear is pushed forward into the nozzle the amount of water striking the runner is reduced



Nozzle with a spear to regulate flow.

Pelton Wheel or Turbine(Cont…)

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Runner with Buckets

- It consists of a circular disc on the periphery of which a number of buckets evenly spaced are fixed
- The shape of the buckets is of a double hemispherical cup or bowl
- Each bucket is divided into two symmetrical parts by a dividing wall which is known as splitter
- The jet of water strikes on the splitter

Pelton Wheel or Turbine(Cont...)

<u>**Runner with Buckets:**</u>

- The splitter divides the jet into two equal parts and the jet comes out at the outer edge of the bucket
- The buckets are shaped in such a way that the jet gets deflected through 160^0 to 170^0

Pelton Wheel or Turbine(Cont…)

Casing:

- The function of the casing is to prevent the splashing of the water and to discharge water to tail race
- It also acts as safeguard against accidents

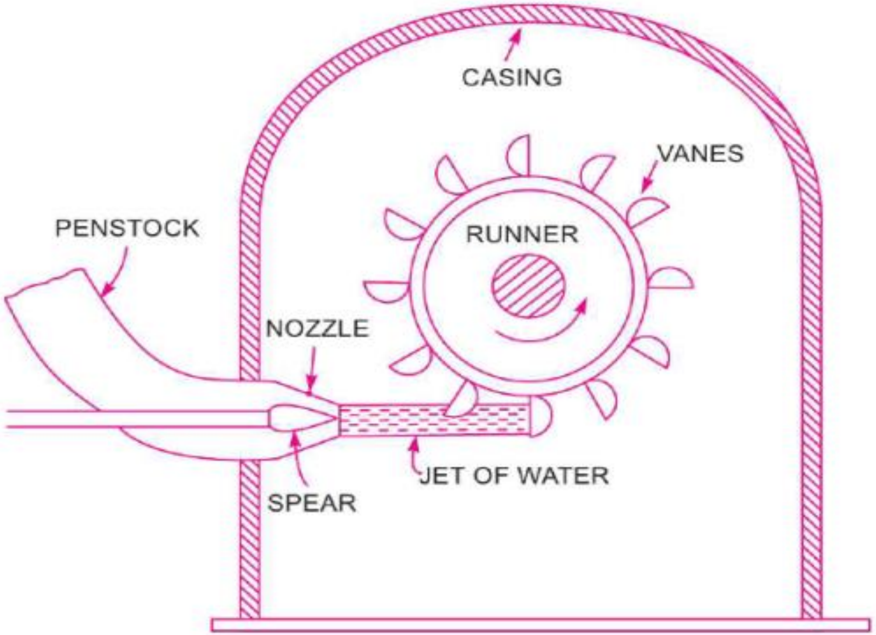
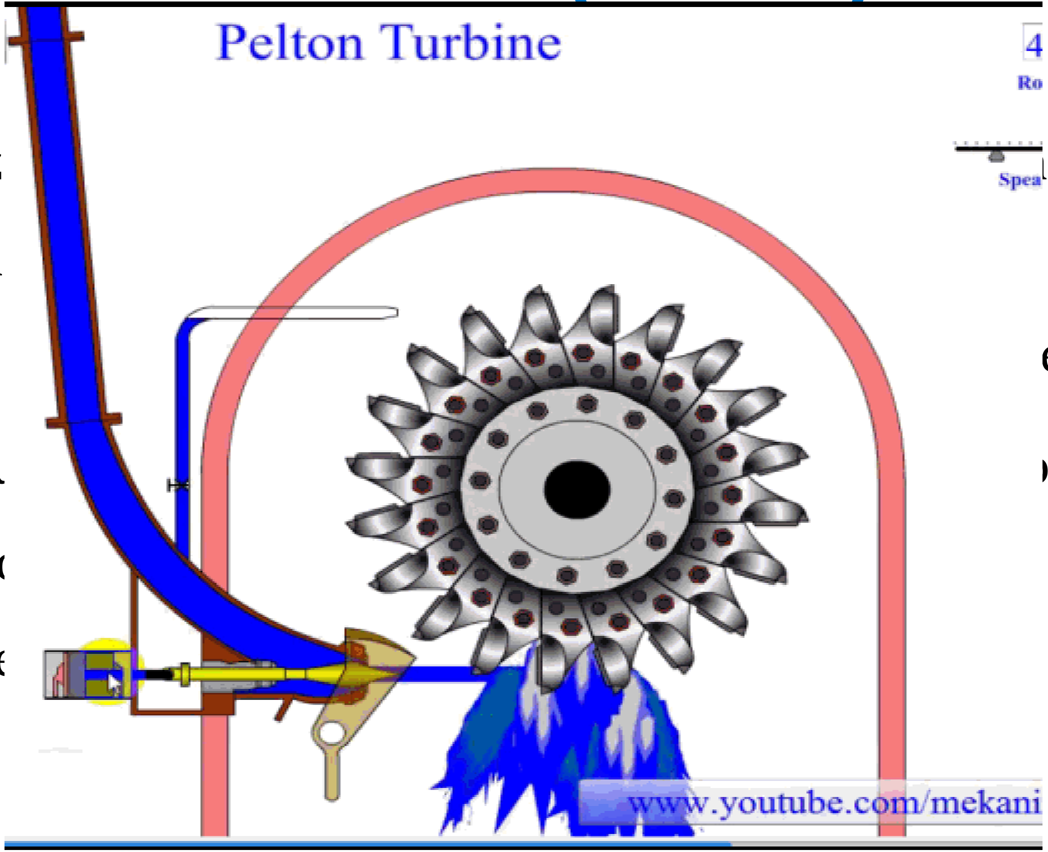


Fig. 18.4 *Pelton turbine.*

Pelton Wheel or Turbine(Cont…)

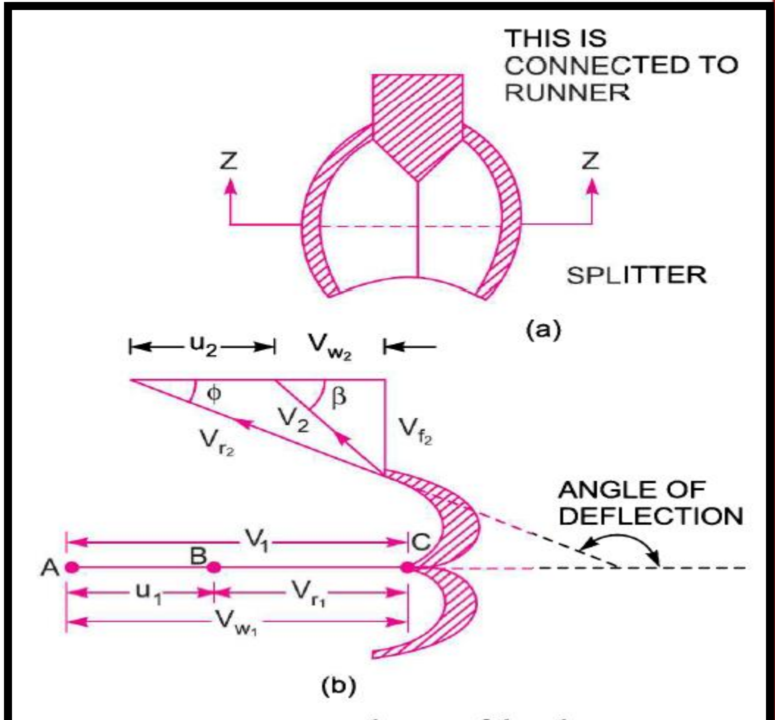
Breaking Jet:

- When the nozz … the forward
  direction, the a
- But the runner … e
- To stop the ru … ovided which
  directs the jet
- This jet of wate



Velocity Triangles and Work done for Pelton Wheel

- The jet of water from the nozzle strikes the bucket at the splitter, which splits up the jet into two parts
- These parts of the jet, glides over the inner surfaces and comes out at the outer edge
- Let H= Net head acting on the Pelton wheel= $H_g - h_f$



Shape of bucket.

Velocity Triangles and Work done for Pelton Wheel(Cont...)

Where H_g =Gross head and $h_f = \frac{4fLV^2}{D^* X 2g}$

D^* =Dia. Of Penstock,

N= Speed of the wheel in r.p.m,

D= Diameter of the wheel ,

d= Diameter of the jet,

V_1 =Velocity of jet at inlet= $\sqrt{2gH}$

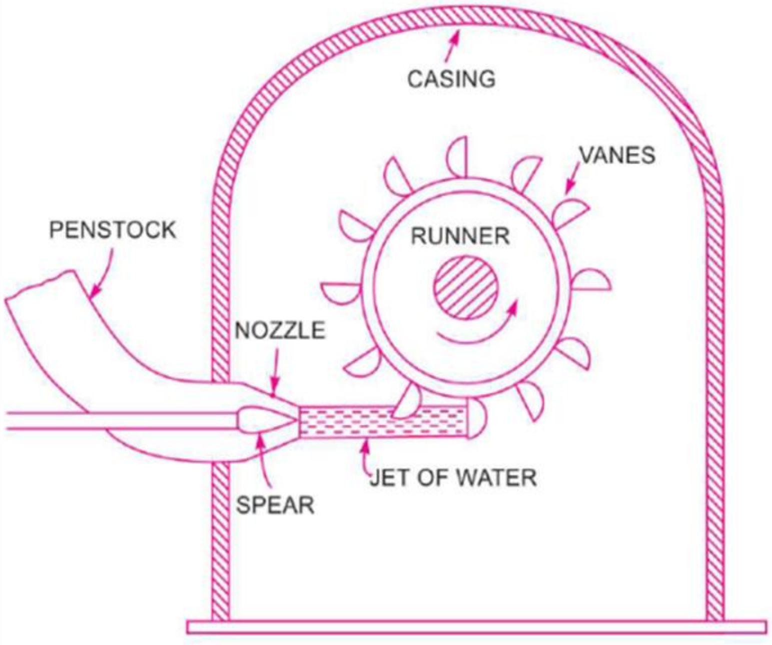


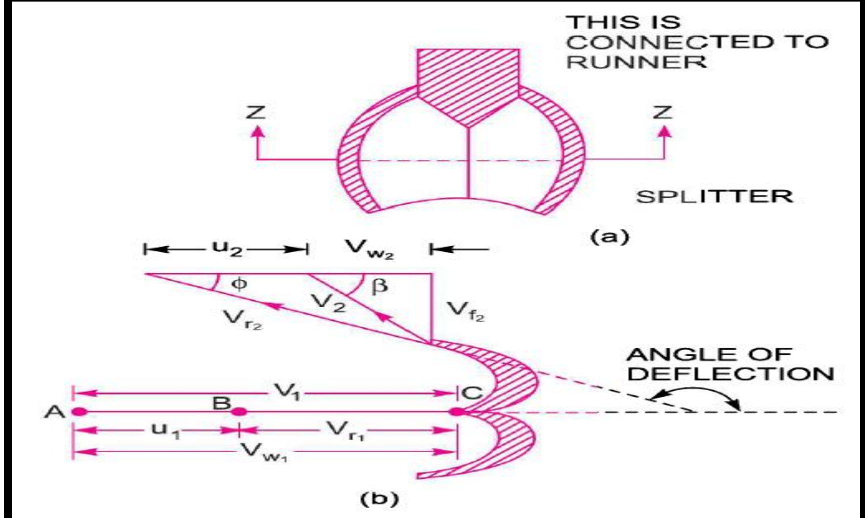
Fig. 18.4 *Pelton turbine.*

Velocity Triangles and Work done for Pelton Wheel(Cont…)

- u= $u_1 = u_2 = \frac{\pi DN}{60}$
- The velocity triangle at inlet will be a straight line where

$V_{r_1} = V_1 - u_1 = V_1 - u$

$V_{w_1} = V_1, \alpha = 0^0$ and $\theta = 0^0$



Shape of bucket.

Velocity Triangles and Work done for Pelton Wheel(Cont...)

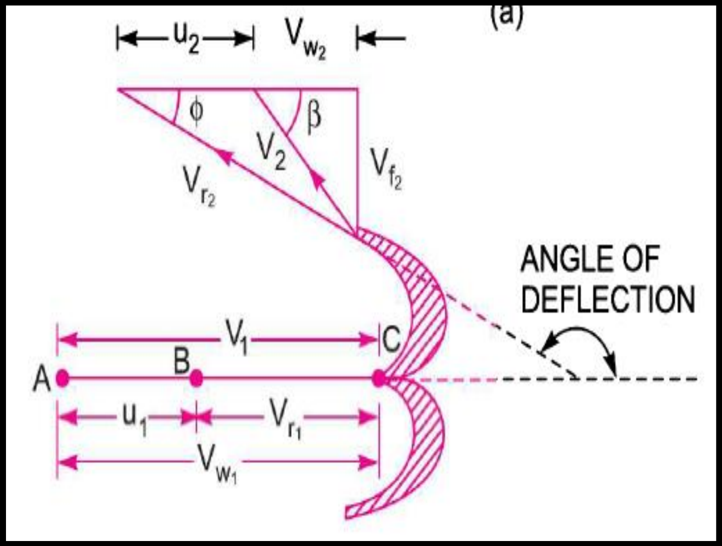
- From the velocity triangle at outlet , we have

$V_{r_2} = V_{r_1}$ and $V_{w_2} = V_{r_2} cos\emptyset - u_2$

- The force exerted by the jet of water in the direction of motion is given

$$F_x = \rho a V_1 [V_{w_1} + V_{w_2}]$$

- As the angle β is an acute angle, +ve sign should be taken



Velocity Triangles and Work done for Pelton Wheel(Cont…)

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- The mass of water striking is $\rho a V_1$ and not $\rho a V_{r_1}$

$$a = \text{Area of jet} = \frac{\pi}{4} d^2$$

- Now work done by the jet on the runner per second

$$= F_x \, X u = \rho a V_1 \left[V_{w_1} + V_{w_2}\right] X u \text{ Nm/s}$$

- Power given to the runner by the jet

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] X u}{1000} \text{kW}$$

Velocity Triangles and Work done for Pelton Wheel(Cont…)

- Work done/s per unit weight of water striking/s

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] X u}{Weight\ of\ water\ striking/s}$$

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] X u}{\rho a V_1 X g} = \frac{1}{g}\left[V_{w_1} + V_{w_2}\right] X u$$

- The energy supplied to the jet at inlet is in the form of K.E $=\frac{1}{2} m V^2$

K.E of jet per second $= \frac{1}{2}(\rho a V_1){V_1}^2$

Velocity Triangles and Work done for Pelton Wheel(Cont…)

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$$\textbf{Hydraulic Efficiency, } \eta_h = \frac{Work\ done\ per\ second}{K.E\ of\ jet\ per\ second}$$

$$= \frac{\rho a V_1 [V_{w_1} + V_{w_2}] X u}{\frac{1}{2}(\rho a V_1) {V_1}^2} = \frac{2[V_{w_1} + V_{w_2}] X u}{{V_1}^2}$$

- Now $\quad V_{w_1} = V_1, V_{r_1} = V_1 - u_1 = (V_1 - u)$

$$V_{r_2} = (V_1 - u)$$

$$V_{w_2} = V_{r_2} cos\phi - u_2 = V_{r_2} cos\phi - u = (V_1 - u) cos\phi - u$$

Velocity Triangles and Work done for Pelton Wheel(Cont…)

- Substituting the values of V_{w_1} and V_{w_2} in $eq^n(\frac{2[V_{w_1}+V_{w_2}]Xu}{{V_1}^2})$

$$\eta_h = \frac{2[V_1 + (V_1 - u)cos\phi - u]Xu}{{V_1}^2}$$

$$= \frac{2[V_1 - u + (V_1 - u)cos\phi]Xu}{{V_1}^2} = \frac{2(V_1 - u)[1 + cos\phi]u}{{V_1}^2}$$

The efficiency will be maximum for the given value of V_1 when

Velocity Triangles and Work done for Pelton Wheel(Cont…)

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$$\frac{d}{du}(\eta_h) = 0$$

$$\frac{d}{du}\left[\frac{2u(V_1 - u)(1 + cos\phi)}{{V_1}^2}\right] = 0$$

$$\frac{(1 + cos\phi)}{{V_1}^2}\frac{d}{du}(2uV_1 - 2u^2) = 0$$

$$\frac{d}{du}(2uV_1 - 2u^2) = 0 \quad \left(\frac{1+cos\phi}{{V_1}^2} \neq 0\right)$$

Velocity Triangles and Work done for Pelton Wheel(Cont…)

$$2V_1 - 4u = 0, \text{u}=\frac{V_1}{2}$$

- Thus hydraulic efficiency of pelton wheel will be maximum when the velocity of the wheel is half the velocity of the jet of the water at inlet

$$\boldsymbol{\eta_{h,max}} = \frac{2\left(V_1 - \frac{V_1}{2}\right)[1+cos\phi]X\frac{V_1}{2}}{{V_1}^2}$$

$$\boldsymbol{\eta_{h,max}} = \frac{2\frac{V_1}{2}[1+cos\phi]\frac{V_1}{2}}{{V_1}^2} = \frac{[1+cos\phi]}{2}$$

Points to be Remembered for Pelton Wheel

(i) The velocity of the jet at inlet $V_1 = C_v\sqrt{2gH}$

Where C_v =Co-efficient of velocity= 0.98 or 0.99, H= Net head on turbine

(ii) The velocity of wheel u= $\phi\sqrt{2gH}$

Where ϕ = Speed ratio= o.43 to 0.48

(iii) The angle of deflection of the jet through buckets is taken at 165^0 if no angle of deflection is given

Points to be Remembered for Pelton Wheel

(iv) The mean diameter of pitch diameter D of the pelton wheel is given by

$$u = \frac{\pi D N}{60} \text{ or } D = \frac{60u}{\pi N}$$

(v) **Jet Ratio(m):** Defined as the ratio of the pitch diameter(D) of the pelton wheel to the diameter of the jet(d) , m= $\frac{D}{d}$ $(= 12\ for\ most\ cases)$

(vi) Number of buckets on a runner Z=15+ $\frac{D}{2d} = 15 + o.5\text{m}$

(vii) **Number of Jets:** It is obtained by dividing the total rate of flow through the turbine by the rate of flow of water through a single jet

Problem:1

A Pelton wheel has a mean bucket speed of 10 metres per second with a jet of water flowing at the rate of 700 litres/s under a head of 30 metres. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume co-efficient of velocity as 0.98.

Solution. Given :

Speed of bucket, $u = u_1 = u_2 = 10$ m/s

Discharge, $Q = 700$ litres/s $= 0.7$ m^3/s, Head of water, $H = 30$ m

Angle of deflection $= 160°$

$\therefore$ Angle, $\phi = 180° - 160° = 20°$

Co-efficient of velocity, $C_v = 0.98$.

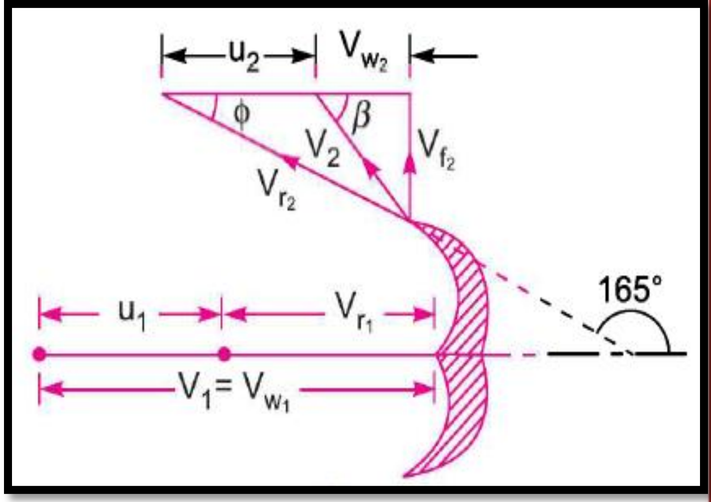
The velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 30} = 23.77$ m/s

Problem:1(Cont…)

$$V_{r_1} = V_1 - u_1 = 23.77 - 10$$
$$= 13.77 \text{ m/s}$$
$$V_{w_1} = V_1 = 23.77 \text{ m/s}$$

From outlet velocity triangle,

$$V_{r_2} = V_{r_1} = 13.77 \text{ m/s}$$
$$V_{w_2} = V_{r_2} \cos \phi - u_2$$
$$= 13.77 \cos 20^\circ - 10.0 = 2.94 \text{ m/s}$$



Problem:1(Cont…)

Work done by the jet per second on the runner is given by equation

$$= \rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u$$

$$= 1000 \times 0.7 \times [23.77 + 2.94] \times 10$$

$$= 186970 \text{ Nm/s} \qquad (\because \quad aV_1 = Q = 0.7 \text{ m}^3\text{/s})$$

$\therefore$ Power given to turbine $= \dfrac{186970}{1000} =$ **186.97 kW. Ans.**

The hydraulic efficiency of the turbine is given by equation

$$\eta_h = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} = \frac{2\left[23.77 + 2.94\right] \times 10}{23.77 \times 23.77}$$

$= \mathbf{0.9454}$ or **94.54%. Ans.**

Problem:2

Problem 18.2 *A Pelton wheel is to be designed for the following specifications :*
Shaft power = 11,772 kW ; Head = 380 metres ; Speed = 750 r.p.m. ; Overall efficiency = 86% ; Jet diameter is not to exceed one-sixth of the wheel diameter. Determine :

(i) *The wheel diameter,* (ii) *The number of jets required, and*

(iii) *Diameter of the jet.*

Take $K_{v_1} = 0.985$ *and* $K_{u_1} = 0.45$

Solution. Given :

Shaft power,	S.P. = 11,772 kW
Head ,	$H = 380$ m
Speed,	$N = 750$ r.p.m.

Problem:2(Cont…)

Overall efficiency, $\eta_0 = 86\%$ or 0.86

Ratio of jet dia. to wheel dia. $= \dfrac{d}{D} = \dfrac{1}{6}$

Co-efficient of velocity, $K_{v_1} = C_v = 0.985$

Speed ratio, $K_{u_1} = 0.45$

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.985\sqrt{2 \times 9.81 \times 380} = 85.05$ m/s

The velocity of wheel, $u = u_1 = u_2$

$= \text{Speed ratio} \times \sqrt{2gH} = 0.45 \times \sqrt{2 \times 9.81 \times 380} = 38.85$ m/s

Problem:2(Cont...)

But $$u = \frac{\pi D N}{60} \quad \therefore \quad 38.85 = \frac{\pi D N}{60}$$

$$D = \frac{60 \times 38.85}{\pi \times N} = \frac{60 \times 38.85}{\pi \times 750} = \mathbf{0.989\ m.\ Ans.}$$

But $$\frac{d}{D} = \frac{1}{6}$$

$\therefore$ Dia. of jet, $$d = \frac{1}{6} \times \text{D} = \frac{0.989}{6} = \mathbf{0.165\ m.\ Ans.}$$

Problem:2(Cont…)

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Discharge of one jet, q = Area of jet × Velocity of jet

$$= \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4}(.165) \times 85.05 \text{ m}^3/\text{s} = 1.818 \text{ m}^3/\text{s}$$

Now

$$\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{11772}{\frac{\rho g \times Q \times H}{1000}}$$

$$0.86 = \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 380}, \text{ where } Q = \text{Total discharge}$$

Problem:2(Cont...)

$\therefore$ Total discharge, $Q = \dfrac{11772 \times 1000}{1000 \times 9.81 \times 380 \times 0.86} = 3.672 \text{ m}^3\text{/s}$

$\therefore$ Number of jets $= \dfrac{\text{Total discharge}}{\text{Discharge of one jet}} = \dfrac{Q}{q} = \dfrac{3.672}{1.818} =$ **2 jets. Ans.**

Design of Pelton Wheel

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- Design of pelton wheel means the following data is to be determined

1. Diameter of the jet(d)
2. Diameter of wheel(D)
3. Width of the buckets which is =5Xd
4. Depth of the buckets on the wheel=1.2Xd
5. Number of buckets on the wheel

Size of buckets means the width and depth of the buckets

Problem:3

A Pelton wheel is to be designed for a head of 60 m when running at 200 r.p.m. The Pelton wheel develops 95.6475 kW shaft power. The velocity of the buckets = 0.45 times the velocity of the jet, overall efficiency = 0.85 and co-efficient of the velocity is equal to 0.98.

Solution. Given :

Head,	$H = 60$ m
Speed	$N = 200$ r.p.m
Shaft power,	S.P. = 95.6475 kW
Velocity of bucket,	$u = 0.45 \times$ Velocity of jet
Overall efficiency,	$\eta_o = 0.85$
Co-efficient of velocity,	$C_v = 0.98$

Problem:3(Cont…)

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Design of Pelton wheel means to find diameter of jet (d), diameter of wheel (D), Width and depth of buckets and number of buckets on the wheel.

(*i*) *Velocity of jet,* $V_1 = C_v \times \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 60} = 33.62$ m/s

$\therefore$ Bucket velocity, $u = u_1 = u_2 = 0.45 \times V_1 = 0.45 \times 33.62 = 15.13$ m/s

But $u = \dfrac{\pi D N}{60}$, where D = Diameter of wheel

$$\therefore \quad 15.13 = \frac{\pi \times D \times 200}{60} \quad \text{or} \quad D = \frac{60 \times 15.13}{\pi \times 200} = \mathbf{1.44\ m.\ Ans.}$$

Problem:3(Cont...)

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(ii) *Diameter of the jet* (d)

Overall efficiency $\eta_o = 0.85$

But $$\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{95.6475}{\left(\dfrac{\text{W.P.}}{1000}\right)} = \frac{95.6475 \times 1000}{\rho \times g \times Q \times H} \qquad (\because \quad \text{W.P.} = \rho g Q H)$$

$$= \frac{95.6475 \times 1000}{1000 \times 9.81 \times Q \times 60}$$

$\therefore$ $$Q = \frac{95.6475 \times 1000}{\eta_o \times 1000 \times 9.81 \times 60} = \frac{95.6475 \times 1000}{0.85 \times 1000 \times 9.81 \times 60} = 0.1912 \text{ m}^3/\text{s}.$$

Problem:3(Cont…)

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But the discharge, Q = Area of jet × Velocity of jet

$$\therefore \quad 0.1912 = \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4} d^2 \times 33.62$$

$$\therefore \quad d = \sqrt{\frac{4 \times 0.1912}{\pi \times 33.62}} = 0.085 \text{ m} = \mathbf{85 \text{ mm. Ans.}}$$

(iii) *Size of buckets*

Width of buckets $= 5 \times d = 5 \times 85 = 425$ mm

Depth of buckets $= 1.2 \times d = 1.2 \times 85 =$ **102 mm. Ans.**

(iv) *Number of buckets on the wheel* is given by equation (18.17) as

$$Z = 15 + \frac{D}{2d} = 15 + \frac{1.44}{2 \times .085} = 15 + 8.5 = \mathbf{23.5 \text{ say } 24. \text{ Ans.}}$$

Summary

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- The pelton turbine is a tangential flow impulse turbine
- The main parts of the pelton turbine are: Nozzle and flow regulating arrangement(Spear), Runner and buckets, Casing , Breaking Jet
- Now work done by the jet on the runner per second $= \rho a V_1 \left[V_{w_1} + V_{w_2}\right] X u$ Nm/s
- Power given to the runner by the jet $= \frac{\rho a V_1 [V_{w_1} + V_{w_2}] X u}{1000}$ kW
- $\boldsymbol{\eta_h} = \frac{2(V_1 - u)[1 + cos\phi]u}{{V_1}^2}$
- $\boldsymbol{\eta_{h,max}} = \frac{[1 + cos\phi]}{2}$

RADIAL FLOW REACTION TURBINES:PART-1

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GIET(A)

Lecture Details:

Unit-IV (Hydraulic Turbines) , Radial flow reaction turbines: Part-1, FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
- FLUID DYNAMICS
- CLOSED CONDUIT FLOW
- BOUNDARY LAYER THEORY AND APPLICATIONS
- BASICS OF TURBO MACHINERY
- HYDRAULIC TURBINES
- PERFORMANCE OF HYDRAULIC TURBINES
- CENTRIFUGAL PUMPS
- RECIPROCATING PUMPS

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RADIAL FLOW REACTION TURBINES

Contents

- Introduction to Radial Flow Reaction Turbines
- Main parts of the Radial Flow Reaction Turbine
- Inward Radial Flow Turbine
- Degree of Reaction
- Important Definitions for Reaction Radial Flow Turbine
- Summary

Introduction to Radial Flow Reaction Turbines

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- Water flows in the radial direction
- Water may flow radially from outwards to inwards(i.e., towards the axis of rotation) or from inwards to outwards
- If water flows from outwards to inwards through the runner, the turbine is known as inward radial flow turbine
- If the water flows from inwards to outwards, the turbine is known as outward radial flow turbine

Introduction to Radial Flow Reaction Turbines(Cont…)

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- Reaction turbine means that the water at the inlet of the turbine possesses K.E as well as pressure energy
- As the water flows through the runner, a part of pressure energy goes on changing into K.E

Main parts of the Radial Flow Reaction Turbine

1.Casing

2.Guide Mechanism

3.Runner

4.Draft tube

Main parts of the Radial Flow Reaction Turbine(Cont…)

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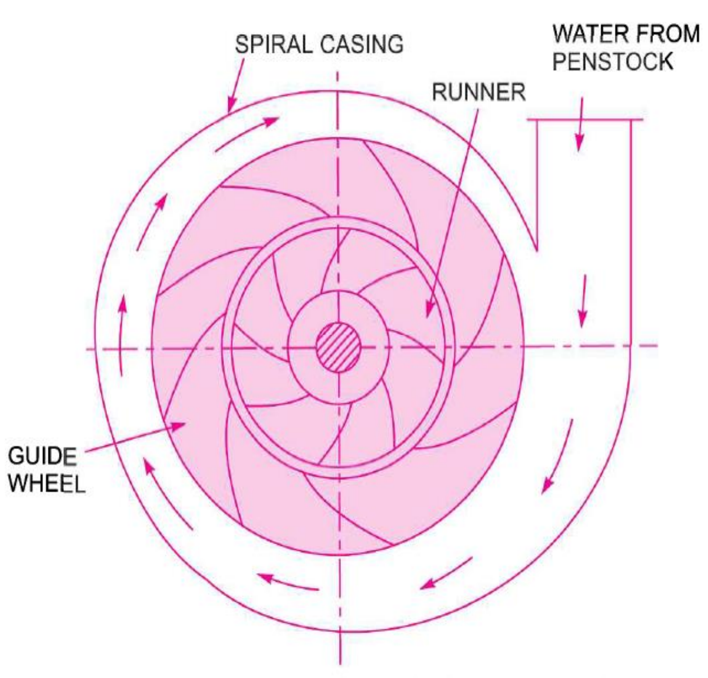
1. Casing:

- Casing and runner are always full of water
- Spiral in shape , area of cross section goes on decreasing gradually
- Casing completely surrounds the runner of the turbine
- Made of concrete, cast steel or plate steel



Main parts of a radial reaction turbines.

Main parts of the Radial Flow Reaction Turbine(Cont...)

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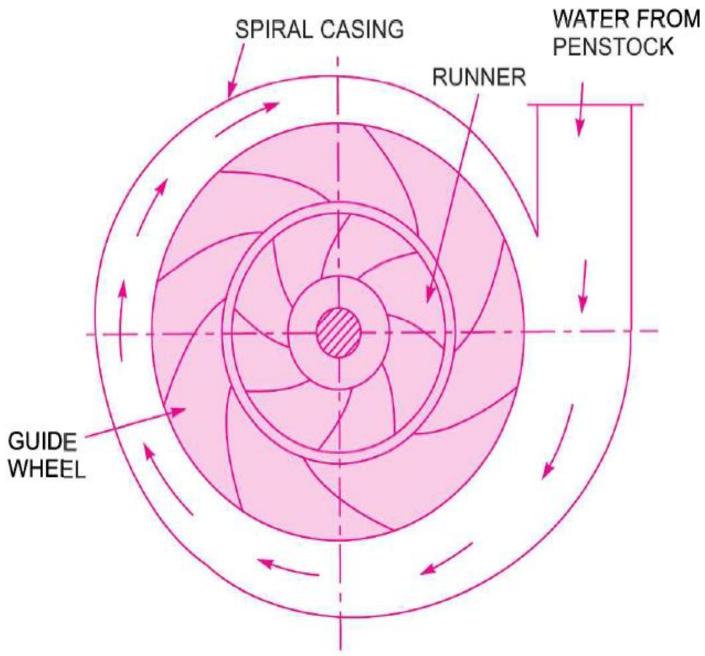
2. Guide Mechanism:

- Consists of a stationary circular wheel all round the runner of the turbine
- The stationary guide vanes are fixed on the guide mechanism
- The guide vanes allow the water to strike the vanes fixed on the runner without shock at inlet

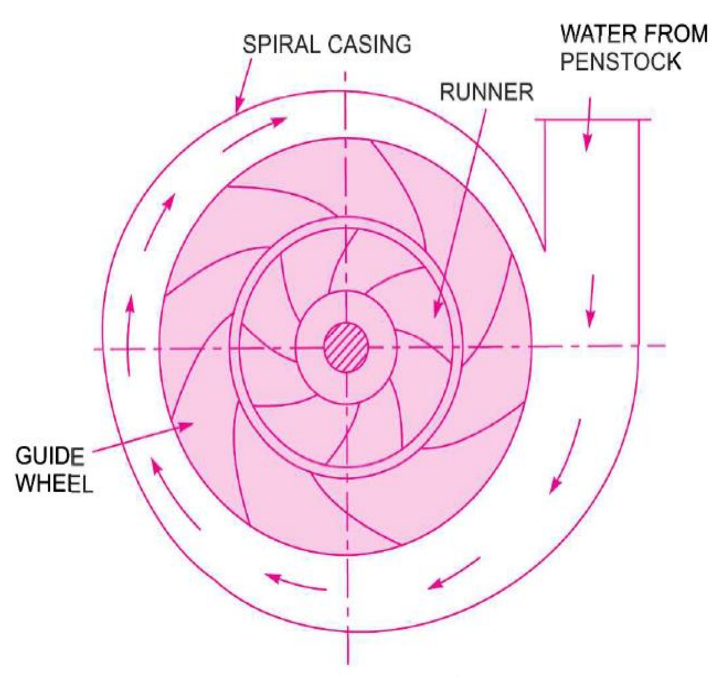


Main parts of a radial reaction turbines.

Main parts of the Radial Flow Reaction Turbine(Cont...)

3. Runner:

- It is a circular wheel on which a series of radial curved vanes are fixed
- The radial curved vanes are so shaped that the water enters and leaves the runner without shock
- Made of cast steel, cast iron or stainless steel



Main parts of a radial reaction turbines.

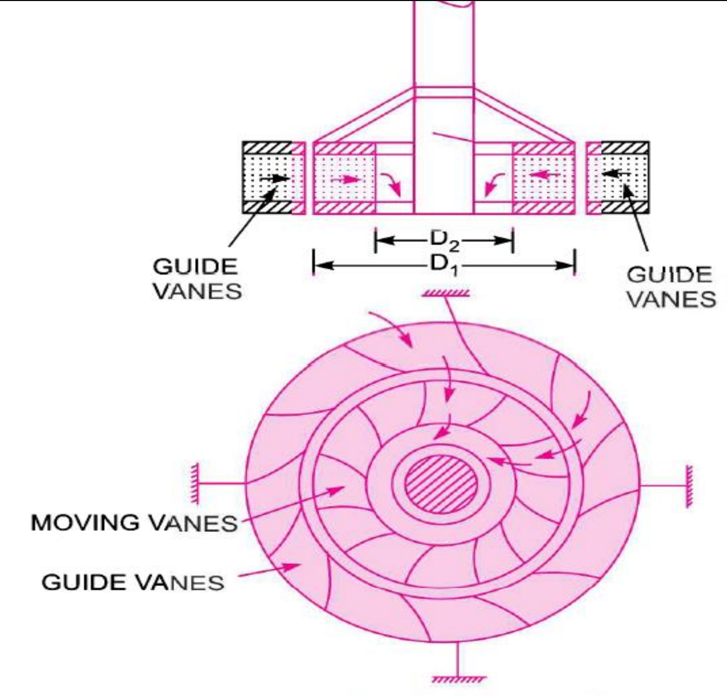
Main parts of the Radial Flow Reaction Turbine(Cont...)

4. Draft -tube:

- The pressure at the exit of the runner of a reaction turbine is generally less than atmospheric pressure
- The water at exit cannot be directly discharged to the tail race
- A tube or pipe of gradually increasing area is used for discharging water from the exit of the turbine to the tail race
- This tube of increasing area is called draft tube

Inward Radial Flow Turbine

- The water from the casing enters the stationary guiding wheel
- The guiding wheel consists of guide vanes which direct the water to enter the runner
- The water flows over the moving vanes in the inward radial direction and is discharged at the inner diameter of the runner



Inward radial flow turbine.

Inward Radial Flow Turbine(Cont…)

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➢ The work done per second on the runner by water is given by:

$$= \rho a V_1 \left[V_{w_1} u_1 \pm V_{w_2} u_2 \right]$$

$$= \rho Q \left[V_{w_1} u_1 \pm V_{w_2} u_2 \right]$$

Where $u_1 (Tangential\ velocity\ of\ wheel\ at\ inlet) = \frac{\pi D_1 X N}{60}$, D_1 = Outer dia. of runner

$u_2 (Tangential\ velocity\ of\ wheel\ at\ outlet) = \frac{\pi D_2 X N}{60}$, D_2 = Inner dia. of runner, N= Speed of the turbine in r.p.m

Inward Radial Flow Turbine(Cont...)

- The work done per second per unit weight of water per second

$$= \frac{Work\ done\ per\ second}{Weight\ of\ water\ striking\ per\ second}$$

$$= \frac{\rho Q[V_{w_1}u_1 \pm V_{w_2}u_2]}{\rho Q g} = \frac{1}{g}[V_{w_1}u_1 \pm V_{w_2}u_2] \text{-----}(1)$$

- Eq(1) is known as **Euler's Equation or Fundamental Equation** of hydrodynamics machines
- In eq(1) +ve sign is taken if angle β is an acute angle, -ve sign is taken if angle β is an obtuse angle

Inward Radial Flow Turbine(Cont...)

- If $\beta = 90^0$, then $V_{w_1} = 0$ and word done per second per unit weight striking/s become as$= \frac{1}{g} V_{w_1} u_1$

Hydraulic Efficiency,$\boldsymbol{\eta_h} = \frac{\boldsymbol{R.P}}{\boldsymbol{W.P}} = \frac{\frac{\boldsymbol{W}}{\boldsymbol{1000g}}[V_{w_1}u_1 \pm V_{w_2}u_2]}{\frac{\boldsymbol{WXH}}{\boldsymbol{1000}}} = \frac{[V_{w_1}u_1 \pm V_{w_2}u_2]}{\boldsymbol{gH}}$

Where R.P= Runner power i.e power delivered by water to the runne

W.P= Water power

If the discharge is radial at outlet, then $V_{w_2} = 0$, $\boldsymbol{\eta_h} = \frac{V_{w_1}u_1}{\boldsymbol{gH}}$

Degree of Reaction(R)

$$R = \frac{Change\ of\ pressure\ energy\ inside\ the\ runner}{Change\ of\ total\ energy\ inside\ the\ runner}$$

$$R = 1 - \frac{({V_1}^2 - {V_1}^2)}{2gH_e}$$

Where H_e =Change of total energy per unit weight inside the runner

$$H_e = \frac{1}{g}[V_{w_1}u_1 \pm V_{w_2}u_2]$$

Degree of Reaction(Cont...)

Value of R for Pelton Turbine and other Actual Reaction Turbines

(i) For a Pelton turbine

$$u_1 = u_2 \text{ and } V_{r_2} = V_{r_1}$$

$$R = 1 - \frac{(V_1{}^2 - V_2{}^2)}{(V_1{}^2 - V_2{}^2)} = 1 - 1 = 0$$

(ii) For an actual reaction turbine, the angle $\beta = 90^0$, so that the loss of K.E at outlet is minimum (i.e., V_2 is minimum)

Hence in outlet velocity triangle, $V_{w_2} = 0$, $R = 1 - \frac{cot\alpha}{2(cot\alpha - cot\theta)}$

Important Definitions for Reaction Radial Flow Turbine

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The following terms are generally used in case of reaction radial flow turbines :

(i) **<u>Speed Ratio</u>:** $= \frac{u_1}{\sqrt{2gH}}$, $u_1 =$ Tangential velocity of wheel at inlet

(ii) **<u>Flow Ratio</u>:** The ratio of the velocity of flow at inlet(V_{f_1}) to the velocity given $\sqrt{2gH}$, Flow ration= $\frac{V_{f_1}}{\sqrt{2gH}}$, H= head on turbine

Important Definitions for Reaction Radial Flow Turbine(Cont...)

(iii) <u>Discharge of the turbine (Q)</u> $= \pi D_1 B_1 X V_{f_1} = \pi D_2 B_2 X V_{f_2}$

Where D_1 =Diameter of runner at inlet, B_1 =Width of runner at inlet

V_{f_1} =Velocity of flow at inlet, D_2, B_2, V_{f_2} =Corresponding values at outlet

- If the thickness of vanes are taken into consideration, then area through which flow takes place is given by $(\pi D_1 - nXt)$

Where n= number of vanes on runner and t= thickness of each vane

The discharge Q=$(\pi D_1 - nXt)XB_1XV_{f_1}$

Important Definitions for Reaction Radial Flow Turbine(Cont...)

(iv) **<u>The Head(H) on the turbine:</u>** $H = \frac{p_1}{\rho X g} + \frac{{V_1}^2}{2g}$

(v) **<u>Radial Discharge:</u>** This means the angle made by absolute velocity with the tangent on the wheel is$(\beta =)\ 90^0$ and the component of the whirl velocity $(V_{w_1} =)$is zero.

(vi) If there is no loss of energy when water flows through the vanes then we have $\mathrm{H} - \frac{{V_2}^2}{2g} = \frac{1}{g}[V_{w_1}u_1 \pm V_{w_2}u_2]$

Problem:1

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An inward flow reaction turbine has external and internal diameters as 1 m and 0.5 m respectively. The velocity of flow through the runner is constant and is equal to 1.5 m/s. Determine :

(i) Discharge through the runner, and

(ii) Width of the turbine at outlet if the width of the turbine at inlet = 200 mm.

Solution. Given :

External diameter of turbine, $D_1 = 1$ m

Internal diameter of turbine, $D_2 = 0.5$ m

Velocity of flow at inlet and outlet, $V_{f_1} = V_{f_2} = 1.5$ m/s

Width of turbine at inlet, $B_1 = 200$ mm $= 0.20$ m

Let the width at outlet $= B_2$

Problem:1(Cont…)

$$Q = \pi D_1 B_1 \times V_{f_1} = \pi \times 1 \times 0.20 \times 1.5 = \mathbf{0.9425\ m^3/s.\ Ans.}$$

$$\pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2} \text{ or } D_1 B_1 = D_2 B_2 \qquad \left(\because \quad \pi V_{f_1} = \pi V_{f_2}\right)$$

$$B_2 = \frac{D_1 \times B_1}{D_2} = \frac{1 \times 0.20}{0.5} = 0.40 \text{ m} = \mathbf{400\ mm.\ Ans.}$$

Problem:2

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Problem 18.15 *An inward flow reaction turbine has external and internal diameters as 0.9 m and 0.45 m respectively. The turbine is running at 200 r.p.m. and width of turbine at inlet is 200 mm. The velocity of flow through the runner is constant and is equal to 1.8 m/s. The guide blades make an angle of 10° to the tangent of the wheel and the discharge at the outlet of the turbine is radial. Draw the inlet and outlet velocity triangles and determine:*

(i) *The absolute velocity of water at inlet of runner,*
(ii) *The velocity of whirl at inlet,* (iii) *The relative velocity at inlet,*
(iv) *The runner blade angles,* (v) *Width of the runner at outlet,*
(vi) *Mass of water flowing through the runner per second,*
(vii) *Head at the inlet of the turbine,*
(viii) *Power developed and hydraulic efficiency of the turbine.*

Problem:2(Cont…)

Solution. Given :

External Dia., $D_1 = 0.9$ m

Internal Dia., $D_2 = 0.45$ m

Speed, $N = 200$ r.p.m.

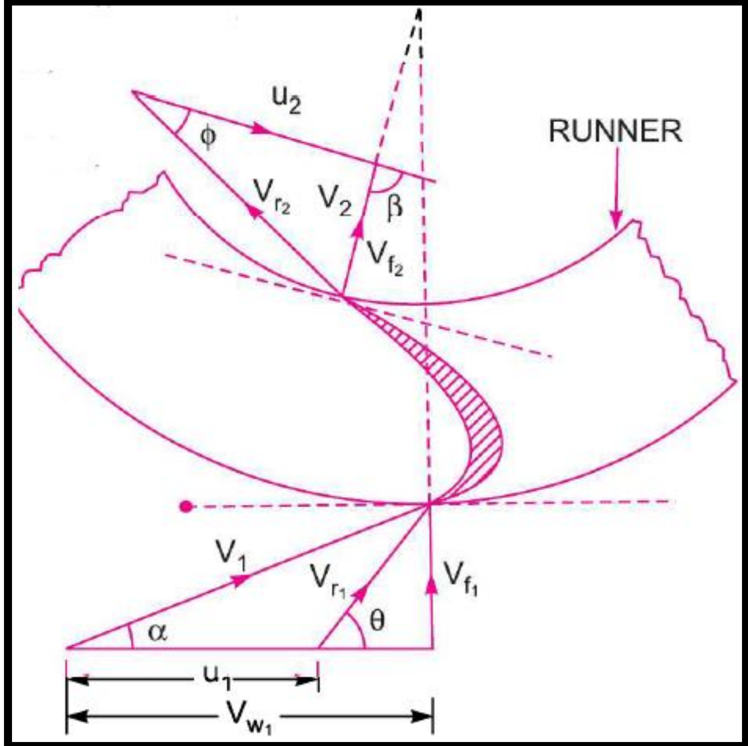
Width at inlet, $B_1 = 200$ mm $= 0.2$ m

Velocity of flow, $V_{f_1} = V_{f_2} = 1.8$ m/s

Guide blade angle, $\alpha = 10°$

Discharge at outlet = Radial

$\therefore$ $\beta = 90°$ and $V_{w_2} = 0$



Problem:2(Cont…)

Tangential velocity of wheel at inlet and outlet are :

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times .9 \times 200}{60} = 9.424 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times .45 \times 200}{60} = 4.712 \text{ m/s.}$$

(i) Absolute velocity of water at inlet of the runner i.e., V_1

From inlet velocity triangle,

$$V_1 \sin \alpha = V_{f_1}$$

$$\therefore \quad V_1 = \frac{V_{f_1}}{\sin \alpha} = \frac{18}{\sin 10^\circ} = \mathbf{10.365 \text{ m/s. Ans.}}$$

Problem:2(Cont…)

(ii) *Velocity of whirl at inlet, i.e.,* V_{w_1}

$$V_{w_1} = V_1 \cos\alpha = 10.365 \times \cos 10^\circ = \mathbf{10.207\ m/s.\ Ans.}$$

(iii) *Relative velocity at inlet, i.e.,* V_{r_1}

$$V_{r_1} = \sqrt{{V_{r_1}}^3 + (V_{w_1} - u_1)^2} = \sqrt{1.8^2 + (10.207 - 9.424)^2}$$

$$= \sqrt{3.24 + .613} = \mathbf{1.963\ m/s.\ Ans.}$$

(iv) *The runner blade angles* means the angle θ and ϕ

Now $$\tan\theta = \frac{V_{f_1}}{(V_{w_1} - u_1)} = \frac{1.8}{(10.207 - 9.424)} = 2.298$$

$\therefore$ $$\theta = \tan^{-1} 2.298 = \mathbf{66.48^\circ \text{ or } 66^\circ\ 29'.\ Ans.}$$

Problem:2(Cont...)

From outlet velocity triangle, we have

$$\tan\phi = \frac{V_{f_2}}{u_2} = \frac{1.8}{4.712} = \tan 20.9^\circ$$

$$\therefore \qquad \phi = \mathbf{20.9^\circ} \quad \text{or} \quad \mathbf{20^\circ\ 54.4'.\ Ans.}$$

(v) *Width of runner at outlet, i.e., B_2*

From equation

$$\pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2} \text{ or } D_1 B_1 = D_2 B_2 \qquad \left(\because \quad \pi V_{f_1} = \pi V_{f_2} \text{ as } V_{f_1} = V_{f_2}\right)$$

$$\therefore \qquad B_2 = \frac{D_1 B_1}{D_2} = \frac{0.90 \times 0.20}{0.45} = 0.40 \text{ m} = \mathbf{400\ mm.\ Ans.}$$

Problem:2(Cont…)

(vi) *Mass of water flowing through the runner per second.*

The discharge, $Q = \pi D_1 B_1 V_{f_1} = \pi \times 0.9 \times 0.20 \times 1.8 = 1.0178 \text{ m}^3\text{/s.}$

$\therefore$ $\text{Mass} = \rho \times Q = 1000 \times 1.0178 \text{ kg/s} = \mathbf{1017.8 \text{ kg/s. Ans.}}$

(vii) *Head at the inlet of turbine, i.e., H.*

Using equation

$$H - \frac{V_2^2}{2g} = \frac{1}{g}(V_{w_1}u_1 \pm V_{w_2}u_2) = \frac{1}{g}(V_{w_1}u_1) \qquad (\because \text{ Here } V_{w_2} = 0)$$

$$H = \frac{1}{g}V_{w_1}u_1 + \frac{V_2^2}{2g} = \frac{1}{9.81} \times 10.207 \times 9.424 + \frac{1.8^2}{2 \times 9.81} \quad (\because V_2 = V_{f_2})$$

$$= 9.805 + 0.165 = \mathbf{9.97 \text{ m. Ans.}}$$

Problem:2(Cont...)

(viii) *Power developed, i.e.,* $P = \dfrac{\text{Work done per second on runner}}{1000}$

$$= \frac{\rho Q\left[V_{w_1} u_1\right]}{1000}$$

$$= 1000 \times \frac{1.0178 \times 10.207 \times 9.424}{1000} = \mathbf{97.9\ kW.\ Ans.}$$

Hydraulic efficiency is given by equation (18.20*B*) as

$$\eta_h = \frac{V_{w_1} u_1}{gH} = \frac{10.207 \times 9.424}{9.81 \times 9.97} = 0.9834 = \mathbf{98.34\%.\ Ans.}$$

Summary

- If water flows from outwards to inwards through the runner, the turbine is known as outward radial flow turbine
- If the water flows from inwards to outwards, the turbine is known as outward radial flow turbine
- The work done per second per unit weight of water per second$= \frac{1}{g}\left[V_{w_1}u_1 \pm V_{w_2}u_2\right]$
- **Hydraulic Efficiency,** $\boldsymbol{\eta_h = \frac{\left[V_{w_1}u_1 \pm V_{w_2}u_2\right]}{gH}}$
- **Degree of Reaction** $R = 1 - \frac{(V_1{}^2 - V_1{}^2)}{2gH_e}$

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RADIAL FLOW REACTION TURBINES:PART-2

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Lecture Details:
Unit-IV (Hydraulic Turbines) , Radial flow reaction turbines: Part-2, FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
  - FLUID KINEMATICS
  - FLUID DYNAMICS
  - CLOSED CONDUIT FLOW
  - BOUNDARY LAYER THEORY AND APPLICATIONS
  - BASICS OF TURBO MACHINERY
  - HYDRAULIC TURBINES
  - PERFORMANCE OF HYDRAULIC TURBINES
  - CENTRIFUGAL PUMPS
  - RECIPROCATING PUMPS

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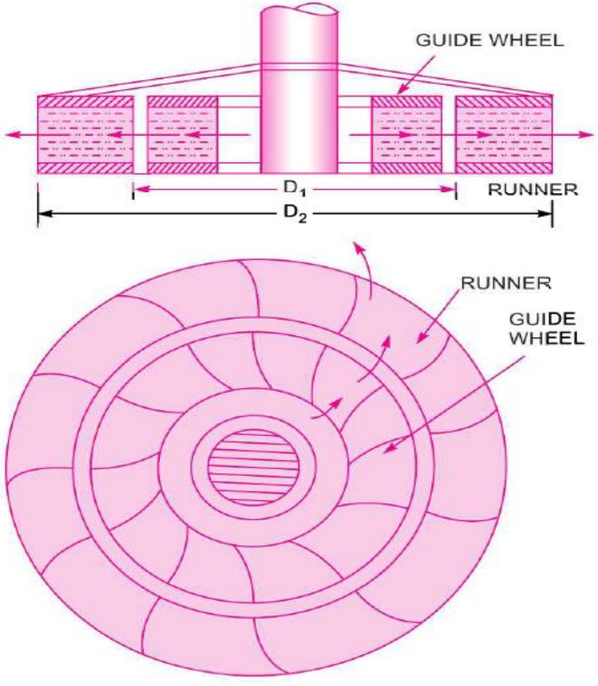
RADIAL FLOW REACTION TURBINES:PART-2

Contents

- **Outward Radial Flow Reaction Turbine**
- **Francis Turbine**
- **Important Relations for Francis Turbines**
- **Summary**

Outward Radial Flow Reaction Turbine

- Water from casing enters the stationary guide wheel
- The guide wheel consists of guide vanes which direct water to enter the runner which is around the stationary guide wheel
- The water flows through the vanes of the runner in the outward radial direction and is discharged at the outer diameter of the runner

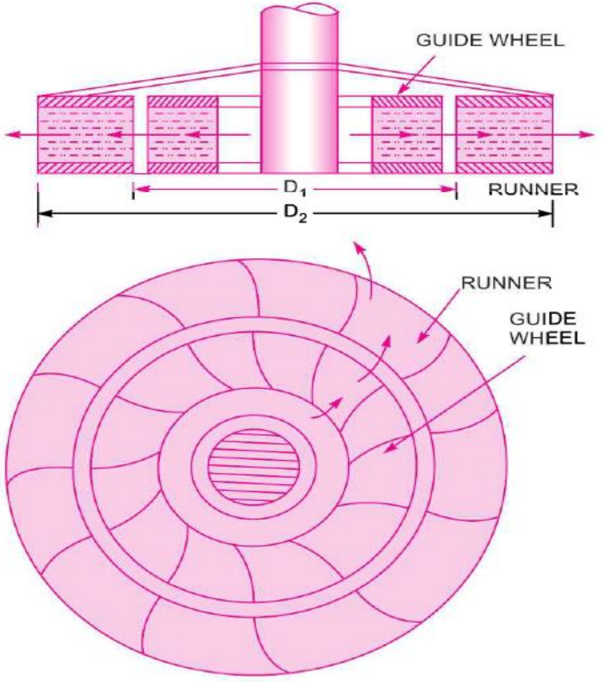


Outward radial flow turbine.

Outward Radial Flow Reaction Turbine(Cont…)

- The inner diameter of the runner is inlet and outlet diameter is the outlet
- In this case as inlet of the runner is at the inner diameter of the runner, the tangential velocity at inlet will be less than that of at outlet i.e

$$u_1 < u_2 \text{ as } D_1 < D_2$$



Outward radial flow turbine.

Problem:1

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An outward flow reaction turbine has internal and external diameters of the runner as 0.6 m and 1.2 m respectively. The guide blade angle is 15° and velocity of flow through the runner is constant and equal to 4 m/s. If the speed of the turbine is 200 r.p.m., head on the turbine is 10 m and discharge at outlet is radial, determine :

(i) The runner vane angles at inlet and outlet,

(ii) Work done by the water on the runner per second per unit weight of water striking per second ,

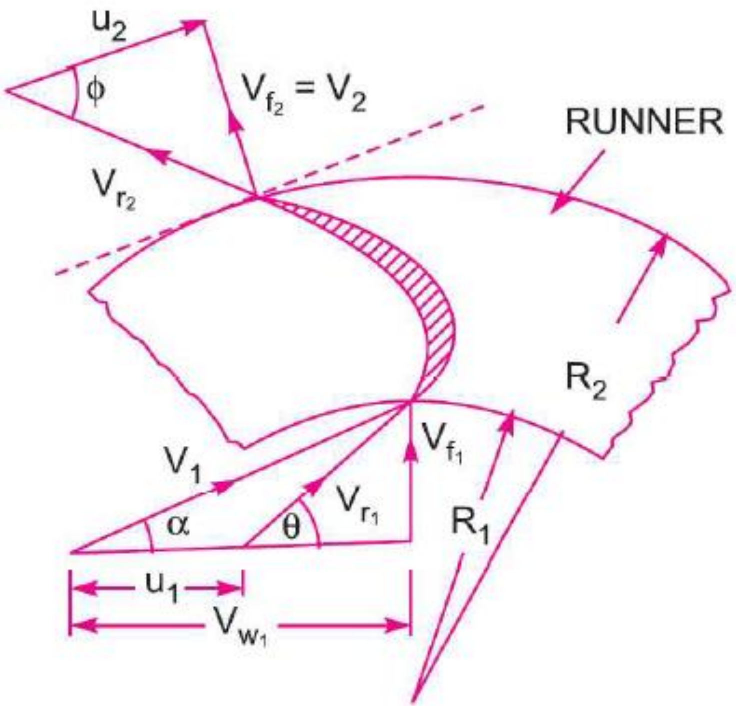
(iii) Hydraulic efficiency, and

(iv) The degree of reaction.

Problem:1(Cont…)

Solution. Given :

Internal diameter,	$D_1 = 0.6$ m
External diameter,	$D_2 = 1.2$ m
Guide blade angle,	$\alpha = 15°$
Velocity of flow,	$V_{f_1} = V_{f_2} = 4$ m/s
Speed,	$N = 200$ r.p.m.
Head,	$H = 10$ m
Discharge at outlet	= Radial
$\therefore$	$V_{w_2} = 0,\ V_{f_2} = V_2$



Problem:1(Cont…)

Tangential velocity of runner at inlet and outlet are :

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.6 \times 200}{60} = 6.283 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.2 \times 200}{60} = 12.566 \text{ m/s.}$$

From the inlet velocity triangle, $\tan \alpha = \dfrac{V_{f_1}}{V_{w_1}}$

$$\therefore \qquad V_{w_1} = \frac{V_{f_1}}{\tan \alpha} = \frac{4.0}{\tan 15^\circ} = 14.928 \text{ m/s.}$$

Problem:1(Cont…)

(*i*) *Runner Vane Angles at inlet and outlet* are θ and ϕ

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{4.0}{(14.928 - 6.283)} = 0.4627$$

$$\theta = \tan^{-1} .4627 = 24.83 \text{ or } \mathbf{24° \ 49.8'}. \ \textbf{Ans.}$$

From outlet velocity triangle, $\tan\phi = \dfrac{V_{f_2}}{u_2} = \dfrac{4.0}{12.566} = 0.3183$

$$\therefore \quad \phi = \tan^{-1} .3183 = \mathbf{17.65°} \text{ or } \mathbf{17° \ 39.4'}. \ \textbf{Ans.}$$

Problem:1(Cont...)

(ii) *Work done by water per second per unit weight of water striking per second*

$$= \frac{1}{g} V_{w_1} u_1 \qquad (\because \; V_{w_2} = 0)$$

$$= \frac{1}{9.81} \times 14.928 \times 6.283 = \mathbf{9.561\ Nm/N.\ Ans.}$$

(iii) *Hydraulic efficiency* is given by equation

$$\eta_h = \frac{V_{w_1} u_1}{gH} = \frac{14.928 \times 6.283}{9.81 \times 10} = \mathbf{0.9561} \text{ or } \mathbf{95.61\%.\ Ans.}$$

Problem:1(Cont…)

(*iv*) Given : In this question, the velocity of flow is constant through the runner (*i.e.*, $V_{f_1} = V_{f_2}$) and the discharge is radial at outlet (*i.e.*, $\beta = 90°$ or $V_{w_2} = 0$), the degree of reaction (R) is given by equation

$$R = 1 - \frac{\cot\alpha}{2(\cot\alpha - \cot\theta)}$$

Here $\alpha = 13.928°$ and $\theta = 41.09°$ (calculated)

Substituting the value of α and θ, we get

$$R = 1 - \frac{\cot 13.928°}{2(\cot 13.928° - \cot 41.09°)} = 1 - \frac{4.032}{2(4.032 - 1.146)}$$

$$= 1 - 0.698 = 0.302 \simeq \mathbf{0.3.\ Ans.}$$

For Francis turbine, the degree of reaction varies from 0 to 1 *i.e.*, $0 \le R \le 1$.

Francis Turbine

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- The inward flow reaction turbine having radial discharge at outlet is known as Francis Turbine
- In the modern Francis turbine, the water enters the runner of the turbine in the radial direction at outlet and leaves in the axial direction at the inlet of the runner
- Thus the modern Francis turbine is a mixed flow type turbine
- The velocity triangle at inlet and outlet of the Francis turbine are drawn in the same way as in case of inward flow reaction turbine

Francis Turbine(Cont...)

- The discharge is radial at outlet, the velocity of whirl at outlet(i.e V_{w_2}) will be zero
- Hence the work done by water on the runner per second will be

$$= \rho Q [V_{w_1} u_1]$$

- And work done per second per unit weight of water striking/s= $\frac{1}{g}[V_{w_1} u_1]$
- Hydraulic efficiency will be given by , $\eta_h = \frac{V_{w_1} u_1}{gH}$

Important Relations for Francis Turbines

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1. The ratio of width of the wheel to its diameter is given as n= $\frac{B_1}{D_1}$. The value of n varies from 0.1 to 0.4

2. The flow ratio $= \frac{V_{f_1}}{\sqrt{2gH}}$ and varies from 0.15 to 0.3

3. The speed ratio= $\frac{u_1}{\sqrt{2gH}}$ varies from 0.6 to 0.9

Problem:2

A Francis turbine with an overall efficiency of 75% is required to produce 148.25 kW power. It is working under a head of 7.62 m. The peripheral velocity = $0.26\sqrt{2gH}$ and the radial velocity of flow at inlet is $0.96\sqrt{2gH}$. The wheel runs at 150 r.p.m. and the hydraulic losses in the turbine are 22% of the available energy. Assuming radial discharge, determine :

(i) The guide blade angle, (ii) The wheel vane angle at inlet,

(iii) Diameter of the wheel at inlet, and (iv) Width of the wheel at inlet.

Solution. Given :

Overall efficiency $\eta_o = 75\% = 0.75$

Power produced, S.P. = 148.25 kW

Head, $H = 7.62$ m

Problem:2(Cont…)

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Peripheral velocity, $u_1 = 0.26\sqrt{2gH} = 0.26 \times \sqrt{2 \times 9.81 \times 7.62} = 3.179$ m/s

Velocity of flow at inlet, $V_{f_1} = 0.96\sqrt{2gH} = 0.96 \times \sqrt{2 \times 9.81 \times 7.62} = 11.738$ m/s.

Speed, $N = 150$ r.p.m.

Hydraulic losses = 22% of available energy

Discharge at outlet = Radial

$$V_{w_2} = 0 \text{ and } V_{f_2} = V_2$$

Hydraulic efficiency is given as

$$\eta_h = \frac{\text{Total head at inlet} - \text{Hydraulic loss}}{\text{Head at inlet}}$$

Problem:2(Cont…)

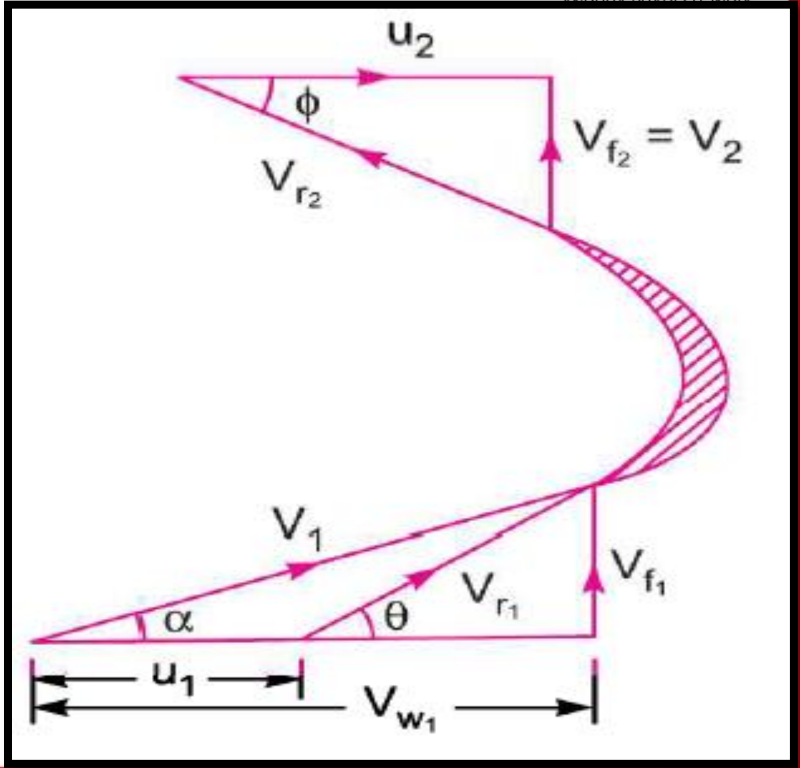
$$= \frac{H - .22\,H}{H} = \frac{0.78\,H}{H} = 0.78$$

$$\eta_h = \frac{V_{w_1} u_1}{gH}$$

$$\frac{V_{w_1} u_1}{gH} = 0.78$$

$$V_{w_1} = \frac{0.78 \times g \times H}{u_1}$$

$$= \frac{0.78 \times 9.81 \times 7.62}{3.179} = 18.34 \text{ m/s.}$$



Problem:2(Cont…)

(*i*) *The guide blade angle, i.e.*, α. From inlet velocity triangle,

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{11.738}{18.34} = 0.64$$

$$\therefore \qquad \alpha = \tan^{-1} 0.64 = \mathbf{32.619^\circ \text{ or } 32^\circ\ 37'.\ Ans.}$$

(*ii*) *The wheel vane angle at inlet, i.e.*, θ

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{11.738}{18.34 - 3.179} = 0.774$$

$$\therefore \qquad \theta = \tan^{-1} .774 = \mathbf{37.74 \text{ or } 37^\circ\ 44.4'.\ Ans.}$$

Problem:2(Cont…)

(iii) *Diameter of wheel at inlet* (D_1).

Using the relation, $$u_1 = \frac{\pi D_1 N}{60}$$

$$D_1 = \frac{60 \times u_1}{\pi \times N} = \frac{60 \times 3.179}{\pi \times 50} = \mathbf{0.4047\ m.\ Ans.}$$

(iv) *Width of the wheel at inlet* (B_1)

$$\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{148.25}{\text{W.P.}}$$

But $$\text{W.P.} = \frac{\text{WH}}{1000} = \frac{\rho \times g \times Q \times H}{1000} = \frac{1000 \times 9.81 \times Q \times 7.62}{1000}$$

Problem:2(Cont…)

$$\eta_o = \frac{148.25}{\frac{1000 \times 9.81 \times Q \times 7.62}{1000}} = \frac{148.25 \times 1000}{1000 \times 9.81 \times Q \times 7.62}$$

$$Q = \frac{148.25 \times 1000}{1000 \times 9.81 \times 7.62 \times \eta_o} = \frac{148.25 \times 1000}{1000 \times 9.81 \times 7.62 \times 0.75} = 2.644 \text{ m}^3/\text{s}$$

$$Q = \pi D_1 \times B_1 \times V_{f_1}$$

$$2.644 = \pi \times .4047 \times B_1 \times 11.738$$

$$B_1 = \frac{2.644}{\pi \times .4047 \times 11.738} = \mathbf{0.177 \text{ m. Ans.}}$$

Summary

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- For Outward Radial Flow Reaction Turbine , the water flows through the vanes of the runner in the outward radial direction and is discharged at the outer diameter of the runner
- For Outward Radial Flow Reaction Turbine , $u_1 < u_2$ as $D_1 < D_2$
- The inward flow reaction turbine having radial discharge at outlet is known as Francis Turbine
- For Francis turbine, the flow ratio $= \frac{V_{f_1}}{\sqrt{2gH}}$ and varies from 0.15 to 0.3
- For Francis turbine, the speed ratio$= \frac{u_1}{\sqrt{2gH}}$ varies from 0.6 to 0.9

AXIAL FLOW REACTION TURBINE

Presented By:

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Lecture Details:

Unit-IV (Hydraulic Turbines) , Axial Flow Reaction Turbine, FM & HM /Mechanical, I -Semester.

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AXIAL FLOW REACTION TURBINE

Contents

- Axial Flow Reaction Turbine
- Some Important Point for Propeller (Kaplan Turbine)
- Summary

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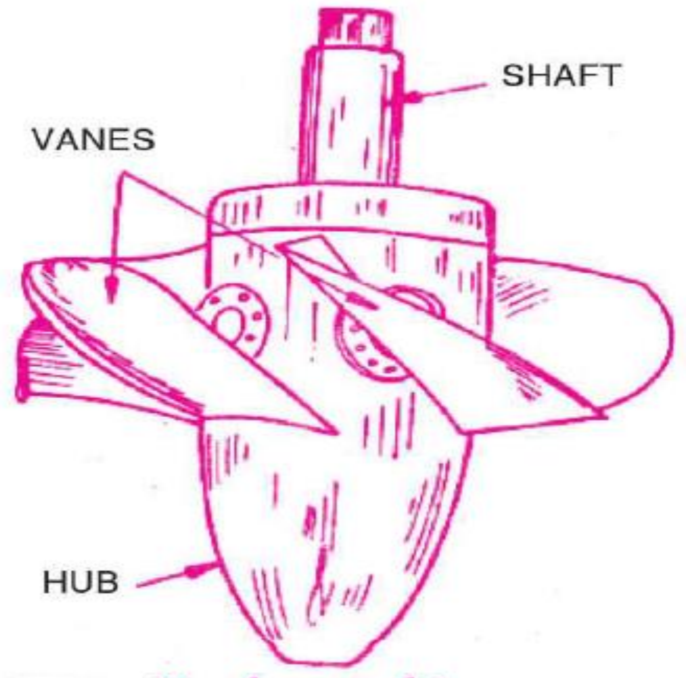
Axial Flow Reaction Turbine

- Water flows parallel to the axis of the rotation of the shaft, the turbine is known as axial flow turbine
- If the head at the inlet of the turbine is the sum of pressure energy and kinetic energy and during the flow of water through runner a part of pressure energy is converted into K.E is known as reaction turbine

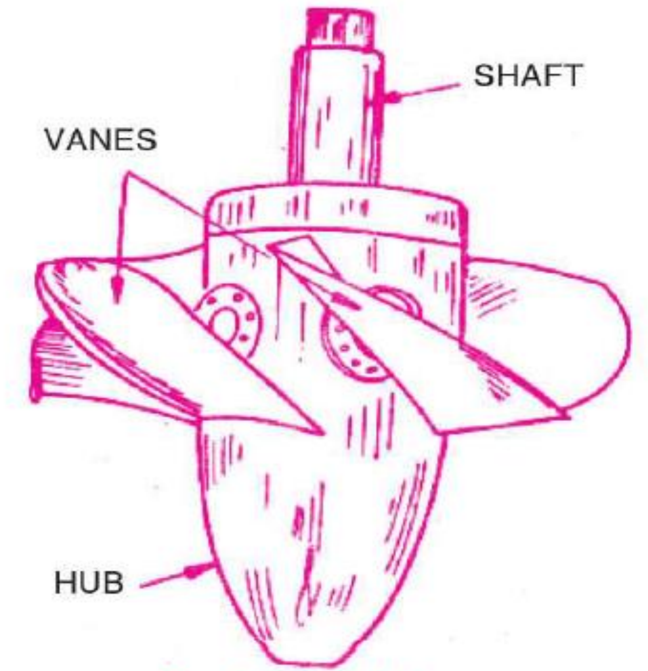


Kaplan turbine runner.

Axial Flow Reaction Turbine(Cont...)

- The shaft of the turbine is vertical
- The lower end of the shaft is made larger which is known as ‘hub’ or ‘boss’
- The vanes are fixed on the hub and hence hub acts as a runner for axial flow reaction turbine
- The following are the important type of axial flow reaction turbines

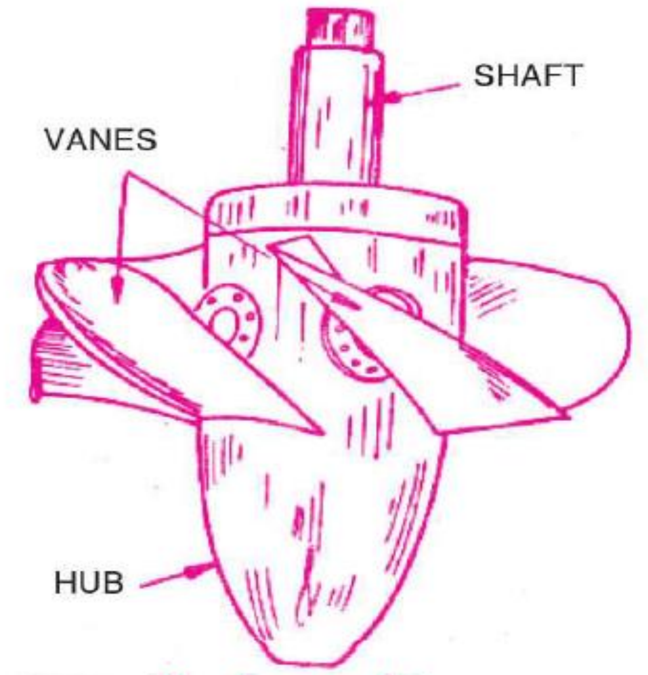
1. Propeller Turbine 2. Kaplan Turbine



Kaplan turbine runner.

Axial Flow Reaction Turbine(Cont…)

- When the vanes are fixed to the hub and they are not adjustable, the turbine is known as propeller turbine
- But if the vanes on the hub are adjustable, the turbine is known as a Kaplan Turbine
- Kaplan turbine is suitable where a large quantity of water at low head is available

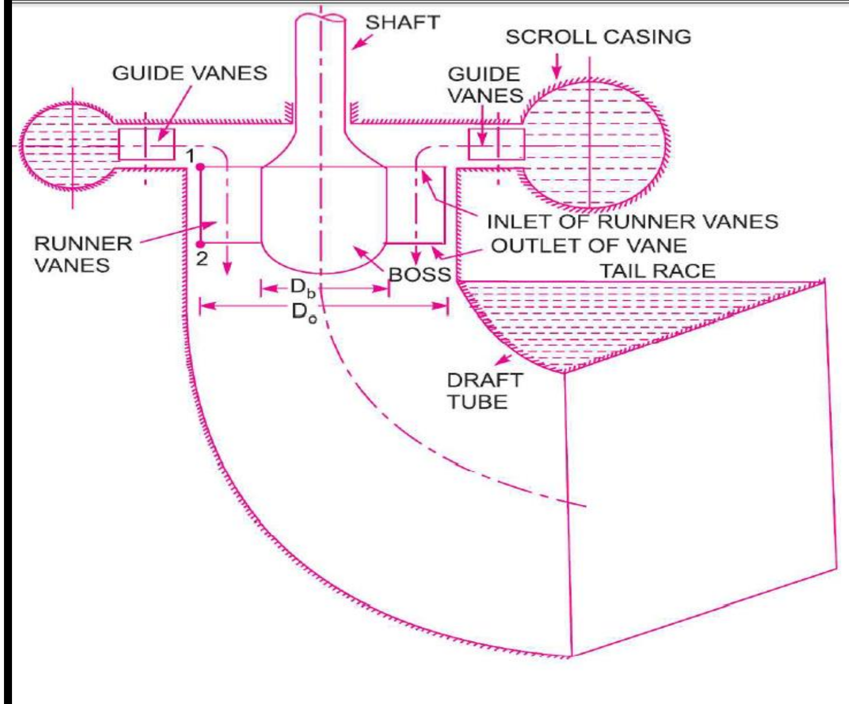


Kaplan turbine runner.

Axial Flow Reaction Turbine(Cont…)

- The main parts of a Kaplan turbine are:

1. Scroll casing
2. Guide vanes mechanism
3. Hub with vanes or runner of the turbine
4. Draft tube



Main components of Kaplan turbine.

Axial Flow Reaction Turbine(Cont...)

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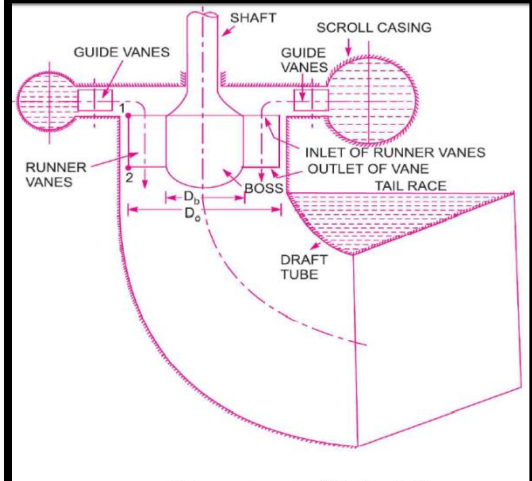
- The water from penstock enters the scroll casing and then moves to the guide vanes
- From the guide vanes, the water turns through 90^0 and flows axially through the runner
- The discharge through the runner is obtained as

$$Q = \frac{\pi}{4}\left({D_0}^2 - {D_b}^3\right) X {V_f}_1$$

Where D_0 =Outer diameter of the runner

, D_b=Diameter of hub ,${V_f}_1$ =Velocity of flow at inlet



Main components of Kaplan turbine.

Some Important Point for Propeller (Kaplan Turbine)

1. The peripheral velocity at inlet and outlet are equal

$u_1 = u_1 = \frac{\pi D_0 N}{60}$, wherer D_0 =Outer dia. of runner

2. Velocity of flow at inlet and outlet are equal

$$V_{f_1} = V_{f_2}$$

3. Area of flow at inlet=Area of flow at outlet

$$= \frac{\pi}{4}\left({D_0}^2 - {D_b}^2\right)$$

Problem:1

A Kaplan turbine working under a head of 20 m develops 11772 kW shaft power. The outer diameter of the runner is 3.5 m and hub diameter is 1.75 m. The guide blade angle at the extreme edge of the runner is 35°. The hydraulic and overall efficiencies of the turbines are 88% and 84% respectively. If the velocity of whirl is zero at outlet, determine :

(i) Runner vane angles at inlet and outlet at the extreme edge of the runner, and

(ii) Speed of the turbine.

Solution. Given :

Head,	$H = 20$ m
Shaft power,	S.P. = 11772 kW
Outer dia. of runner,	$D_o = 3.5$ m
Hub diameter,	$D_b = 1.75$ m

Problem:1(Cont…)

Guide blade angle, $\alpha = 35°$

Hydraulic efficiency, $\eta_h = 88\%$

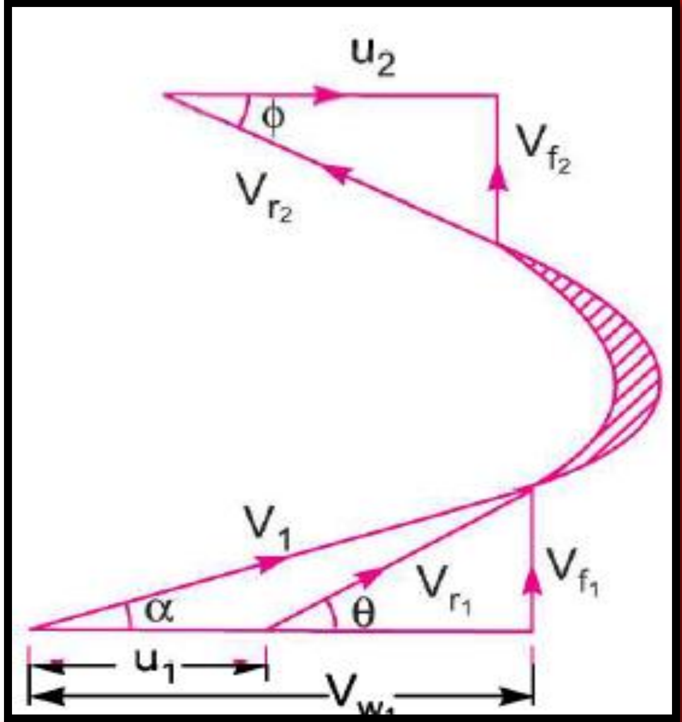
Overall efficiency, $\eta_o = 84\%$

Velocity of whirl at outlet $= 0.$

Using the relation, $\eta_o = \dfrac{\text{S.P.}}{\text{W.P.}}$

where $\text{W.P.} = \dfrac{\text{W.P.}}{1000} = \dfrac{\rho \times g \times Q \times H}{1000}$, we get

$$0.84 = \frac{11772}{\dfrac{\rho \times g \times Q \times H}{1000}}$$



Problem:1(Cont...)

$$= \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 20}$$

$$Q = \frac{11772 \times 1000}{0.84 \times 1000 \times 9.81 \times 20} = 71.428 \text{ m}^3/\text{s}.$$

Using equation

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1}$$

$$71.428 = \frac{\pi}{4}(3.5^2 - 1.75^2) \times V_{f_1} = \frac{\pi}{4}(12.25 - 3.0625)\ V_{f_1}$$

$$= 7.216\ V_{f_1}$$

$$\therefore \quad V_{f_1} = \frac{71.428}{7.216} = 9.9 \text{ m/s}.$$

Problem:1(Cont…)

From inlet velocity triangle, $\tan \alpha = \dfrac{V_{f_2}}{V_{w_1}}$

$$\therefore \qquad V_{w_1} = \frac{V_{f_1}}{\tan \alpha} = \frac{9.9}{\tan 35^\circ} = \frac{9.9}{.7} = 14.14 \text{ m/s}$$

Using the relation for hydraulic efficiency,

$$\eta_h = \frac{V_{w_1} u_1}{gH} \qquad (\because \; V_{w_2} = 0)$$

$$0.88 = \frac{14.14 \times u_1}{9.81 \times 20}$$

$$\therefore \qquad u_1 = \frac{0.88 \times 9.81 \times 20}{14.14} = 12.21 \text{ m/s.}$$

Problem:1(Cont…)

(*i*) Runner vane angles at inlet and outlet at the extreme edge of the runner are given as :

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{9.9}{(14.14 - 12.21)} = 5.13$$

$$\therefore \qquad \theta = \tan^{-1} 5.13 = \mathbf{78.97^\circ} \text{ or } \mathbf{78^\circ\ 58'}. \ \ \textbf{Ans.}$$

For Kaplan turbine, $u_1 = u_2 = 12.21$ m/s and $V_{f_1} = V_{f_2} = 9.9$ m/s

$\therefore$ From outlet velocity triangle, $\tan\phi = \dfrac{V_{f_2}}{u_2} = \dfrac{9.9}{12.21} = 0.811$

$$\therefore \qquad \phi = \tan^{-1}.811 = \mathbf{39.035^\circ} \text{ or } \mathbf{39^\circ\ 2'}. \ \ \textbf{Ans.}$$

Problem:1(Cont…)

(*ii*) Speed of turbine is given by $u_1 = u_2 = \dfrac{\pi D_o N}{60}$

$$12.21 = \frac{\pi \times 3.5 \times N}{60}$$

$$\therefore \quad N = \frac{60 \times 12.21}{\pi \times 3.50} = \mathbf{66.63\ r.p.m.}\ \ \textbf{Ans.}$$

Summary

- Water flows parallel to the axis of the rotation of the shaft, the turbine is known as axial flow turbine
- When the vanes are fixed to the hub and they are not adjustable, the turbine is known as propeller turbine
- If the vanes on the hub are adjustable, the turbine is known as a Kaplan Turbine
- The discharge through the runner is obtained as $Q = \frac{\pi}{4}\left(D_0{}^2 - D_b{}^3\right) X V_{f_1}$

DRAFT -TUBE

Presented By:
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Lecture Details:
Unit-IV (Hydraulic Turbines) , Draft- Tube, FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
- FLUID KINEMATICS
- FLUID DYNAMICS
- CLOSED CONDUIT FLOW
- BOUNDARY LAYER THEORY AND APPLICATIONS
- BASICS OF TURBO MACHINERY
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- CENTRIFUGAL PUMPS
- RECIPROCATING PUMPS

DRAFT -TUBE

Contents

Draft-Tube

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- The draft-tube is a pipe of gradually increasing area which connects the outlet of the runner to the tail race
- It is used for discharging water from the exit of the turbine to the tail race
- The draft tube

1. Discharges water
2. It permits a negative head to be established at the outlet of the runner and thereby increase the net head on the turbine

Draft-Tube(Cont…)

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3. It converts a large proportion of the K.E rejected at the outlet of the turbine into useful pressure energy. Without draft tube, K.E rejected at the outlet of the turbine will go waste to the tail race

- Hence by using draft-tube, the net head on the turbine increases
- The turbine develops more power and also the efficiency of the turbine increases
- If a reaction turbine is not fitted with a draft –tube, the pressure at the outlet of the runner will be equal to atmospheric pressure

Draft-Tube(Cont...)

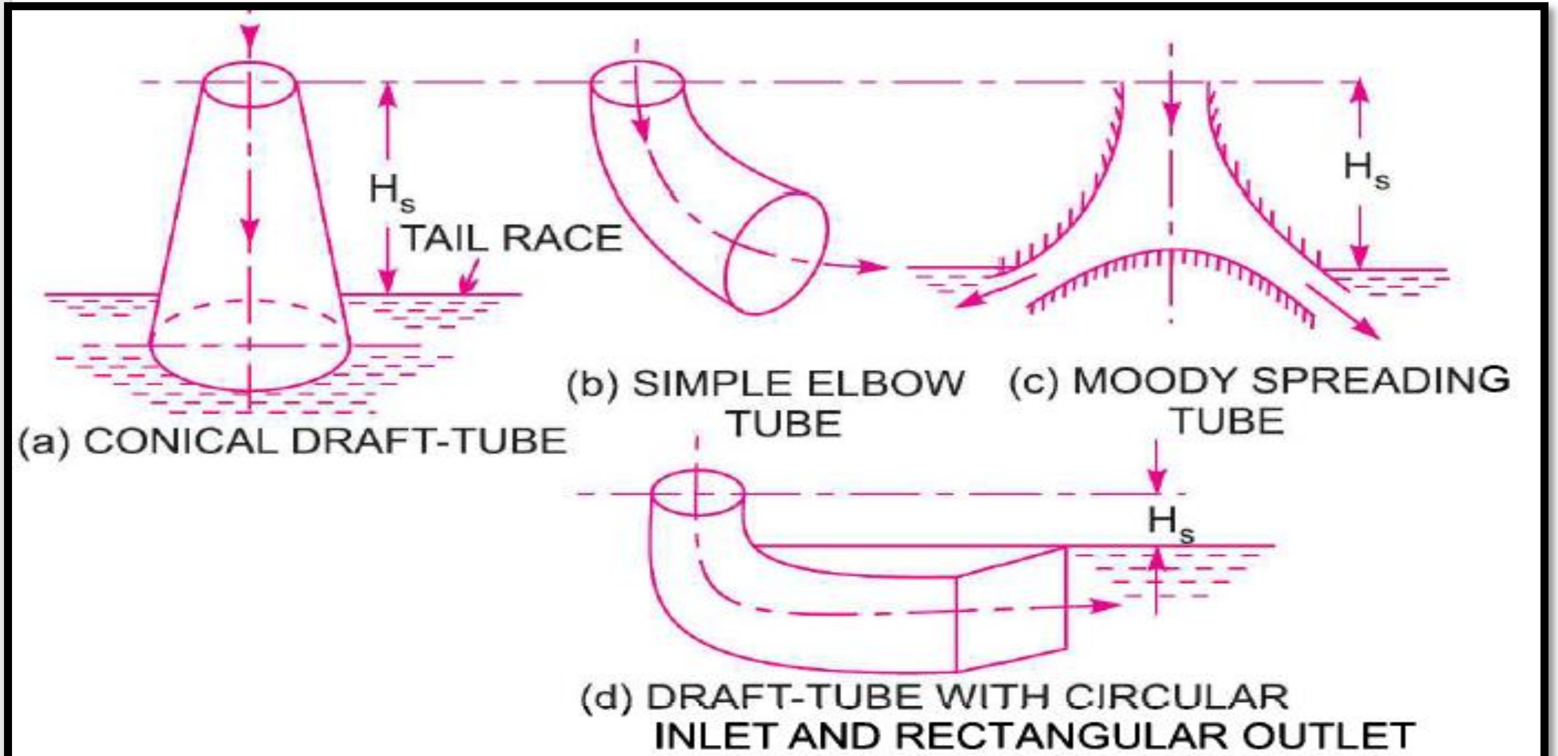
- The water from the outlet of the runner will discharge freely into the tail race
- The net head on the turbine will be less than that of a reaction turbine fitted with a draft-tube
- Without a draft tube, the K.E rejected at the outlet of the runner will go waste to the tail race

Types of Draft-Tubes

Types of Draft-Tubes

- 1. Conical Draft-Tubes
- 2. Simple Elbow Tubes
- 3. Moody Spreading Tubes
- 4.Elbow Draft-Tubes with Circular Inlet and Rectangular Outlet

Types of Draft-Tubes(Cont…)



Types of draft-tubes.

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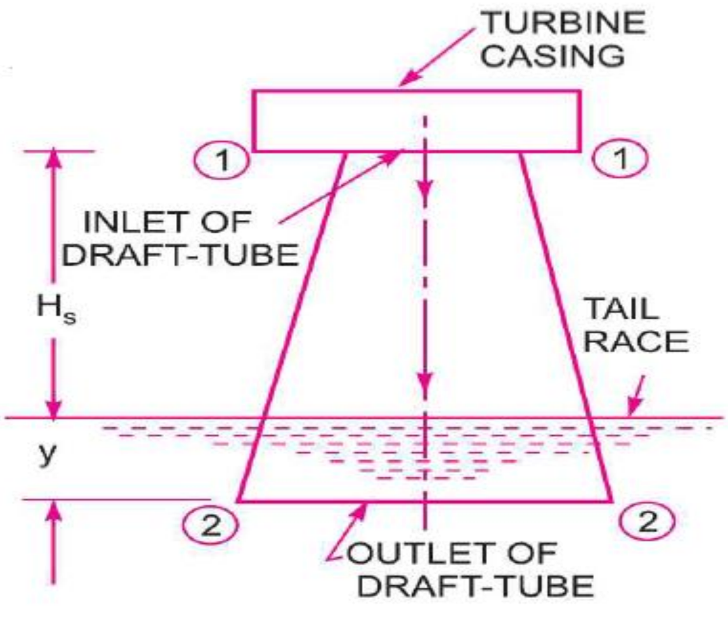
Draft-Tube Theory

- Let H_s = Vertical height of draft tube above the tail race

y= Distance of bottom of draft tube from tail race

- Applying bernoulli's equation to inlet (section 1-1) and outlet(section 2-2) of the draft tube
- Taking section 2-2 as the datum line



Draft-tube theory.

Draft-Tube Theory(Cont…)

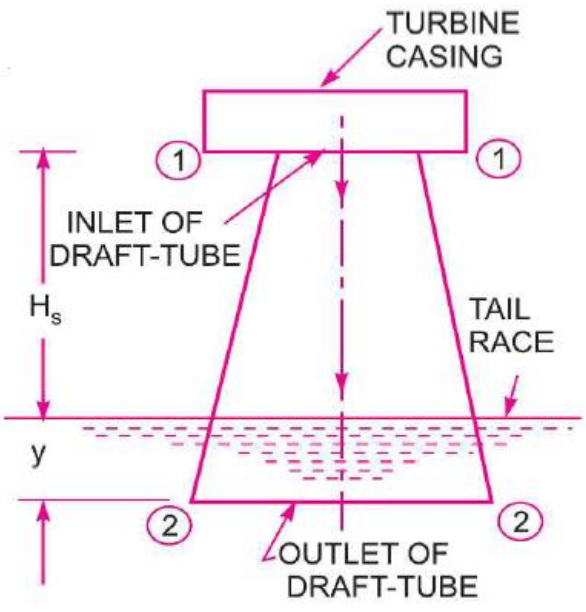
$$\frac{p_1}{\rho g}+\frac{{V_1}^2}{2g}+(H_s+y)=\frac{p_2}{\rho g}+\frac{{V_2}^2}{2g}+0+h_f \text{.........}(1)$$

- Where h_f =loss of energy between section 1-1 and 2-2

But $\frac{p_2}{\rho g}$ =Atmospheric pressure +y

$$\frac{p_2}{\rho g}=\frac{p_a}{\rho g}+y$$

Substituting $\frac{p_2}{\rho g}$ value in equation (1)



Draft-tube theory.

Draft-Tube Theory(Cont…)

$$\frac{p_1}{\rho g}+\frac{{V_1}^2}{2g}+(H_s+y)=\frac{p_a}{\rho g}+y+\frac{{V_2}^2}{2g}+h_f$$

$$\frac{p_1}{\rho g}+\frac{{V_1}^2}{2g}+H_s=\frac{p_a}{\rho g}+\frac{{V_2}^2}{2g}+h_f$$

$$\frac{p_1}{\rho g}=\frac{p_a}{\rho g}-H_s-\left(\frac{{V_1}^2}{2g}-\frac{{V_2}^2}{2g}-h_f\right)$$

Efficiency of Draft-Tube

$$\eta_d = \frac{Actual\ conversion\ of\ kinetic\ head\ into\ pressure\ head}{Kinetic\ head\ at\ the\ inlet\ of\ draft\ tube}$$

Let V_1 =Velocity of water at inlet of draft tube

V_2 =Velocity of water at outlet of draft tube

h_f =Loss of head in the draft tube

➢ Theoretical conversion of kinetic head into pressure head in draft tube= $\left(\frac{{V_1}^2}{2g} - \frac{{V_2}^2}{2g}\right)$

Efficiency of Draft-Tube(Cont…)

- Actual conversion of kinetic head into pressure head $= \left(\frac{{V_1}^2}{2g} - \frac{{V_2}^2}{2g}\right) - h_f$

$$\eta_d = \frac{\left(\frac{{V_1}^2}{2g} - \frac{{V_2}^2}{2g}\right) - h_f}{\frac{{V_1}^2}{2g}}$$

Problem:1

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A water turbine has a velocity of 6 m/s at the entrance to the draft-tube and a velocity of 1.2 m/s at the exit. For friction losses of 0.1 m and a tail water 5 m below the entrance to the draft-tube, find the pressure head at the entrance.

Solution. Given :

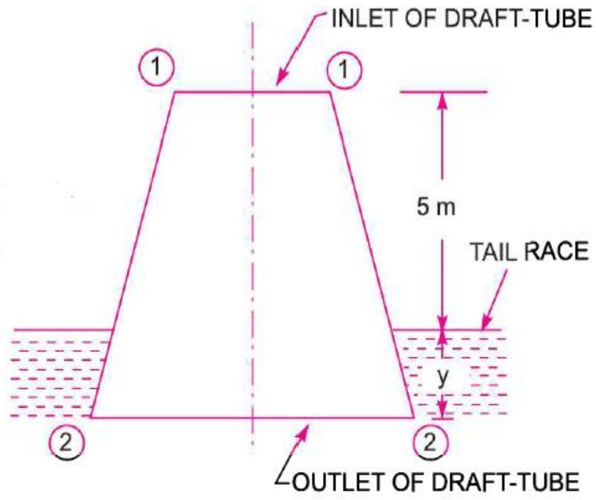
Velocity at inlet, $V_1 = 6$ m/s

Velocity at outlet, $V_2 = 1.2$ m/s

Friction loss, $h_f = 0.1$ m

Vertical height between tail race and inlet of draft-tube = 5 m

Let y = Vertical height between tail race and outlet of draft-tube.



Problem:1(Cont…)

Applying Bernoulli's equation at the inlet and outlet of draft-tube and taking reference line passing through section (2-2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{\rho g} + Z_2 + h_f$$

where $Z_1 = (5 + y)$; $V_1 = 6$ m/s ; $V_2 = 1.2$ m/s,
$h_f = 0.1$

$$\frac{p_2}{\rho g} = \text{Atmospheric pressure head} + y = \frac{p_a}{\rho g} + y$$

$$Z_2 = 0$$

Problem:1(Cont...)

Substituting the values, we get

$$\frac{p_1}{\rho g} + \frac{6^2}{2 \times 9.81} + (5 + y) = \left(\frac{p_a}{\rho g} + y\right) + \frac{1.2^2}{2 \times 9.81} + 0 + 0.1$$

$$\frac{p_1}{\rho g} + 1.835 + 5 + y = \frac{p_a}{\rho g} + y + 0.0734 + 0.1$$

$$\frac{p_1}{\rho g} + 6.835 = \frac{p_a}{\rho g} + 0.1734$$

If $\frac{p_a}{\rho g}$ (*i.e.*, atmospheric pressure head) is taken zero, then we will get $\frac{p_1}{\rho g}$ as vacuum pressure head at inlet of draft-tube.

But if $\frac{p_a}{\rho g}$ = 10.3 m of water, then we will get $\frac{p_1}{\rho g}$ as absolute pressure head at inlet of draft-tube.

Problem:1(Cont…)

Taking $\frac{p_a}{\rho g} = 0$ and substituting this value in equation $\frac{p_1}{\rho g} + 6.835 = \frac{p_a}{\rho g} + 0.1734$

$$\frac{p_1}{\rho g} + 6.835 = 0 + 0.1734$$

$$\therefore \qquad \frac{p_1}{\rho g} = -6.835 + 0.1734 = \mathbf{-6.6616\ m.\ Ans.}$$

Negative sign means vacuum pressure head.

Problem:2

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A conical draft-tube having inlet and outlet diameters 1 m and 1.5 m discharges water at outlet with a velocity of 2.5 m/s. The total length of the draft-tube is 6 m and 1.20 m of the length of draft-tube is immersed in water. If the atmospheric pressure head is 10.3 m of water and loss of head due to friction in the draft-tube is equal to 0.2 × velocity head at outlet of the tube, find :

(i) Pressure head at inlet, and (ii) Efficiency of the draft-tube.

Solution. Given :

Diameter at inlet, $D_1 = 1.0$ m

Diameter at outlet $D_2 = 1.5$ m

Velocity at outlet, $V_2 = 2.5$ m/s

Total length of tube, $H_s + y = 6.0$ m

Problem:2(Cont...)

Length of tube in water, $y = 1.20$ m

$\therefore$ $H_s = 6.0 - 1.20 = 4.80$ m

Atmospheric pressure head, $\dfrac{p_a}{\rho g} = 10.3$ m

Loss of head due to friction, $h_f = 0.2 \times$ Velocity head at outlet

$$= 0.2 \times \frac{V_2^2}{2g}$$

Discharge through tube, $Q = A_2 V_2 = \dfrac{\pi}{4} D_2^2 \times 2.5 = \dfrac{\pi}{4} (1.5)^2 \times 2.5 = 4.4178 \text{ m}^3/\text{s}$

Problem:2(Cont...)

Velocity at inlet, $$V_1 = \frac{Q}{A_1} = \frac{4.4178}{\frac{\pi}{4} \times 1^2} = 5.625 \text{ m/s}$$

(i) *Pressure head at inlet* $\left(\dfrac{p_1}{\rho g}\right)$

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - H_s - \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_f\right)$$

$$= 10.3 - 4.8 - \left(\frac{5.625^2}{2 \times 9.81} - \frac{2.5^2}{2 \times 9.81} - 0.2 \times \frac{V_2^2}{2g}\right)$$

Problem:2(Cont...)

$$= 10.3 - 4.8 - \left(1.6126 - .3185 - \frac{0.2 \times 2.5^2}{2 \times 9.81}\right)$$

$$= 10.3 - 4.8 - (1.6126 - .3185 - .0637) = 5.5 - (1.2304) = 4.269$$

$\simeq$ **4.27 m (abs.) Ans.**

(ii) *Efficiency of Draft-tube* (η_d)

$$\eta_d = \frac{\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - h_f}{\frac{V_1^2}{2g}} = \frac{\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - \frac{0.2\, V_2^2}{2g}}{\frac{V_1^2}{2g}}$$

Problem:2(Cont…)

GLOBAL STANDARDS GIET INSTITUTIONS ANDHRA PRADESH, INDIA

$$= \frac{V_1^2 - 1.2\,V_2^2}{V_1^2} = 1 - 1.2\left(\frac{V_2}{V_1}\right)^2 = 1 - 1.2\left(\frac{2.5}{5.625}\right)^2 = 1 - 0.237$$

$= \mathbf{0.763}$ or $\mathbf{76.3\%}$. **Ans.**

Summary

GLOBAL STANDARDS
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- The draft-tube is a pipe of gradually increasing area which connects the outlet of the runner to the tail race
- Without a draft tube, the K.E rejected at the outlet of the runner will go waste to the tail race
- Efficiency of Draft-Tube $\eta_d = \dfrac{\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - h_f}{\frac{V_1^2}{2g}}$

UNIT QUANTITIES

Presented By:
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Lecture Details:
Unit-IV (Performance of Hydraulic Turbines) , Unit Quantities, FM & HM /Mechanical, I -Semester.

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UNIT QUANTITIES

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Contents

Unit Quantities

- In order to predict the behaviour of a turbine working under varying conditions of head, speed, output and gate opening, the results are expressed in terms of quantities which may be obtained when the head on the turbine is reduced to unity
- The conditions of the turbine under unit head are such that the efficiency of the turbine remains unaffected

Unit Quantities(Cont...)

The three important unit quantities which must be studied under unit head

1. Unit speed

2.Unit discharged

3. Unit power

Unit Speed(N_u)

- Defined as the speed of a turbine working under a unit head(i.e under a head of 1 m)
- Let, N= Speed of a turbine under a head H, H=Head under which a turbine is working, u= Tangential velocity
- The tangential velocity, absolute velocity of water and head on the turbine are related as u$\propto$ V, where V $\propto \sqrt{H}$

Then, u$\propto \sqrt{H}$

- Also tangential velocity(u) $= \frac{\pi DN}{60}$, where D= Diameter of turbine

Unit Speed(N_u)(Cont…)

- For a given turbine, the diameter (D) is constant

$$u \propto N \text{ or } N \propto u \text{ or } N \propto \sqrt{H}$$

$$N = K_1\sqrt{H} ------- -(1)$$

- Where K_1 is constant of proportionality
- If head on the turbine becomes unity, the speed becomes unit speed or

  H=1, N=N_u
- But from eq(1) N= $K_1\sqrt{H}$, $N_u = K_1\sqrt{1} \Rightarrow N_u = K_1$
- Substituting in equation (1) N= $K_1\sqrt{H}$, N= $N_u\sqrt{H} \Rightarrow \boldsymbol{N_u = \frac{N}{\sqrt{H}}}$

Unit Discharge(Q_u)

- Defined as the discharge passing through a turbine, which is working under a unit head(i.e 1m)
- Let H= Head of water on the turbine, Q= Discharge passing through turbine when head is H on the turbine, a=Area of flow of water
- The discharge passing through a given turbine under a head 'H' is

Q=Area of flow X Velocity

- But for a turbine, area of flow is constant and velocity is proportional to $\sqrt{H}$

Unit Discharge(Q_u)(Cont...)

$$Q \propto \text{Velocity} \propto \sqrt{H}$$

$$Q = K_2\sqrt{H} \text{-----} (2)$$

- Where K_2 is constant of proportionality
- If H=1, Q=Q_u, substituting in eq(2)

$$Q_u = K_2\sqrt{1} \Longrightarrow Q_u = K_2$$

- Substituting in eq(2)

$$Q = Q_u\sqrt{H}$$

$$Q_u = \frac{Q}{\sqrt{H}}$$

Unit Power(P_u)

- Defined as the power developed by a turbine, working under a unit head(i.e under a head or 1 m)
- Let H= Head of water on the turbine, P= Power developed by the turbine under a head of H, Q= Discharge through turbine under a head H
- The overall efficiency $(\eta_0) = \frac{Power\ developed}{Water\ power} = \frac{P}{\frac{\rho X g X Q X H}{1000}}$

$$P = \eta_0 X \frac{\rho X g X Q X H}{1000}$$

$$P \propto QXH$$

Unit Power(P_u)(Cont...)

$$\text{But } Q \propto \sqrt{H}$$

$$P \propto \sqrt{H} X H$$

$$P \propto H^{3/2}$$

$$P = K_3 H^{3/2} \text{--------}(3)$$

- Where K_3 is constant of proportionality
- When H=1m , P=$P_u \Rightarrow P_u = K_3(1)^{3/2} \Rightarrow P_u = K_3$
- Substituting the K_3 value in equation (3)

$$P = P_u H^{3/2} \Rightarrow P_u = \frac{P}{H^{3/2}}$$

Use of Unit Quantities(N_u, Q_u, P_u)

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- If a turbine is working under different heads, the behaviour of the turbine can be easily known from the values of the unit quantities, i.e from the values of unit speed, unit discharge and unit power
- Let H_1, H_2…….are the heads under which a turbine works, N_1, N_2……. are the corresponding speeds, Q_1, Q_2……. Are the discharge , P_1, P_2…….are the power developed by the turbine
- But from equations $\boldsymbol{N_u} = \frac{\boldsymbol{N}}{\sqrt{\boldsymbol{H}}}$, $Q_u = \frac{Q}{\sqrt{H}}$, $P_u = \frac{P}{H^{3/2}}$

Use of Unit Quantities(N_u, Q_u, P_u)(Cont…)

$$N_u = \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}},$$

$$P_u = \frac{P_1}{{H_1}^{3/2}} = \frac{P_2}{{H_2}^{3/2}}$$

Hence, if the speed, discharge and power developed by a turbine under a head are known, then by using above relations the speed, discharge and power developed by the same turbine under a different head can be obtained easily

Problem:1

A turbine develops 9000 kW when running at 10 r.p.m. The head on the turbine is 30 m. If the head on the turbine is reduced to 18 m, determine the speed and power developed by the turbine.

Solution. Given :

Power developed,	$P_1 = 9000$ kW
Speed,	$N_1 = 100$ r.p.m.
Head,	$H_1 = 30$ m
Let for a head,	$H_2 = 18$ m
Speed	$= N_2$
Power	$= P_2$

Problem:1(Cont…)

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$$\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$N_2 = \frac{N_1\sqrt{H_2}}{\sqrt{H_1}} = \frac{100\sqrt{18}}{\sqrt{30}} = \frac{100 \times 4.2426}{5.4772} = \textbf{77.46 r.p.m. Ans.}$$

Also we have $$\frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$$

$$\therefore \quad P_2 = \frac{P_1 H_2^{3/2}}{H_1^{3/2}} = \frac{9000 \times 18^{3/2}}{30^{3/2}} = \frac{687307.78}{164.316} = \textbf{4182.84 kW. Ans.}$$

Problem:2

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A turbine is to operate under a head of 25 m at 200 r.p.m. The discharge is 9 cumec. If the efficiency is 90%, determine the performance of the turbine under a head of 20 metres.

Solution. Given :

Head on turbine,	$H_1 = 25$ m
Speed,	$N_1 = 200$ r.p.m.
Discharge,	$Q_1 = 9\ m^3/s$
Overall efficiency,	$\eta_o = 90\%$ or 0.90.

Performance of the turbine under a head, $H_2 = 20$ m, means to find the speed, discharge and power developed by the turbine when working under the head of 20 m.

Problem:2(Cont...)

Let for the head, $H_2 = 20$ m, Speed $= N_2$, discharge $= Q_2$ and power $= P_2$

Using the relation,
$$\eta_o = \frac{P}{\text{W.P.}} = \frac{p_1}{\dfrac{\rho \times g \times Q_1 \times H_1}{1000}}$$

$$P_1 = \frac{\eta_o \times \rho \times g \times Q_1 \times H_1}{1000} = \frac{0.90 \times 1000 \times 9.81 \times 9 \times 25}{1000} = 1986.5 \text{ kW}$$

Using equation
$$\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$\therefore$
$$N_2 = \frac{N_1\sqrt{H_2}}{\sqrt{H_1}} = 200 \times \frac{\sqrt{20}}{\sqrt{25}} = \mathbf{178.88 \text{ r.p.m. Ans.}}$$

Problem:2(Cont…)

Also $$\frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$$

∴ $$Q_2 = Q_1 \times \frac{\sqrt{H_2}}{\sqrt{H_1}} = 9.0 \times \sqrt{\frac{20}{25}} = \mathbf{8.05\ m^3/s.\ Ans.}$$

And $$\frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$$

∴ $$P_2 = \frac{P_1 H_2^{3/2}}{H_1^{3/2}} = P_1 \left(\frac{H_2}{H_1}\right)^{3/2} = 1986.5 \left(\frac{20}{25}\right)^{3/2} = \mathbf{1421.42\ kW.\ Ans.}$$

Summary

- The conditions of the turbine under unit head are such that the efficiency of the turbine remains unaffected
- Unit Speed($\boldsymbol{N_u}$) can be expressed as $\boldsymbol{N_u} = \frac{\boldsymbol{N}}{\sqrt{\boldsymbol{H}}}$
- Unit Discharge($\boldsymbol{Q_u}$) can be expressed as $Q_u = \frac{Q}{\sqrt{H}}$
- Unit Power($\boldsymbol{P_u}$) can be expressed as $P_u = \frac{P}{H^{3/2}}$

CHARACTERISTIC CURVES AND GOVERNING OF TURBINES

Presented By:
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Lecture Details:
Unit-IV (Performance of Hydraulic Turbines) , Characteristic Curves and Governing of Turbines, FM & HM /Mechanical, I -Semester.

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CHARACTERISTIC CURVES AND GOVERNING OF TURBINES

Contents

- Characteristic Curves of Hydraulic Turbines
- Main Characteristic Curves or Constant Head Curves
- Operating Characteristic Curves or Constant Speed Curves
- Constant Efficiency Curves or Muschel Curves or Iso-Efficiency Curves
- Governing of Turbines
- Governing of Pelton Turbine
- Summary

Characteristic Curves of Hydraulic Turbines

- With the help of these curves , the exact behaviour and performance of the turbine under different working conditions can be known
- These curves are plotted from the results of the tests performed on the turbine under different working conditions
- The important parameters which are varied during a test on a turbine are:

1. Speed(N) 2. Head(H) 3. Discharge(Q) 4. Power(P) 5. Overall efficiency(η_0) 6. Gate opening

Characteristic Curves of Hydraulic Turbines(Cont…)

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- The parameters speed(N), head(H) and discharge(Q) are independent parameters
- Out of the three independent parameters, (N,H,Q) one of the parameter is kept constant(say H) and the variation of the other four parameters w.r.t any one of the remaining two independent variables (say N and Q) are plotted and various curves are obtained
- These curves are called characteristic curves

Characteristic Curves of Hydraulic Turbines(Cont…)

- The following are the important characteristic curves of a turbine

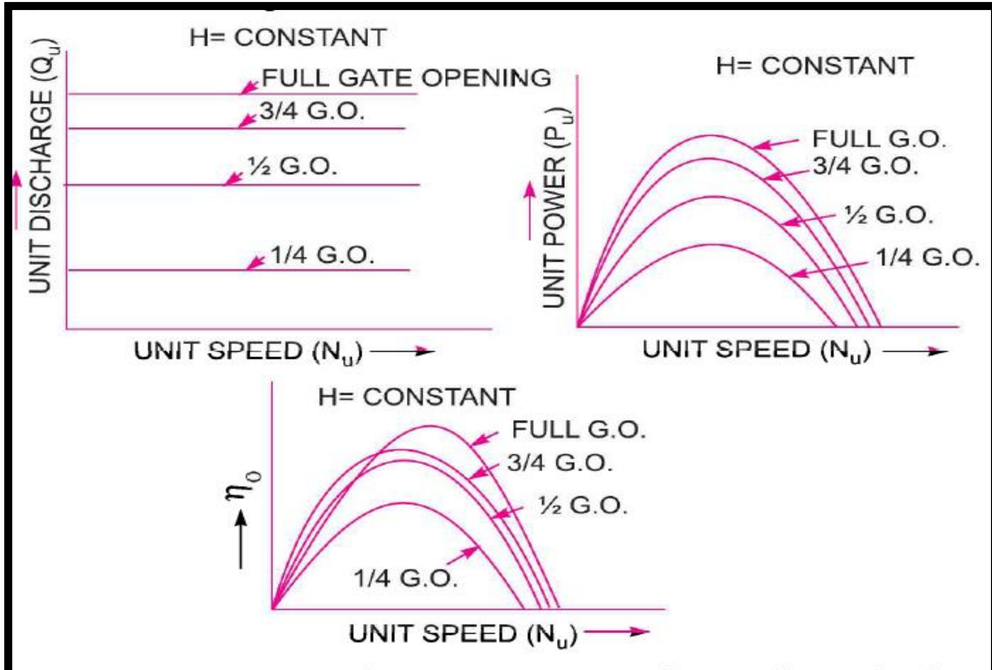
1. Main Characteristic curves or Constant Head Curves
2. Operating Characteristic Curves or Constant Speed Curves
3. Muschel Curves or Constant Efficiency Curves

Main Characteristic Curves or Constant Head Curves

- Main characteristic curves are obtained by maintaining a constant head and a constant gate opening(G.O) on the turbine
- The speed of the turbine is varied by changing load on the turbine
- For each value of the speed, the corresponding values of the power(P) and discharge (Q) are obtained
- Then the overall efficiency(η_0) for each value of the speed is calculated
- From these readings the values of unit speed(N_u), unit power(P_u) and unit discharge(Q_u) are determined

Main Characteristic Curves or Constant Head Curves(Cont…)

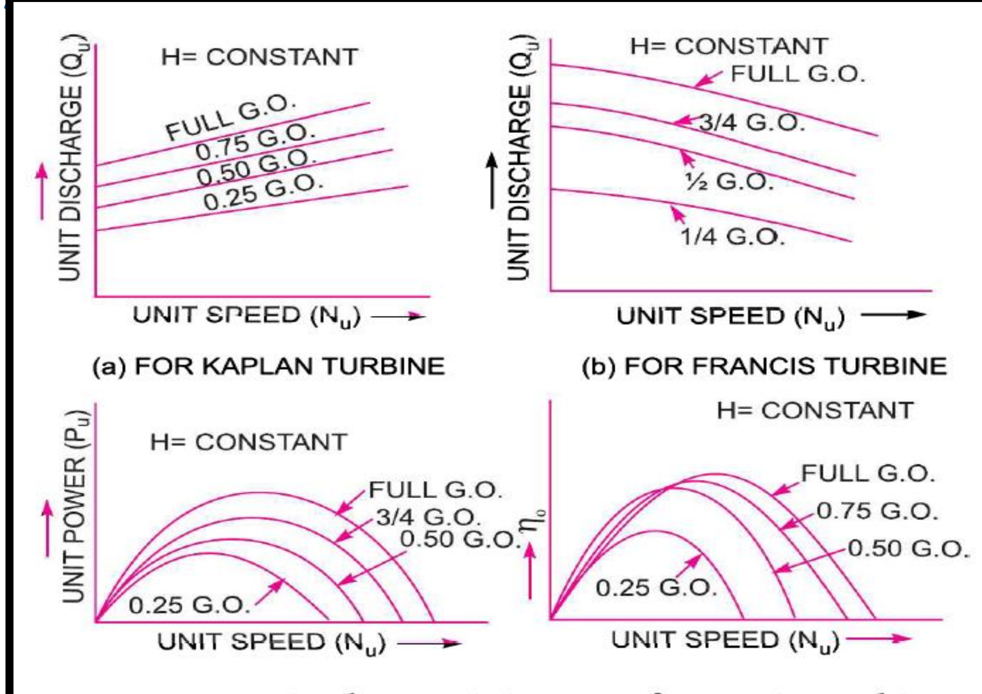
- Taking N_u as abscissa, the values of Q_u, P_u and η_0 are plotted as shown in fig



Main characteristic curves for a Pelton wheel.

Main Characteristic Curves or Constant Head Curves(Cont…)

- By changing the gate opening, the values of Q_u, P_u and η_0 and N_u are determined and taking N_u as abscissa, the values of Q_u, P_u and η_0 are plotted



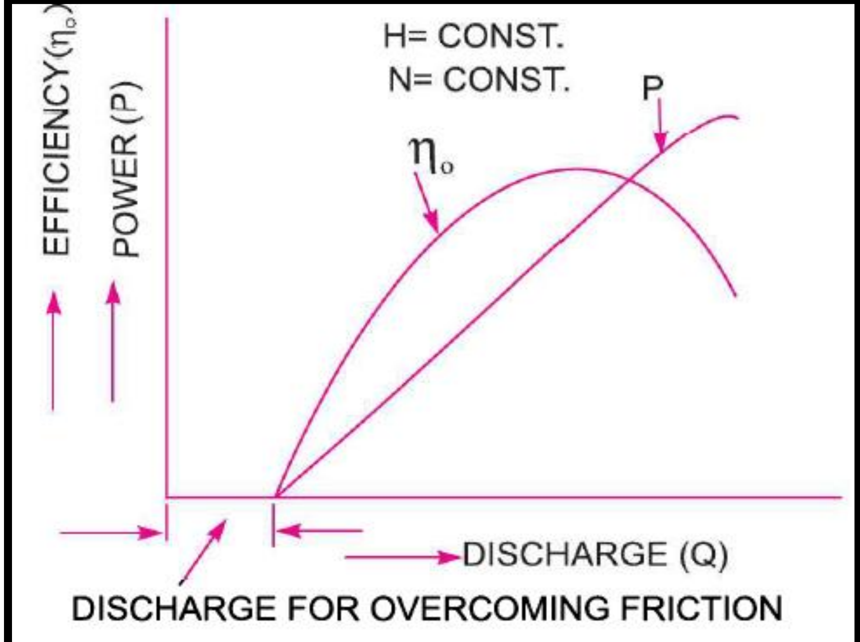
Main characteristic curves for reaction turbine.

Operating Characteristic Curves or Constant Speed Curves

- Plotted when the speed on the turbine is constant
- In case of turbines, the head is generally constant
- For operating characteristics N and H are constant and hence the variation of power and efficiency w.r.t discharge Q are plotted
- The power curve for turbines shall not pass through the origin because certain amount of discharge is needed to produce power to overcome initial friction

Operating Characteristic Curves or Constant Speed Curves(Cont...)

- Hence the power and efficiency curves will be slightly away from the origin on the x-axis, as to overcome initial friction certain amount of discharge will be required



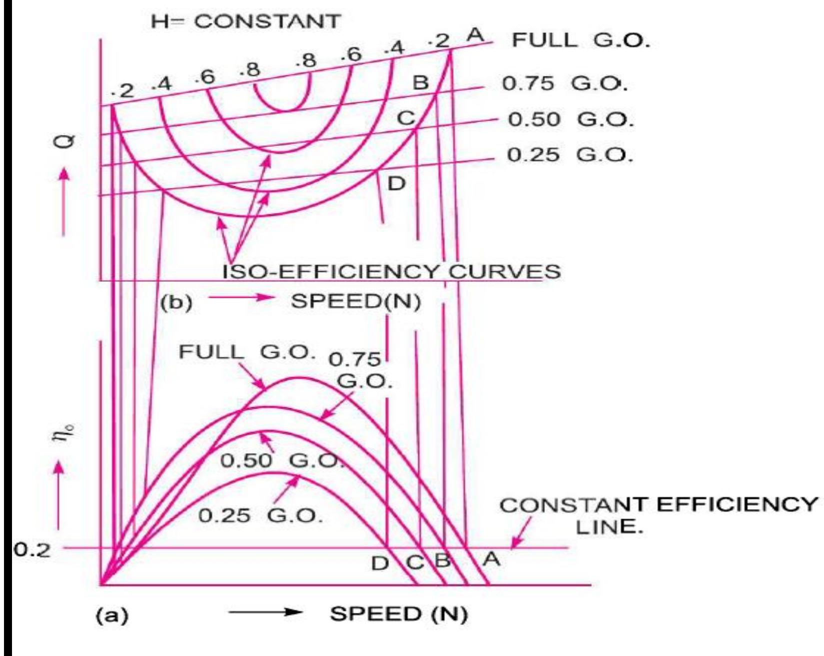
Operating characteristic curves.

Constant Efficiency Curves or Muschel Curves or Iso-Efficiency Curves

- These curves are obtained from the speed vs efficiency and speed vs discharge curves for different gate openings
- For a given efficiency from the N_u vs η_0 curves, there are two speeds
- From the N_u vs Q_u curves, corresponding to two values of speeds there are two values of discharge
- There are two values of discharge for a particular gate opening
- So, there are two values of speeds and two values of the discharge for a given gate opening

Constant Efficiency Curves or Muschel Curves or Iso-Efficiency Curves(Cont…)

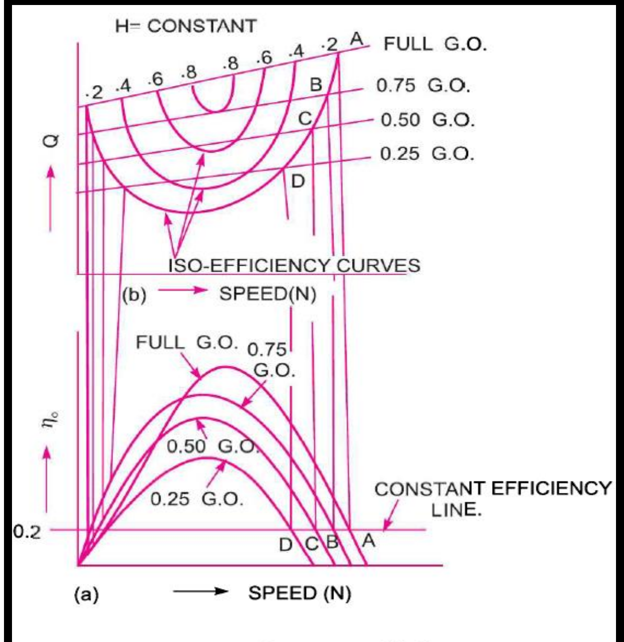
- If the efficiency is maximum there is only one value
- These two values of speed and two values of discharge corresponding to a particular gate opening are plotted
- The procedure is repeated for different gate openings and the curves Q vs N are plotted



Constant efficiency curve.

Constant Efficiency Curves or Muschel Curves or Iso-Efficiency Curves(Cont…)

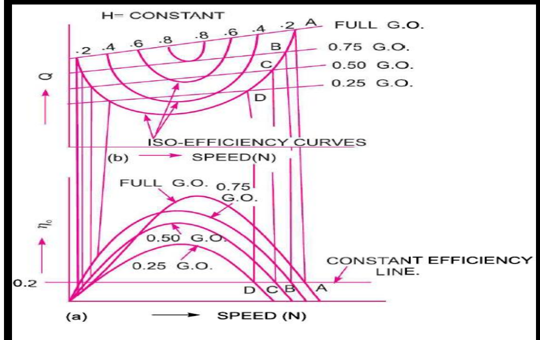
- The points having the same efficiency are joined
- The curves having same efficiency are called iso-efficiency curves
- These curves are helpful for determining the zone of constant efficiency and for predicating the performance of the turbine at various efficiencies



Constant efficiency curve.

Constant Efficiency Curves or Muschel Curves or Iso-Efficiency Curves(Cont…)

- For plotting the iso-efficiency curves, horizontal lines representing the same efficiency are drawn on the η_0 vs speed curves
- The points at which these lines cut the efficiency curves at various gate openings are transferred to the corresponding Q vs speed curves
- The points having the same efficiency are then joined by a smooth curves
- These smooth curves represents the iso-efficiency curve



Constant efficiency curve.

Governing of Turbines

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- Governing of a turbine is defined as the operating by which the speed of the turbine is kept constant under all conditions of working
- It is done automatically by means of a governor, which regulates the rate of flow through the turbines according to the changing load conditions on the turbine
- The speed of the generator will be constant , when the speed of the turbine(which is coupled to the generator) is constant

Governing of Turbines(Cont…)

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- When the load on the generator decreases, the speed of the generator increases beyond the normal speed(constant speed)
- Then the speed of the turbine also increases beyond the normal speed
- If the turbine or the generator is to run at constant(normal) speed, the rate of water to the turbine should be decreased till the speed becomes normal
- This process by which the speed of the turbine is kept constant under varying condition of load is called governing

Governing of Pelton Turbine(Impulse Turbine)

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- Governing of pelton turbine is done by means of oil pressure governor, which consists of the following parts:

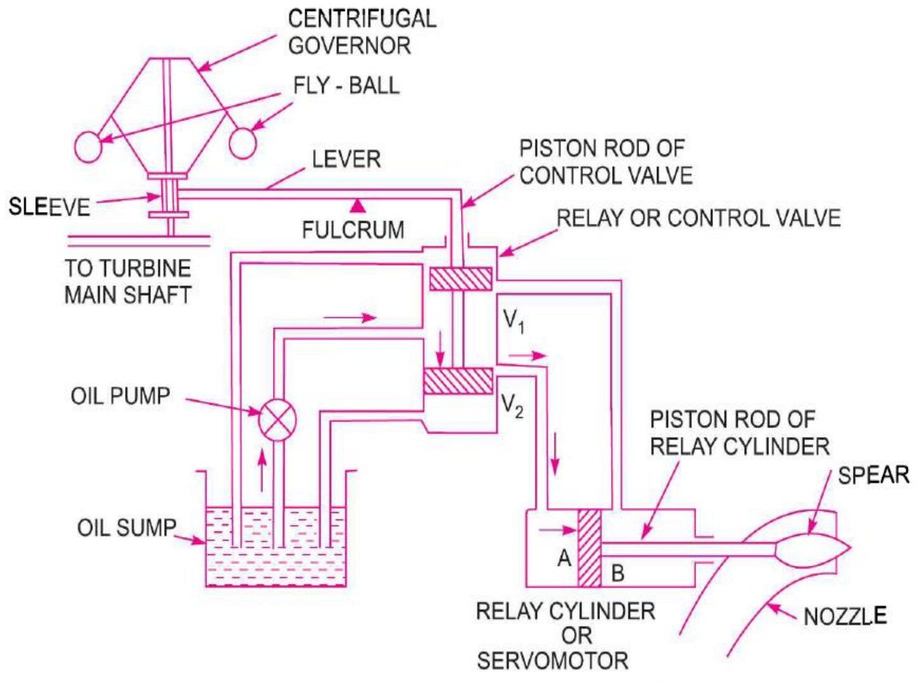
1. Oil sump
2. Gear pump also called oil pump, which is driven by the power obtained from turbine shaft
3. The Servomotor also called the relay cylinder
4. The control valve or the distribution valve or relay valve

Governing of Pelton Turbine(Impulse Turbine)(Cont…)

5. The centrifugal governor or pendulum which is driven by belt or gear from the turbine shaft
6. Pipes connecting the oil sump with the control valve and control valve with servomotor
7. The spear rod or needle

Governing of Pelton Turbine(Impulse Turbine)(Cont…)

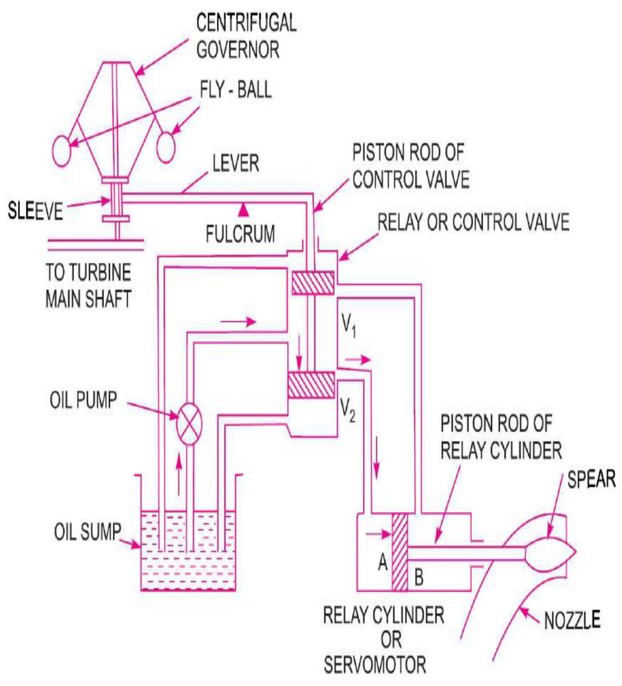
- When the load on the generator decreases, the speed of the generator increases
- The centrifugal governor, which is connected to the turbine main shaft, will be rotating at an increased speed



Governing of Pelton turbine.

Governing of Pelton Turbine(Impulse Turbine)(Cont...)

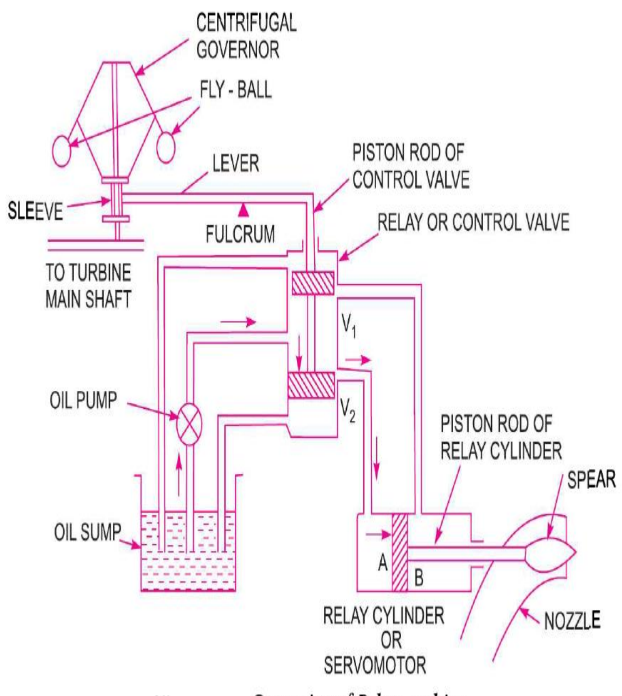
- Due to increase in the speed of the centrifugal governor, the fly balls move upward due to the increased centrifugal force on them
- Due to the upward movement of the fly-balls, the sleeve will also move upward
- As the sleeve move up, the lever turns about the fulcrum and piston rod of the control valve moves downward



Governing of Pelton turbine.

Governing of Pelton Turbine(Impulse Turbine)(Cont…)

- This closes the valve V_1 and opens the valve V_1 as shown in fig
- The piston along with piston rod and spear will move towards right
- This will decrease the area of flow of water at the outlet of nozzle, and reduce the rate of flow of water to the turbine which consequently reduce the speed of the turbine



Governing of Pelton turbine.

Governing of Pelton Turbine(Impulse Turbine)(Cont…)

- When the speed of the turbine becomes normal, the fly-balls, sleeve, lever and piston rod of control valve come to its normal position
- When the load on the generator increases, the speed of the generator and hence of the decreases
- The speed of the centrifugal governor also decreases and hence centrifugal force acting on the fly-balls also reduces
- This brings the fly-balls in the downward direction

Governing of Pelton Turbine(Impulse Turbine)(Cont…)

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- This closes the valve V_2 and opens the V_1
- Piston move along with the piston rod and spear towards left, increasing the area of flow of water at the outlet of the nozzle
- This will increase the rate of flow of water to the turbine and consequently, the speed of the turbine will also increase, till the speed of the turbine becomes normal

Summary

- Characteristic curves are plotted from the results of the tests performed on the turbine under different working conditions
- The following are the important characteristic curves of a turbine 1)Main Characteristic curves or Constant Head Curves 2)Operating Characteristic Curves or Constant Speed Curves 3)Muschel Curves or Constant Efficiency Curves
- Governing of a turbine is defined as the operating by which the speed of the turbine is kept constant under all conditions of working

TURBINE SELECTION AND CAVITATION PHENOMENON

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Lecture Details:
Unit-IV (Performance of Hydraulic Turbines) ,
Turbine selection and cavitation phenomenon, FM & HM /Mechanical, I -Semester.

Fluid Mechanics & Hydraulic Machinery

- FLUID STATICS
  - FLUID KINEMATICS
  - FLUID DYNAMICS
  - CLOSED CONDUIT FLOW
  - BOUNDARY LAYER THEORY AND APPLICATIONS
  - BASICS OF TURBO MACHINERY
  - HYDRAULIC TURBINES
  - PERFORMANCE OF HYDRAULIC TURBINES
  - CENTRIFUGAL PUMPS
  - RECIPROCATING PUMPS

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TURBINE SELECTION AND CAVITATION PHENOMENON

Contents

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Selection of Types of Turbine

Selection of a suitable type of turbine is usually governed by

i) Head and Specific Speed

ii) Part load Operation

Head and Specific Speed

- It has been found that there is a range of head and specific speed for which each type of a turbine is most suitable

S. No	Head in meters	Types of Turbine	Specific Speed
1	300 or more	Pelton wheel single or multiple Jet	8.5 to 47
2	150-300	Pelton or Francis	30 to 85
3	60-150	Francis	85 to 188
4	Less than 60	Kaplan or Propeller	188 to 860

Head and Specific Speed(Cont…)

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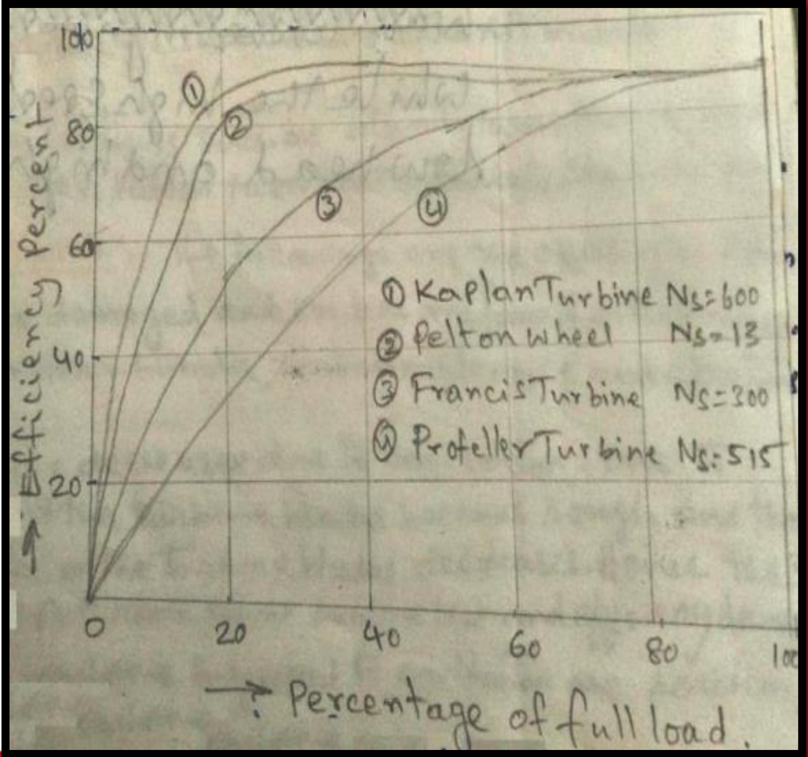
- A turbine with highest permissible specific speed should be chosen which will be cheapest and relatively small in size and high rotational speed will reduce the size of the generator as well as the power house
- But the specific speed cannot be increased indefinitely because it results is cavitations
- The cavitations may be avoided by installing the turbine at a lower level with respect to the tail race

Part Load Operation

- As the load deviates from the normal working load, the efficiency would also vary
- At part load the performance of Kaplan and pelton turbines is better in comparison to that of Francis and Propeller turbines
- The variability of load will influence the choice of type of turbine



Part Load Operation(Cont…)

- If the head lies between 150m to 300m or lies below 30m
- For higher range of heads pelton wheel is preferable for part load operation in comparison to Francis turbine
- For heads below 30m Kaplan turbine is preferable for part load operation in comparison to propeller turbine
- In addition to the above factors the overall cost, which includes the initial cost and running cost should be considered
- The cavitations characteristics of the turbine should be considered

Cavitation

- Cavitation is defined as the phenomenon of formation of vapour bubbles of a flowing liquid in a region, where the pressure of the liquid falls below its vapour pressure and the sudden collapsing of these vapour bubbles in a region of higher pressure
- When the vapour bubbles collapse, a very high pressure is created
- The metallic surfaces, above which these vapour bubbles collapse, is subjected to these high pressures, which cause pitting action on the surface

Cavitation(Cont…)

- Thus cavities are formed on the metallic surface and also considerable noise and vibrations are produced
- When the pressure of the flowing liquid is less than its vapour pressure, the liquid starts boiling and the vapour bubbles are formed
- These vapour bubbles are carried along with the flowing liquid to higher pressure zones, where these vapour condense and the bubbles collapse
- Due to sudden collapsing of the bubbles on the metallic surface, high pressure is produced and metallic surfaces are subjected to high local stress

Precaution against Cavitation

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- The pressure of the flowing liquid in any part of the hydraulic system should not be allowed to fall below its vapour pressure
- If the flowing liquid is water, then the absolute pressure head should not be below 2.5m of water
- The special materials or coatings such as Aluminum-bronze and stainless steel, which are cavitation resistant materials, should be used

Effects of Cavitation

- The metallic surfaces are damaged and cavities are formed on the surfaces
- Due to sudden collapse of vapour bubbles, considerable noise and vibrations are produced
- The efficiency of a turbine decreases due to cavitation
- Due to pitting action, the surface of the turbine blades becomes rough and the force exerted by the water on the turbine blades decreases
- Hence, the work done by water or output horse power becomes less and efficiency decreases

Hydraulic Machines Subjected to Cavitation

- Only reaction turbine and centrifugal pumps are subjected to cavitation
- In reaction turbines the cavitation may occur at the outlet of the runner or at the inlet of the draft tube where the pressure is considerably reduced
- Due to cavitation, the metal of the runner vanes and draft tube is gradually eaten away, which results in lowering the efficiency of the turbine
- In order to determine whether cavitation will occur in any portion of a reaction turbine, the critical value of Thoma‟s cavitation factors sigma(σ) is calculated

Hydraulic Machines Subjected to Cavitation(Cont...)

$$\sigma = \frac{H_b - H_s}{H} = \frac{(H_{atm} - H_v) - H_s}{H}$$

- Where H_b =Barometric pressure head in m of water,

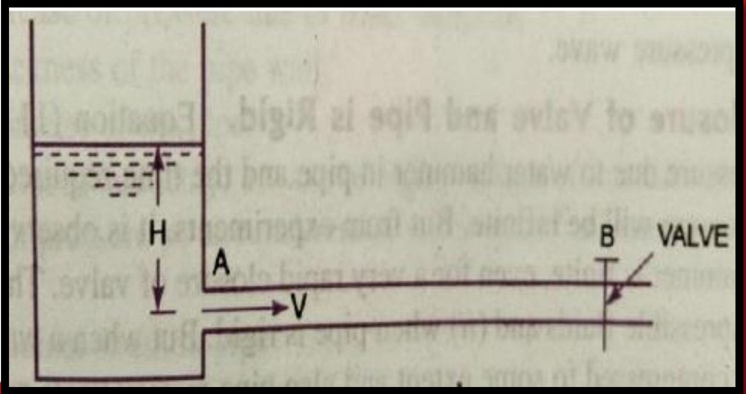
H_{atm} =Atmospheric pressure head in m of water,

H_v =Vapour pressure head in m of water,

H = Net head on the turbine in m

Water Hammer

- When the valve is completely open, the water is flowing with a velocity, V in the pipe
- If now the valve is suddenly closed, the momentum of the flowing water will be destroyed and consequently a wave of high pressure will be set up
- This wave of high pressure will be transmitted along the pipe with a velocity equal to the velocity of sound wave and may create noise called knocking



Water Hammer(Cont…)

- Also this wave of high pressure has the effect of hammering action on the walls of the pipe and hence it is known as water hammer
- The pressure rise due to water hammer depends up on:

Velocity of flow of water in pipe, The length of pipe, Time taken to close the valve, Elastic properties of the material of the pipe, Gradual closure of valve Sudden closure of valve considering pipe in rigid, Sudden closer of valve considering pipe elastic

Summary

- Cavitations may be avoided by installing the turbine at a lower level with respect to the tail race
- At part load the performance of Kaplan and pelton turbines is better in comparison to that of Francis and Propeller turbines
- In cavitation, formation of vapour bubbles of a flowing liquid in a region, where the pressure of the liquid falls below its vapour pressure and the sudden collapsing of these vapour bubbles in a region of higher pressure
- Only reaction turbine and centrifugal pumps are subjected to cavitation